# Modeling juror decay in a high profile criminal trial

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## Introduction

On April 6, 1995, the *Boston Globe* reported on the four month old first OJ Simpson trial that

> . . . most legal analysts agreed that the danger of a mistrial due to juror attrition is growing exponentially . . .

The jurors brought it a verdict in early October, 10 months after they were selected and sequestered. But on April 6 there was no way to know what would happen. Concern about a mistrial was reasonable. How reasonable? What might the *Globe's* assertion mean? For a legal analyst or a reporter "exponentially" is probably just a synonym for "frighteningly fast" or "out of control", a cliche — used the way "quantum" is to describe many a leap or a jump. But although no real mathematics was implied by the newspaper report, we took it as an invitation to supply some, and found that a little quantitative thinking could carry us long way. Before we start that journey, though, we must acknowledge the the overarching sadness of the event — two people dead, two children half orphaned. No mathematics can soothe those losses.

If "the danger of a mistrial" means "the probability of a mistrial" it can't grow exponentially for long, since probabilities can never exceed 100%. But there may have been reason to worry about how high that probability was at various times during the trial, which would explain why analysts looked for dramatic language to express their opinions. In order to think dispassionately and quantitatively about the question we will construct a mathematical model, analyze it, compare it with observations, and, finally temper our conclusions by discussing the extent to which the model represents reality accurately.

Our model is that of radioactive decay. Each juror is like an atom, with a certain probability of disappearing on any given day. The probability is the same for each item (juror or atom), is the same from day to day, and what happens to one item has no effect on what happens to the others. Physicists trust these assumptions when the items are atoms of Radium, but they may not apply to jurors; we will return to this point later.

An estimate based on this model shows that on April 6 it would have been reasonable to predict that the jury pool would have just 12 members remaining just about when the trial actually ended. In fact there were 14 on that date.

To work out the probability on April 6 (or any other date) of an eventual mistrial we use use the pattern of juror dismisals up until that date to estimate the likelihood of any given juror's decaying in any fixed period of time. With that likelihood and an estimate of the length of the trial we can compute the desired mistrial probability. We conclude from our model that on April 6 the probability of a mistrial for a 10 month trial was 44%. For a hypothetical one year trial that probability was 73%.

## Half life calculations on the back of an envelope

The trial began with a pool of 24 potential jurors: twice the necessary number. The *Globe* article appeared when the pool fell to 18. At that time the trial was about 4 months old. Those numbers are particularly convenient for a back of an envelope computation. We wish to compute the half life of the juror pool: the time it takes for the number of jurors to drop from 24 to 12. Half life for radioactive atoms is analogous to doubling time for investments earning compound interest. The bankers' "Rule of 70" says that the doubling time for money invested at $r\%$ annual interest regularly compounded is $70/r$ years. This rough rule also gives the time to halve your money if you are losing $r\%$ annually.

Thus in our model the half life of the jury pool is $70/r$ months, where $r$ is the monthly juror decay rate — the analogue of the interest rate. But we don't know $r$. We can bypass the need for it. The rule of 30 says an investment earning $r\%$ interest annually grows by a factor of $4/3$ ( a 33% increase) in $30/r$ years.

There is a "rule of 30" for $3/4$ life, and we know that the $3/4$ life of the jury pool is 4 months.

Combining the rules of 70 and 30 shows that the the ratio of half to $3/4$ life is $7/3$. Thus when the *Globe* article appeared we estimated that the jury would have just 12 people left toward the end of September, when the trial was $(7/3) \times 4 = 9+$ months old.

Finally, we can use this now known half life in the rule of 70 to estimate the decay rate $r$. Since $70/r \approx (7/3) \times 4$, the jurors were disappearing at the rate $r \approx 30/4 = 7.5\%/\text{month}$ on April 6.

Anyone can use these rules without knowing why they are true. But it's not hard to prove them with just a little precalculus algebra. To derive the rule of 70, solve the equation $e^{rt} = 2$ for $t$ in terms of $r$ and notice that $\ln(2) = 0.6931 \approx 0.7$. Expressing $r$ as a percent instead of a fraction moves the decimal point two places, converting 0.7 to 70. The rule of 30 follows from the fact that $\ln(4/3) \approx 0.3$.

In *Money Angles* Andrew Tobias writes [5]

> All you really need to know — or a good start, anyway, — is the rule of 72. It says: To determine approximately how fast an investment will double, divide the interest rate into 72. ...Why does this work? *No one knows.* It works because it's a *rule.*

The emphasis is Tobias'. He then adds a footnote:

> Actually, it has something to do with the natural logarithm of 2, but I skipped that class.

As we have seen, it has everything to do with the fact that $100 \times \ln(2) \approx 70$. Dividing the interest rate (expressed as a %) into $(100 \times \ln(2))$ gives the doubling time exactly, when the quoted rate is the effective rate for continuous compounding. Why 72 instead of 70? Probably because it has more integral divisors, making quick calculations easier at the cost of a mere

3% in accuracy. And using 72 instead of 70 tends to correct for the fact that the real interest is not continuously compounded.

It was not always fashionable to brag about skipping math class. In Robert Heinlein's *Time for the Stars* you will find the dialogue [3]:

> "Point six nine three one."
> "That's the natural logarithm of two."
> "What did you think it was? Mabel's telephone number?"

implying that anyone would know ln(2) without consulting a calculator (or a table or slide rule when the book was written).

## The probability of a mistrial

So far this is what Lewis Carroll called a "pillow problem": one to be solved "in the head, while lying awake at night."[2] But we want to know more. Half life calculations are good for large samples where precise counts are unimportant: we weigh Radium rather than counting atoms. In this problem we are dealing with small numbers whose exact values matter. We want to compute the probability of a mistrial on any given day of the trial, and how that probability and our perception of it change over time.

We start by estimating the unknown juror decay rate by using the history of the trial. The *Globe* tells us that jurors were dismissed on January 19 (two at once), February 7, March 2, March 18, April 5 (the day before the article that prompted this note), May 1, May 25 and June 5 (two at once). (On April 24 alternate juror Betty (Boopsie) Boopstein in Garry Trudeau's *Doonesbury* was booted. We decided not to incorporate this information in our calculations.) The jury was seated on November 3, the alternates on December 8. In our calculations we use the latter date for the start of the trial.

The following theorem tells how to calculate the most likely value of $r$ given any particular history of juror decay.

*Theorem:* At any time $T$ during the course of the trial the *maximum likelihood estimator* $\hat{r}$ for $r$ is given by the formula

$$\hat{r} = \frac{\text{number of jurors disqualified so far}}{\text{total juror months accumulated}}.$$

The proof of the theorem, (discussed in the sidebar) depends on understanding a little calculus and some of the mathematics of the process that models decay. But even without a proof we can see that the theorem is plausible. In the simplest case in which there is just one juror who lasts $m$ months, $\hat{r} = 1/m$, one juror decay per juror per $m$ months, which makes sense.

The theorem yields the value $\hat{r} = 7.05\%$ per month on April 6 — close to the quick estimate of 7.5% we made on the back of our envelope. The numbers differ because the quick estimate depends only on the total number of juror decays, not on the more detailed information specifying when the decays took place.

The value calculated at the end of the 10 month trial was 5.24% per month.

The step function in Figure 1 shows the observed jury size as a function of time. We have superimposed two exponential decay curves approximating that function, one (solid line) using the April 6 value of $\hat{r}$, the other (dashed line) using the value computed at the end of the trial.

With an approximating exponential decay curve we can estimate the time at which jury size will fall to 12. But we want to know something a little more subtle: for any particular future time $T$, what is the probability that the jury will have fewer than 12 members at that time?

To make that calculation we use the single most important consequence of our decay model: the probability that any individual juror survives for t months starting at any fixed time is $e^{-rt}$. If we let $a$ be the age of the trial and $T$ its expected length then the probability that any particular juror survives to the end of the trial is then $p = e^{-r(T-a)}$ and the probability that he or she is disqualified is $1 - p$.

Suppose there are $J$ jurors left at time $a$. The trial will end normally (no mistrial) provided that the number $k$ of jurors disqualified in the next $T - a$ months is at most $J - 12$. Since in our model what happens to one juror has nothing to do with what happens to any other jurors, the juror decay events are independent and we can multiply their probabilities. So for each set of $k$ jurors ($k = 0, \ldots, J - 12$) the probability that those $k$ jurors decay and the other $J - k$ survive is

$$(1-p)^k p^{J-k}$$

Then we sum over the possible sets of disqualified jurors to get the probability of a successful conclusion. The mistrial probability is the complement:

$$\text{Mistrial probability} = 1 - \sum_{k=0}^{J-12} \binom{J}{k} (1-p)^k p^{J-k}$$

The solid line in Figure 2 shows how the probability that a 10 month trial would end with a mistrial varied during the course of the trial, assuming that the value of $r$ in effect for the whole trial was the maximum likelihood estimator $\hat{r}$ computed at the end of the trial, when the most data was available.

This graph shows that during any interval of time in which no juror decays the best estimate of the probability of a mistrial decreases. That is because the time until the end of the trial is decreasing so jurors have less time to decay. When a juror decays the probability of a mistrial jumps suddenly (not a quantum jump) and then begins to decrease again. The graph shows that the probability of a mistrial oscillated between roughly 10% and 25% for most of the trial, with a large spike to almost 60% on June 5, followed by subsequent decay to 0.

Our analysis so far has been based on the complete trial, which in retrospect took 10 months. But when we were making these computations during the trial, we could not know how long the trial would last, and of course could not know the final best estimate of $r$.

We can use the same combinatorics in a different analysis that more accurately reflects the changing *perceptions* of the probability of a mistrial. We need only recalculate $\hat{r}$ each day using the trial history up to that day before computing the mistrial probability. The result is the dotted graph in Figure 2. The oscillations are wider, and their peaks seem to be growing through June. In intervals between disqualifications the probability of a mistrial decreases faster than it did in the previous analysis, because now $\hat{r}$ decreases in those intervals.

This second analysis more accurately captures the uncertainties in the trial. An intuitive sense of the peaks in this graph probably triggered pundits comments on days just after jurors had been disqualified. For example, on May 26 the *Globe* quoted Professor Laurie Levenson of Loyola Law School as saying "We're losing too many jurors too fast".

It is interesting to see how the estimate of $r$ changed as the trial progressed. Figure 3 shows that data. On the average, the disqualification rate was increasing slowly through the first 6 months of the trial from about 5 to 8% per month. The long jump at the first juror decay is a consequence of the fact that before any jurors have been disqualified the best estimate for $r$ is 0. Two simultaneous decays caused the long jump on June 5.

## Conclusions and open questions

So, contrary to the legal analysts' opinions, the probability of a mistrial was never "growing exponentially" during the trial. Whether or not the probability of a mistrial was unacceptably high is another matter. A cost benefit analysis for this multimillion dollar enterprise is beyond the scope of this paper.

How well is juror decay modeled by radioactive decay? Not perfectly, because jurors are not (like atoms) subject to decay at random times for no particular reasons. Unlike atoms, they have histories and hidden variables. Moreover, the assumption that what happens to one juror is independent of what happens to others may be questionable. Some later disqualifications seem to have resulted from juror dissatisfaction at least as much as from new information about juror history. And juror dissatisfaction may well be contagious. That would effectively increase $r$. This is in fact what happens in a chain reaction in a nuclear reactor or explosion. The decay of one atom *causes* others to decay. But the model is a reasonable first approximation. It is the best we can construct with the limited data we have, and it is simple enough to calculate with. (But applied mathematicians must be mindful of a *caveat* attributed to H. L. Mencken: "Every complex problem has a solution that is simple, neat and wrong.")

In our analysis we treat jurors and alternates identically. The decay of either influences the probability of a mistrial in the same way. In fact in some jurisdictions the pool is not divided into jurors and alternates until deliberations begin. At that time 12 jurors are chosen at random and the remainder of the pool is dismissed (or kept on as alternates while the selected jurors deliberate).

Our model treats the decay rate $r$ as a constant we try to estimate with $\hat{r}$. But is $r$ constant? The data suggest that it may have been increasing for the first half of the trial — a cause for concern then. We note that the April 6 *Globe* suggested the opposite. They argued first that, as time passed it would become less likely that new information about a juror would come to light — whatever the ongoing investigations had not found was probably not there to be found. Second, they hinted that the judge might increase the threshold for juror misbehavior warranting dismissal precisely in order to avoid a mistrial. One could argue that the data from June on support this view.

If either the prosecution or the defense were actively seeking a mistrial they might intensify their search for evidence to disqualify jurors. That would cause $r$ to increase. The article suggested that each side wanted to play the hand it had. In that case both might agree to continue with a jury with fewer than 12 members should that be necessary, thus decreasing the probability of a mistrial. *The New Yorker* for June 19 shows the trial judge addressing the "Lady and gentleman of the jury". We doubt that the trial could have continued with just two jurors. [DON: PETER SAYS TRY TO GET NEW YORKER PERMISSION TO REPRINT THIS CARTOON, OR CONSIDER DROPPING THE REFERENCE. WE'D LIKE TO KEEP IT.]

We think of the length of the trial as a parameter in our model, and draw numerical conclusions that depend on its value. We should probably treat it as a random variable with a probability distribution of its own.

It would be interesting to gather statistics and repeat our calculations for other trials in which juror attrition was significant. Perhaps our methods can be helpful in deciding how many alternates to seat in future controversial trials. We might think about calculating confidence intervals for the quantities we calculate. How good do we think our April 6 estimate of $\hat{r} = 7.05\%$ per month and the 44% probability of a mistrial in a 10 month trial really were?

These are serious questions in the statistics of risk analysis: the ambitious reader can consult [1], [4] Dealing with them is well beyond the scope of this paper, which is meant only to show how variations on the "exponential" theme can enlighten us about what we may find in the daily paper.

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## Sidebar

Where does the formula for the maximum likelihood estimate $\hat{r}$ come from? The word "maximum" suggests that we may be able to derive it by setting up an appropriate function and differentiating to find the critical point.

In our model juror decay is a Poisson process with (unknown) rate $r$. We want to find the value of $r$ that maximizes the probability that we *would* observe just the sequence of decays that we *did* observe.

We have already noted that when there are $J$ jurors then the probability that no juror decays in the next $t$ months is is $e^{-Jrt}$. The probability that the first decay occurs between times $t$ and $t + dt$ is then the differential $-Jrte^{-Jrt}dt$.

Now it takes a little notation to write down the function we need. Suppose we started with $N$ jurors and that $n$ dismissals have occurred, separated by time intervals $t_0, \ldots, t_{n-1}$; let $t_n$ be the time from the last dismissal until the time $T$ at which we are estimating $r$. Thus in our case $t_0$ is the time interval from December 8 to January 19 (the first dismissal), $t_1 = 0$ since the second dismissal coincided with the first, $t_3$ is the time from January 19 to February 7, and so on. Let $p_i(r)dt$ be the probability that the $i^{th}$ time interval behaved as it did. Then

$$p_i(r) = r(N-i)e^{r(N-i)t_i}.$$

Since what happens in each interval is independent of what happened in the others, we can multiply those probabilities to find the probability $P(r)$ of observing what we did observe:

$$P(r) = p_1(r)dt \times p_2(r)dt \times \cdots \times p_{n-1}(r)dt \times e^{-r(N-n)t_n}dt$$

The last exponential factor in this product corresponds to the time interval from the last dismissal until $T$.

The *maximum likelihood* estimate for $r$ is the value that maximizes $P(r)$. Finding it is a calculus exercise. It's easier and equivalent to take logarithms before calculating the derivative. We can also ignore the $n+1$ factors $dt$, since as far as $r$ is concerned they are constants and cannot affect the location of the root of the derivative.

$$\ln(P(r)) = \sum_{i=0}^{n-1} (\ln(r) + \ln(N-i) - r(N-i)t_i) - r(N-n)t_n)$$

[CLEAN UP TYPOGRAPHY]

Differentiating kills all terms that don't involve $r$, leaving

$$n/r - \sum_{i=0}^{n-1}((N-i)t_i) - (N-n)t_n = n/r - \sum_{i=0}^{n}(N-i)t_i.$$

The value of $r$ that makes this equal to zero is

$$r = \frac{n}{\sum_{i=0}^{n}(N-i)t_i}$$

We recognize the denominator as the total number of juror-months: the integral over time of the step function giving the size of the jury, thus proving the theorem.

## Figure legends

Figure 1. The actual number of jurors as a function of time and the exponential decay curves approximating that function, one (solid line) using the maximum likelihood value of $r$ computed on April 6, the other (dashed line) the value computed at the end of the trial.

Figure 2. Estimated probability of a mistrial caused by a deficiency of jurors for a 10-month trial. "Retrospective" (dashed line): mistrial probability using the maximum likelihood estimate of $r$ calculated at then end of the trial. "Time-varying" (solid line): mistrial probability using the estimate of $r$ calculated anew each day. (Note that the time-varying estimate is undefined before the first juror drops out, because $r = 0$ until that time.)

Figure 3. Maximum likelihood estimate of $r$ as a function of time since the start of the trial.

## References

[1] D.R. Cox and D. Oakes. *Analysis of survival data.* Chapman and Hall, London, 1984.

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[3] Robert Heinlein. *Time For The Stars.* Ballantine, 11th edition, 1956.

[4] J.D. Kalbfleisch and R.L. Prentice. *The statistical analysis of failure time data.* Wiley, New York, 1980.

[5] Andrew Tobias. *Money Angles.* Linden Press / Simon & Schuster, 1984.