CS/MA320 HW1 Solutions--Parts 1 & 2

1.1 Reading. All of this is relatively important. A lot of it is how to translate between Logic and English sentences -- I will not spend much time on this in class, but will answer questions. When in doubt as to importance of coverage, consider the Exercises with Solutions for which you're responsible (odd-numbered ones are solved starting on page S-1), and of cours Exercises For You To Solve (even-numbered ones).

Exercises With Solutions in 1.1: p 16: 5,7,11,15,23,27,29,33,51,63. NOTE: There is nothing to turn in, but these Exercises are fair game for Quizzes and Exams. You don't have to look at solutions for all parts of an Exercise if you know the answers without looking. Up to you.

1.2 Reading. Example 3 is important, and you should mark with a sticky Table 6 for Open-Book Quizzes and Exams.

Exercises With Solutions in 1.2: p 28: 1,7,9,13,15,19,23,27,31

Exercises For You To Solve in 1.1 and 1.2: As stated, not in Rosen.

1. State the converse, contrapositive, and inverse (labeling clearly) of the following implication: If it snows tonight then I will stay at home. Which of pairs of those forms mean the same thing (two pairs of implications are identical in meaning).

Converse: If I stay at home then it snows tonight.

Inverse: If it doesn't snow tonight then I don't stay at home.

Contrapositive: If I don't stay at home then it doesn't snow tonight.

The last of these is equivalent to the original.

2. Construct a truth table for each of these compound propositions (you can use the same frame for all truth tables). Are any of them Contradictions or Tautologies?

(a) $p \wedge \neg p$

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

(b) $(p \wedge q) \rightarrow (p \vee q)$ (c) $p \oplus (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$	$p \oplus (p \vee q)$
T	T	T	T	T	F
T	F	F	T	T	F
F	T	F	T	T	T
F	F	F	F	T	F

Note that (a) is a Contradiction and (b) is a Tautology.

(d) $p \vee (q \wedge r) \leftrightarrow (p \vee q) \wedge (p \vee r)$ (e) $(p \wedge q) \wedge r \leftrightarrow p \wedge (q \wedge r)$

			(1)	(2)	(3)	(4)	(5)	(d)	(6)	(7)	(8)	(e)
p	q	r	$q \wedge r$	$p \vee (1)$	$p \vee q$	$p \vee r$	$(3) \wedge (4)$	$(2) \leftrightarrow (5)$	$p \wedge q$	$(6) \wedge r$	$p \wedge (1)$	$(7) \leftrightarrow (8)$
T	T	T	T	T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T	T	F	F	T
T	F	T	F	T	T	T	T	T	F	F	F	T
T	F	F	F	T	T	T	T	T	F	F	F	T
F	T	T	T	T	T	T	T	T	F	F	F	T
F	T	F	F	F	T	F	F	T	F	F	F	T
F	F	T	F	F	F	T	F	T	F	F	F	T
F	F	F	F	F	F	F	F	T	F	F	F	T

Note that (d) and (e) are both Tautologies. Can you find either or both of them in Table 6, pg. 24?

3. Note precedence diagram below. (a) Could the parentheses be left out of 2.(b) above without changing the meaning? (b) Same question for 2.(c). Note that the symbol $\Leftrightarrow$ we've been using for tautological equivalence will be $\equiv$ from now on.

2.(b) is: $(p \wedge q) \rightarrow (p \vee q)$; if we leave the parentheses out we get: $p \wedge q \rightarrow p \vee q$. The higher precedence operations are $\wedge$, then $\vee$, then $\rightarrow$. So $\wedge$ binds first and the result is as if we put parentheses around the $\wedge$ term to force it to evaluate first: $(p \wedge q) \rightarrow p \vee q$; then $\vee$ binds, so the result looks like: $(p \wedge q) \rightarrow (p \vee q)$, which is what we were given. Leaving out parentheses will not change the meaning.

2.(c) is: $p \oplus (p \vee q)$, but by the precedence table below, $\oplus$ binds before $\vee$. Therefore these parentheses are necessary to give the desired meaning.

Precedence of Logical Operators

Operator	Precedence
$\neg$	1
$\wedge$ $\oplus$ $\vee$	2 3 4
$\rightarrow$ $\leftrightarrow$	5 6

4. Sorry, I left a statement out of this set, so you will not be marked wrong. I will add the missing statement (in **bold**) and solve it. Determine whether these system requirements are consistent.

"If the file system is not locked, then new messages will be queued."
"If the file system is not locked, then the system is functioning normally, and conversely."
"If new messages are not queued, then they will be sent to the message buffer."
"**If the file system is not locked then new messages will be sent to the message buffer.**"
"New messages will not be sent to the message buffer."

Let *l* mean that the file system is locked; let *q* mean that the messages are queued; let *s* mean that the system is functioning normally; let *b* mean that messages are sent to the message buffer. We have the following requirement statements: $\neg l \rightarrow q$; $\neg l \leftrightarrow s$ (each implies the other -- that's what "and conversely" means); $\boldsymbol{\neg l \rightarrow b}$; $\neg q \rightarrow b$; $\neg b$. ANSWER. Since $\neg b$ is True, b is False, and $\neg q \rightarrow b$ has F on the right, so it must have F on the left which means q is True; $\boldsymbol{\neg l \rightarrow b}$ also has F on the right, so $\neg l$ is F and *l* is True. Now since $\neg l \leftrightarrow s$, s must be False. Thus **(TYPO: *l* is F)** **(Should Be: *l* is T)**: *l* is T, *q* is T, *s* is F and *b* is F.

5. (a) Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent (that they are $\equiv$).

$\neg(p \oplus q) \leftrightarrow (p \leftrightarrow q)$ is a Tautology

p	q	$p \oplus q$	$\neg(p \oplus q)$	$p \leftrightarrow q$	$\neg(p \oplus q) \leftrightarrow (p \leftrightarrow q)$
T	T	F	T	T	T
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	T	T	T

(b) Show that $(p \vee q) \wedge \neg(p \vee r) \rightarrow (q \vee r)$ is a tautology.

			(1)	(2)	(3)	(4)	(5)	(b)
p	q	r	$p \vee q$	$p \vee r$	$\neg$(2)	(1)$\wedge$(3)	$q \vee r$	(4)$\rightarrow$(5)
T	T	T	T	T	F	F	T	T
T	T	F	T	T	F	F	T	T
T	F	T	T	T	F	F	T	T
T	F	F	T	T	F	F	F	T
F	T	T	T	T	F	F	T	T
F	T	F	T	F	T	T	T	T
F	F	T	F	T	F	F	T	T
F	F	F	F	F	T	F	F	T

This is relatively uninteresting, only a Tautology because so many the values on the left are mostly F, and $F \rightarrow X$ is always T for X either F or T.

1.2 Reading. Example 3 is important, and you should mark with a sticky Table 6 for Open-Book Quizzes and Exams.

Exercises With Solutions in 1.2: p 28: 1,7,9,13,15,19,23,27,31

Exercises For You To Solve in 1.2: Again, I will assume this is covered by HW1.

1.3 Reading. Skip "Other Quantifiers" on pp. 37-38. Slip Logic Programming pp. 45-46. You might want to mark with a sticky Table 2. A lot of this is English-Logic translation.

Exercises With Solutions in 1.3: p 46: 5,9,13,23,29,43,45,47,59,61.

Exercises For You To Solve in 1.3: pg. 46: 32ab, 38 a-e, 60 a-c.

Solutions for 1.3.

32.a. "All dogs have fleas." Let F(x) be "x has fleas", and our domain of discourse is dogs. The original statement is $\forall x\, F(x)$; it's negation is $\exists x\, \neg F(x)$, or "There is a dog that does not have fleas". **32.b**. Let H(x) be "x can add." where the domain of discourse is horses. The statement is "There is a horse that can add.", which can be written $\exists x\, H(x)$; the negation is $\forall x\, \neg H(x)$ can be stated as, "No horse can add."

38.a. Some system is open. **38.b**. Every system is either malfunctioning or in a diagnostic state. **38.c**. Some system is open or some system is in a diagnostic state. **38.d**. Some system is unavailable. **38.e**. No system is working. (If one says: "Every system is not working," this can easily be confused with "Not every system is working," which is the same as "It is false that all systems are working," or, "At least one system is not working." But this is wrong.)

60.a. $\forall x\, (P(x) \rightarrow Q(x))$ **60.b**. $\exists x\, (R(x) \wedge \neg Q(x))$ **60.c**. $\exists x\, (R(x) \wedge \neg P(x))$

1.4 Reading. Nested Quantifiers: Examples 1-5 and 14 are sufficient.

Exercises With Solutions in 1.4: p 58: 3,9,17,27,31,39,45.

Exercises For You To Solve in 1.4: pg 58: 12 a-e, j-m, 16 a-e, 40.

Solutions for 1.4.

The answers to **Exercise 12** are NOT unique; there are many ways of expressing these propositions symbolically. Note that C(x, y) and C(y, x) say the same thing. **12.a**. $\neg$ I(Jerry) **12.b**. $\neg$ C(Rachel, Chelsea) **12.c**. $\neg$ C(Jan, Sharon) **12.d**. $\neg \exists$ x C(x, Bob) **12.e**. $\forall$ x (x $\neq$ Joseph $\leftrightarrow$ C(x, Sanjay)) **12.j**. $\forall$ x (I(x) $\rightarrow$ $\exists$ y (x $\neq$ y $\wedge$ C(x, y))) **12.k**. $\exists$ x (I(x) $\wedge$ $\forall$ y (x $\neq$ y $\rightarrow \neg$C(x, y))) **12.l**. $\exists$ x $\exists$ y (x $\neq$ y $\rightarrow$ C(x, y)) **12.m**. $\exists$ x $\forall$ y C(x, y)

In **Exercise 16**, we let P(s, c, m) be the statement that student s has class standing c and is majoring in m. The variable s ranges over all students in the class, the variable c over the four class standings, and the variable m ranges over all possible majors. **16.a**. The proposition is $\exists$ s $\exists$ m P(s, junior, m). It is true from the given information. **16.b**. The proposition $\forall$ s $\exists$ c P(s, c, computer science). This is false, since there are some mathematics majors. **16.c**. The proposition is $\exists$ s $\exists$ c $\exists$ m (P(s, c, m) $\wedge$ (c $\neq$ junior) $\wedge$ (m $\neq$ mathematician)). This is true since there is a sophomore majoring in computer science. **16.d**. The proposition is $\forall$ s ($\exists$ c P(s, c, computer science) $\vee$ $\exists$ m P(s, sophomore, m)). This is false since there is a freshman mathematics major. **16.e**. The proposition is $\exists$ m $\forall$ c $\exists$ s P(s, c, m). This is false. It cannot be that m is mathematics since there is no senior math major, and it cannot be that m is computer science since there is no freshman CS major. These are the only two majors that exist.

40.a. There are many counterexamples. If x = 2, there is no integer y such that 2 = 1/y since we can multiply both sides by y/2 to get y = 1/2. **40.b**. We can rewrite $y^2 - x < 100$ as $y^2 < 100 + x$. If x < -200 (say), that would make $y^2 = -100$, which is impossible. **40.c**. This is not true since sixth powers are both squares and cubes. Thus, for example: $(2^2)^3 = 4^3 = 64 = 8^2 = (2^3)^2$, so if x = 8 and y = 4, $x^2 = y^3$.

1.5 Reading. This Chapter is an introduction to a kind of formal proof. It's good to see how these work but I won't give you any Exam questions about chains of reasoning, just how individual rules work, e.g. to explain some step in Examples 6 or 7. I also will also not ask you to identify errors in proofs. Understand everything through Example 7, then skip to pg. 70, for quantified statements and understand Examples 12 and 13.

Exercises With Solutions in 1.5: pg. 72: 3,13 ab.

Exercises For You To Solve in 1.5: pg. 73: 14.

14. Perhaps the best way to demonstrate these proofs is with **Step**s in one column and **Reasons** in another, but I will be less formal, though I use names for steps from Table 1 on pg. 66 and Table 2 at the bottom of pg. 70.

14.a. Let c(x) be "x is in the class," r(x) be "x owns a red convertible" and t(x) be "x has gotten a speeding ticket." We are given premises: c(Linda), r(Linda), and $\forall$ x (r(x) $\rightarrow$ t(x)). Since we are given the premise $\forall$ x (r(x) $\rightarrow$ t(x)), so we know r(Linda) $\rightarrow$ t(Linda) by Universal Instantiation; we have the premise that r(Linda) is True, and by the former implication, we know t(Linda) is true by Modus Ponens. We are given c(Linda) as a premise, so c(Linda) $\wedge$ t(Linda). Thus by Existential Generalization $\exists$ x (c(x) $\wedge$ t(x)).

I will not give any problems like this on quizzes and Exams, so I won't solve 14.b and 14.c. However you might be asked to name the reason for a step based on the two tables used here.

1.6 Reading. Introduction to Proofs: study through Example 14, pg 83. I won't ask you to find mistakes in proofs (the last part of this section.)

Exercises With Solutions in 1.6: pg. 85: 11, 21, 33.

Exercises For You To Solve in 1.6: pg. 85: 8, 22, 30

8. Since n is a perfect square, $n = m^2$ for some integer m. If m = 0 then n = 0 and the next two squares are 1 and 4, for then the next perfect square after n is not n + 2. Thus we can assume $m \geq 1$ (negative numbers would have the same effect.) The next perfect square after $n = m^2$ is $m^2 + 2m + 1$, but $2m + 1 > 2$ for $m \geq 1$, so there is no perfect square equal to n + 2.

22. We give a proof by contradiction. Suppose we didn't get a pair of either color. Then we drew at most one blue sock and at most one black sock, but that means we drew only two socks and we stated that we drew three. By contradiction, we must have chosen a pair of the same color.

30. We write these statements in symbols: $a < b$, $(a + b)/2 > a$, and $(a + b)/2 < b$. By multiplying the two later inequalities by 2, we get $a + b > 2a$ and $a + b < 2b$. By canceling common terms, these give $b > a$ and $a < b$, so it is clear that all three statements have the same meaning.

1.7 Reading. Proof Methods. Study through Ex 7, skip to Looking for Counterexamples, pg. 96 and Ex. 17. Skip rest of section.

Exercises With Solutions in 1.7: pg. 103: 18.

Exercises For You To Solve in 1.7: pg. 102: 6

Solutions for 1.7.

6. The number 1 has the property since the only positive integers not exceeding 1 is 1 itself, and therefore the sum is 1. This is a constructive proof.