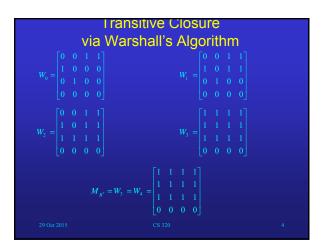
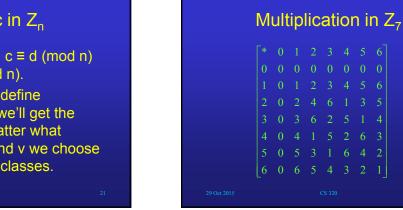


## Proof: We'll use induction. Base case: k=0. W₀ = W<sub>R</sub> because there can be no interior vertices, so just a single edge. Induction step: If true for k-1, show w<sub>ij</sub>[k] = w<sub>ij</sub>[k-1] ∨ (w<sub>ik</sub>[k-1] ∧ w<sub>kj</sub>[k-1]) because there is a path from v<sub>i</sub> to v<sub>1</sub> using interior vertices from {v<sub>1</sub>, v<sub>2</sub>,...,v<sub>k</sub>} iff There is a path without v<sub>k</sub> as an interior vertex (so w<sub>ij</sub><sup>[k-1]</sup> = 1) or There is path with v<sub>k</sub> as an interior vertex, in which case both w<sub>ik</sub><sup>[k-1]</sup> and w<sub>kj</sub><sup>[k-1]</sup> are 1. (there must be a k-1path from v<sub>i</sub> to v<sub>k</sub> and from v<sub>k</sub> to v<sub>j</sub>)

## Using Warshall's Algorithm As shown in the book, the formula giving Warshall's Algorithm easily translates to computer code. If you do it by hand, just note that in $w_{ji}^{[K]} = w_{ji}^{[K-1]} \cdot (w_{jk}^{[K-1]} \cdot w_{kj}^{[K-1]})$ you go from $W_{k-1}$ by by looking at the matrix for $W_{k-1}$ . If you can go from $v_i$ to $v_k$ in $W_{k-1}$ then in $W_k$ by looking at the matrix for $W_{k-1}$ . If you can go from $v_i$ to $v_k$ in $W_{k-1}$ then in $W_k$ you can add an entry if $v_k$ goes to $v_j$ in $W_{k-1}$ . (this is easier than it sounds)





## Arithmetic in Z<sub>n</sub>

- If a ≡ b (mod n) and c ≡ d (mod n) then a\*c ≡ b\*d (mod n).
- This shows we can define

   [u] \* [v] = [u\*v] and we'll get the same answer no matter what representatives u and v we choose for the equivalence classes.