Summations

What does $\sum_{j=m}^{n} a_j$ stand for?

It represents the sum $a_m + a_{m+1} + a_{m+2} + \ldots + a_n$.

The variable j is called the ***index of summation***, running from its ***lower limit*** m to its ***upper limit*** n. We could as well have used any other letter to denote this index.

Summations

How can we express the sum of the first 1000 terms of the sequence $\{a_n\}$ with $a_n=n^2$ for n = 1, 2, 3, … ?

We write it as $\sum_{j=1}^{1000} j^2$.

What is the value of $\sum_{j=1}^{6} j$?

It is 1 + 2 + 3 + 4 + 5 + 6 = 21.

What is the value of $\sum_{j=1}^{100} j$?

It is so much work to calculate this…

Summations

It is said that Friedrich Gauss came up with the following formula:

$$\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$$

When you have such a formula, the result of any summation can be calculated much more easily, for example:

$$\sum_{j=1}^{100} j = \frac{100(100+1)}{2} = \frac{10100}{2} = 5050$$

Double Summations

Corresponding to nested loops in C or Java, there is also double (or triple etc.) summation:

Example:

$$\sum_{i=1}^{5}\sum_{j=1}^{2} ij$$

$$= \sum_{i=1}^{5} (i + 2i)$$

$$= \sum_{i=1}^{5} 3i$$

$$= 3 + 6 + 9 + 12 + 15 = 45$$