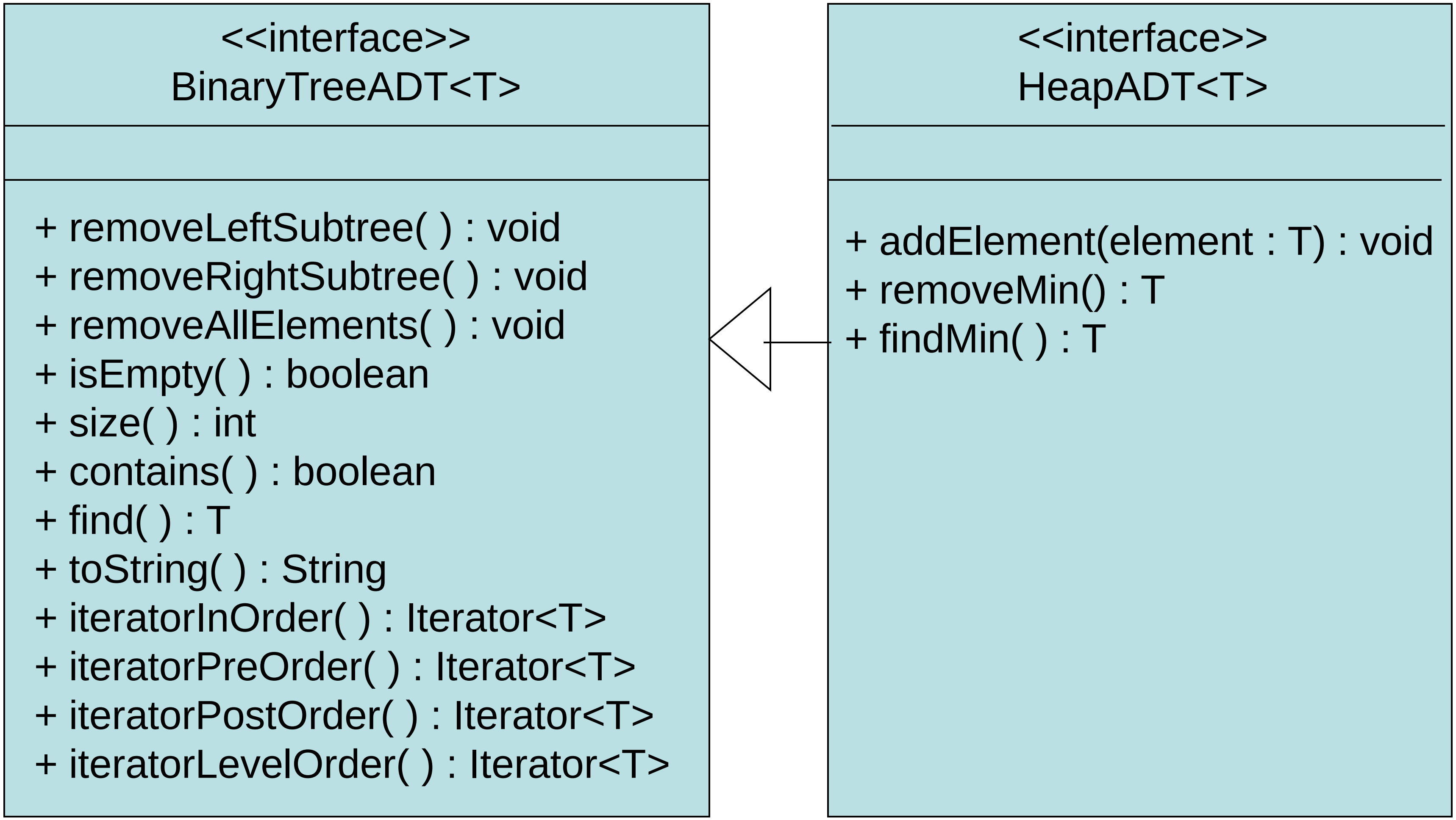
Heaps and Priority Queues

- Heap Definition and Operations
- Using Heaps
  - Heap sort
  - Priority queues
- Implementing heaps
  - With links
  - With arrays
  - Analysis of heap implementations
- Reading: L&C 12.1 – 12.5

Heap Definition

- A heap is a binary tree with added properties
  - It is a complete tree (as defined earlier)
  - For each node, the parent is less than or equal to both its left and its right child (a min heap) OR
  - For each node, the parent is greater than or equal to both its left and its right child (a max heap)
- For simplicity, we will study only a min heap
- Class of objects stored in a heap may implement Comparable interface or a Comparator may be used

Heap Operations



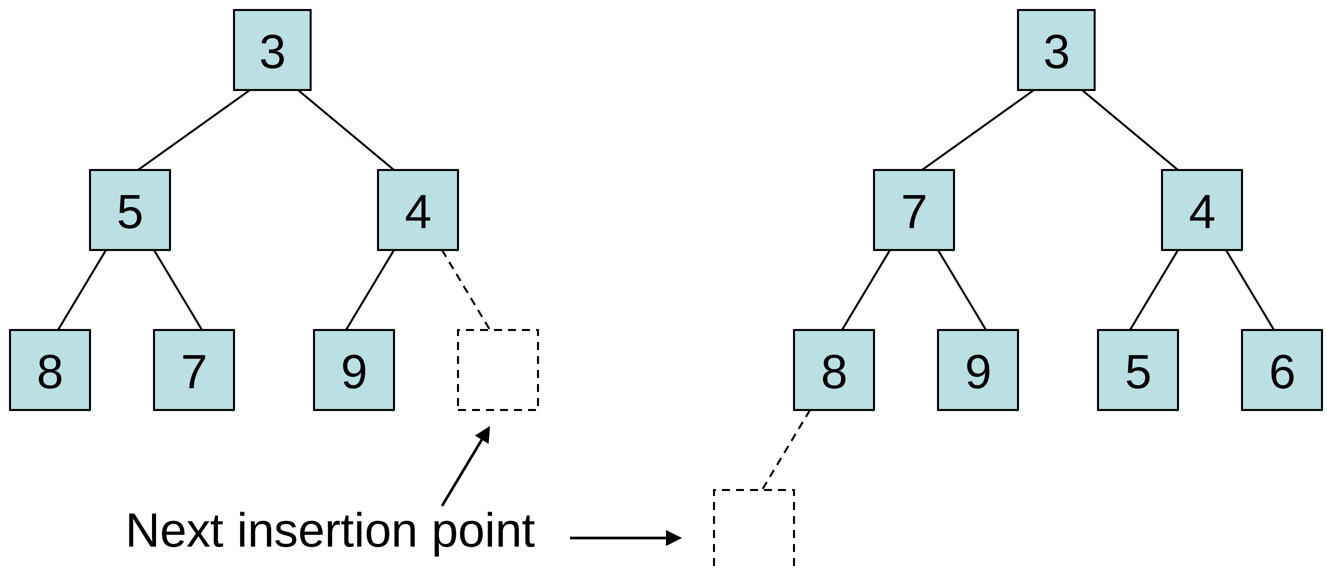
Note: toString is missing in L&C Fig 11.2

Heap Operations

- Notice that we are extending BinaryTreeADT – not BinarySearchTreeADT
- We don't use BinarySearchTree operations
- We define different heap operations to add and remove elements according to the new definition of the storage order for a heap
  - addElement O(log n)
  - RemoveMin O(log n)
  - findMin O(1)

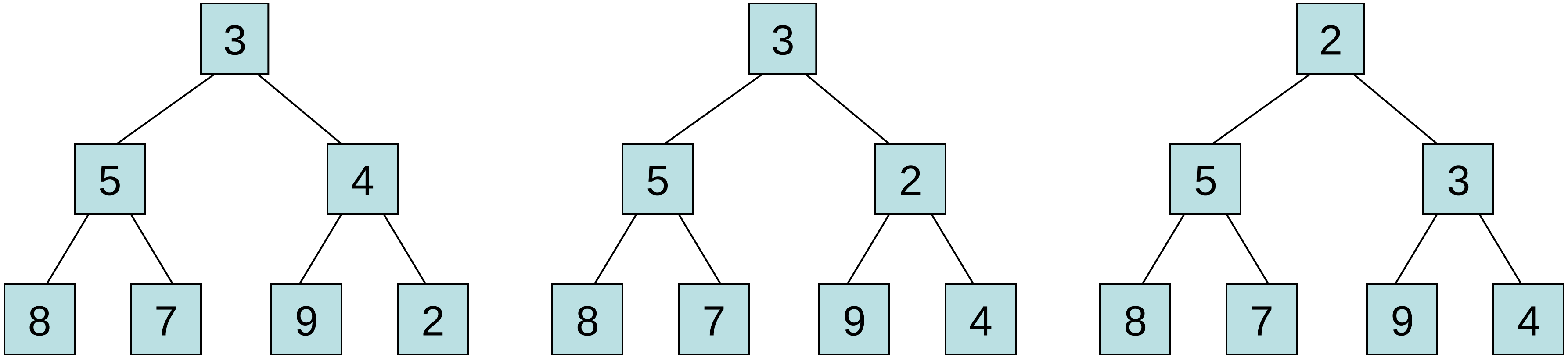
Heap Operations

- Semantics for addElement operation
  - Location of new node



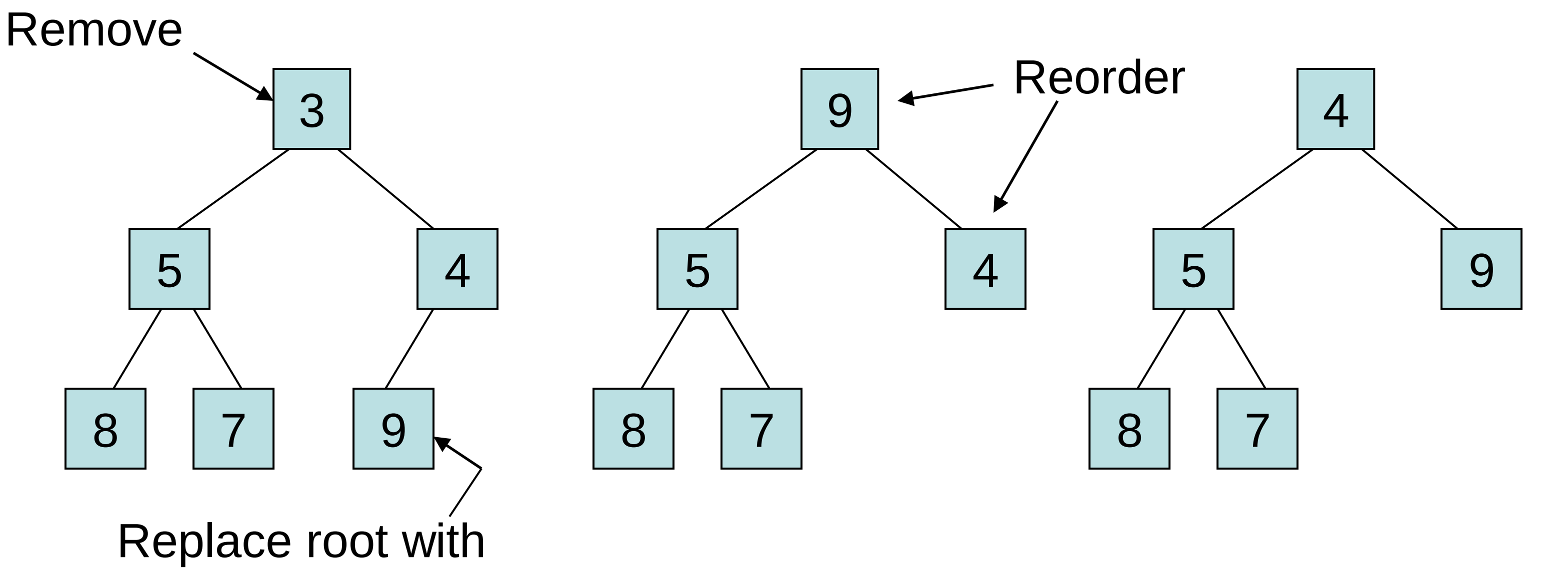
Heap Operations

- Semantics for addElement operation
  - Add 2 in the next available position
  - Reorder (“heapify on add”)
    - Compare new node with parent and swap if needed
    - Continue up the tree until at root if necessary



Heap Operations

- Semantics of removeMin operation
  - Remove root node
  - Replace root with last leaf
  - Reorder the heap (“heapify on remove”)

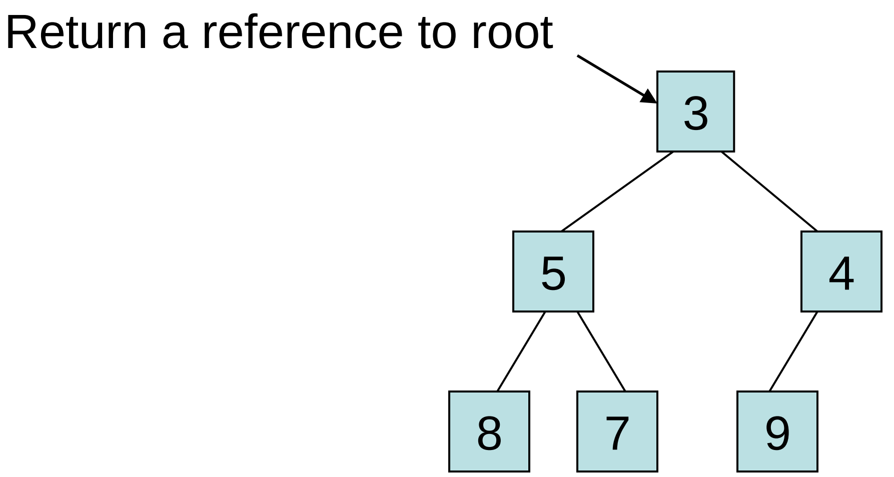


Heap Operations

- Heapify on Remove
  – Start at root
  – Find smallest child
  – If root greater than smallest child (min heap), swap parent with smallest child
  – Continue at child node until no swap needed

Heap Operations

- Semantics of findMin operation
  – The min element is always at the root element
  – Just return a reference to the root's element



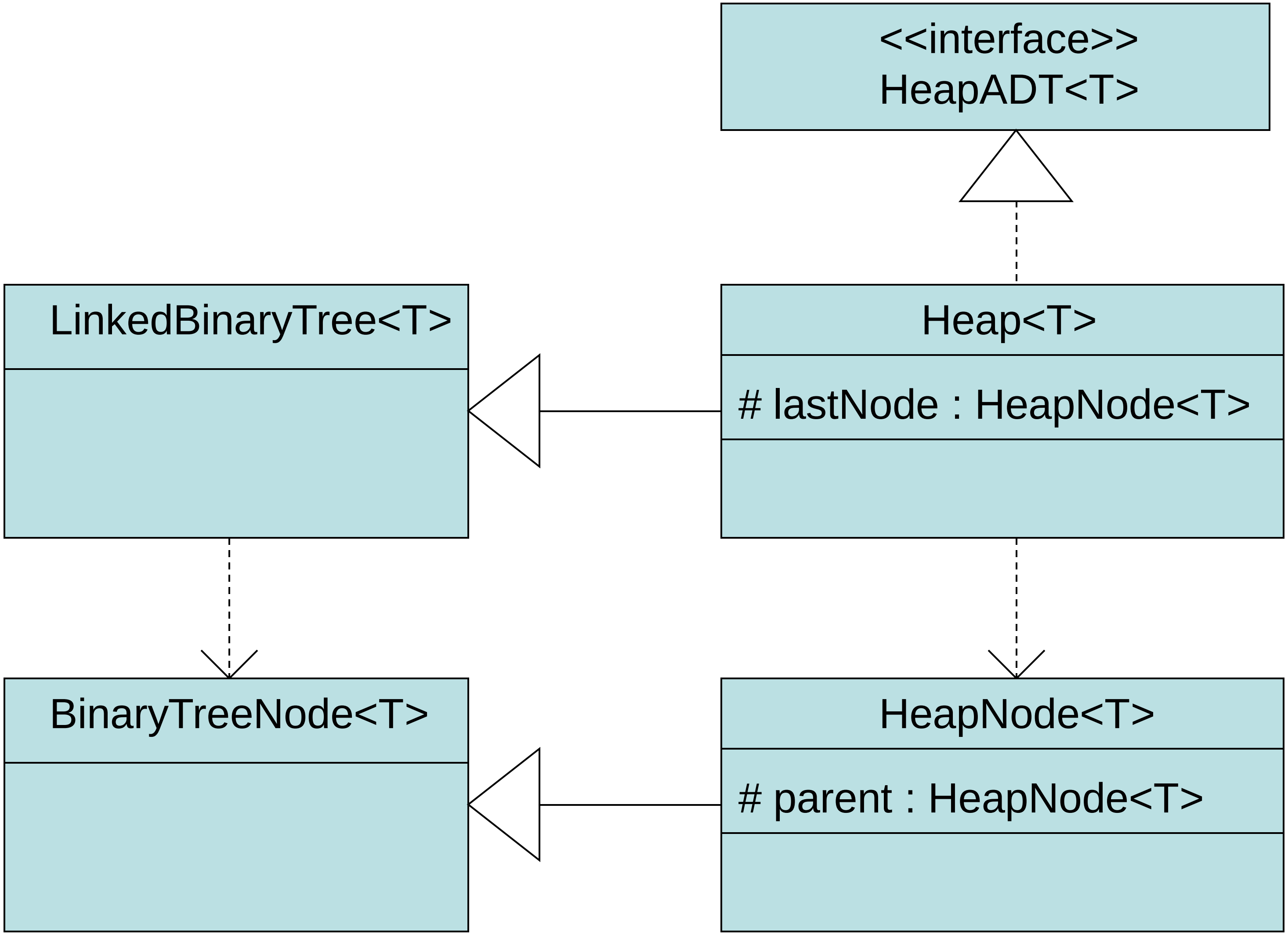
Using Heaps

- Heap Sort
  - Add each element to a heap
  - Remove them one at a time
- Sort Order
  - Min heap → ascending order
  - Max heap → descending order
- Performance
  - O(n log n)

Using Heaps

- Priority Queue Applications
  - Task scheduling in an operating system
  - Traffic scheduling on a network link
  - Job scheduling at a service center
  - Creating a Huffman coding tree
- Write the compareTo method to compare priorities which heap logic uses first
- If there is a tie on priority, heap logic uses order of entry (FIFO within priority)

Implementing Heaps with Links



Implementing Heaps with Links

- Class HeapNode<T>

```
public class HeapNode<T> extends BinaryTreeNode<T>
{
   protected HeapNode<T> parent;
   public HeapNode (T obj);
   {
      super(obj);
      parent = null;
   }
}
```

- Class Heap<T>

```
public class Heap<T> extends LinkedBinaryTree<T>
                           implements HeapADT<T>
{
   private HeapNode<T> lastNode;
```

Implementing Heaps with Links

- Heap<T> addElement operation

```
public void addElement (T obj)
{
  HeapNode<T> node = new HeapNode<T>(obj);

  if (root == null)
    root=node;
  else
  {
     HeapNode<T> next_parent = getNextParentAdd();
     if (next_parent.getLeft() == null;
       next_parent.setLeft(node);
     else
       next_parent.setRight(node);
     node.setParent(next_parent);
  }
```

Implementing Heaps with Links

- Heap<T> addElement operation

```
      lastNode = node;
      count++;
      if (count > 1)
           heapifyAdd();
   }
   private void heapifyAdd()        // a helper method
   {
      HeapNode<T> next = lastNode;
      while ((next != root) &&
            (((Comparable)next.getElement()).compareTo
             (next.getParent().getElement()) < 0))
      {
         T temp = next.getElement();
         next.setElement(next.getParent().getElement());
         next.getParent().setElement(temp);
         next = next.getParent();
      }
   }
```

Implementing Heaps with Links

- Heap<T> addElement operation

```
private HeapNode<T> getNextParentAdd() //a helper
{
  HeapNode<T> result = lastNode;
  // go up right branch as needed
  while (result != root &&
         result.getParent().getLeft() != result)
    result = result.getParent();

  // at top of a left branch or at root now
  if (result == root)
    // at root, go down left branch to lower left
    while (result.getLeft() != null)
      result = result.getLeft();
  else
```

Implementing Heaps with Links

- Heap<T> addElement operation

```
    { // need to go over the hump to a right branch
      if (result.getParent().getRight() == null)
        // parent is the next parent
        result = result.getparent();
      else {
        // parent is an ancestor of the next parent
        result = (HeapNode<T>)result.getParent().
                                            getRight();
        // go down its left branch to bottom level
        while result.getLeft() != null)
          result = (HeapNode<T>) result.getLeft();
      }
    }
    return result;
  }
```

Implementing Heaps with Links

- Heap<T> removeMin operation

```
public T removeMin() throws EmptyCollectionException
{
  if (isEmpty())
   throw new EmptyCollectionException("Empty heap");
  T minElement = root.getElement();
  if (count ==1)
  {
    root = null;
    lastNode = null;
  }
  else
```

Implementing Heaps with Links

- Heap<T> removeMin operation

```
    {
      HeapNode<T> next_last = getNewlastNode();
      if (lastNode.getParent().getLeft() == lastNode)
        lastNode.getparent().setLeft(null);
      else
        lastNode.getParent().setRight(null);
      root.setElement(lastNode.getElement());
      lastNode = next_last;
      heapifyRemove();
    }
    count--;
    return minElement;
  }
```

Implementing Heaps with Links

- Heap<T> removeMin operation

```
private HeapNode<T> getNewLastNode() // a helper
{
  HeapNode<T> result = lastNode;
  while (result != root &&
         result.getParent().getLeft() == result)
    result = result.getParent();
  if (result != root)
    result =
        (HeapNode<T>) result.getParent().getLeft();
  while (result.getRight() != null)
    result = (HeapNode<T>) result.getRight();
  return result;
}
```

Implementing Heaps with Links

- Heap<T> removeMin operation

```
private void heapifyRemove()  // a helper method
{
  if (root == null)
    return;
  HeapNode<T> node = null;
  HeapNode<T> next = (HeapNode<T>)root;
  do
  {
    if (node != null) {       // skip on first pass
      T temp = node.getElement();
      node.SetElement(next.getElement());
      next.setElement(temp);
    }
    node = next;
```

Implementing Heaps with Links

- Heap<T> removeMin operation

```
    HeapNode<T> left = (HeapNode<T>)node.getLeft();
    HeapNode<T> right = (HeapNode<T>)node.getRight();

    if ((left == null) && (right == null))
       next = null;
    else if (left == null)
       next = right;
    else if (right == null)
       next = left;
    else if (((Comparable)left.getElement()).
                compareTo(right.getElement()) < 0)
       next = left;
    else
       next = right;
    } while (next != null &&
             ((Comparable)next.getElement()).
                compareTo(node.getElement()) < 0);
  }
```