CS/MA320 HW2 Solutions

Quiz 2 will be on this material on Wednesday, Oct. 30, in the first 20 minutes.

First, I list Reading Assignment & Solved Exercises to look at.

2.1 Reading: Study everything but skip Exercise 20. Exercises With Solutions in 2.1: pg. 119: 5.a-d, 7.a-e, 15. Exercises For You To Solve in 2.1: pg. 119: 2.a, 8.a-c&g, 16, 20.

Solutions for 2.1.

2. There are multiple correct answers; I give one or two. **2.a**. $\{3n \mid n = 0, 1, 2, 3, 4\}$ or $\{x \mid x \in \mathbf{N} \wedge x \text{ is a multiple of } 3 \wedge 0 \le x \le 12\}$. I ADD A FEW OTHER SOLUTIONS. **2.b**. $\{x \mid -3 \le x \le 3 \wedge x \in \mathbf{Z}\}$, or one can use English: $\{x \mid -3 \le x \le 3\}$ where the domain of x is the set of integers. **2.c**. Possible answer: {x | x is a letter of the word *monopoly* other than *l* or *y*}.

8.a. True. **8.b**. True. **8.c**. False: see 8.a. **8.g**. False. The two sets are equal.

16. Since the empty set is a subset of every set, we just need to take a set B that contains ∅ as an element. Thus we can let A = ∅ and B = {∅} as the simplest example. We could also let A = {1, 2) and B = {1, 2, {1, 2)), for example.

20. The union of all sets in the power set of S is S. Thus we can recover the set S from P(S) and the set T from P(T) and if P(S) = P(T) then S = T.

2.2 Reading: Skip last part on Computer Representation.

Exercises With Solutions in 2.2: pg. 130: 3, 7, 15.a-b, 35

Exercises For You To Solve in 2.2: pg. 130: 4, 14 (Hint: consider (A - B) ∪ (A ∩ B) and another union of two sets considered), 36.

Solutions for 2.2.

4. Note that $A \subseteq B$ (in fact A is a ***proper*** subset of B: $A \subset B$). **4.a**. A ∪ B = B = {a, b, c, d, e, f, g, h}. **4.b**. A ∩ B = A = {a, b, c, d, e}. **4.c**. No elements in A fail to be in B, so A - B = ∅.
4.d. B - A = {f, g, h}.

14. Since (A - B) ∪ (A ∩ B) = A (you can see this with a Venn diagram), A - B = {1, 5, 7, 8} and A ∩ B = {3, 6, 9}, A = the union of these = {1, 3, 5, 6, 7, 8, 9}. Similarly, (B - A) ∪ (A ∩ B) = B, and since B - A = {2, 10}, B = {2, 10} ∪ {3, 6, 9} = {2, 3, 6, 9, 10}. You can now check these answers by deriving A - B, B - A, etc.

36. There are just two ways that an item can be in either A or B but not both (e.g., A ⊕ B). It can be in A but not in B (i.e., A - B) or it can be in B but not in A (B - A), Thus A ⊕ B = (A - B) ∪ (B - A). Another way to show this is by displaying a membership table (this always works for identities such as this).

Membership Table With Two Basic Sets, A & B

A	B	A ⊕ B	A - B	B - A	(A - B) ∪ (B - A)	A ⊕ B = (A - B) ∪ (B - A)
1	1	0	0	0	0	1
1	0	1	1	0	1	1
0	1	1	0	1	1	1
0	0	0	0	0	0	1

2.3. You can skip Example 26, rest is important.

Exercises With Solutions in 2.3: pg. 146: 1, 7.a-b, 19, 29.

Exercises For You To Solve in 2.3: pg. 146: 2, 6.a&b&d., 18, 30 (Hint: Answer is Yes.)

Solutions for 2.3.

2.a. This is not a function because the rule is not well-defined: is it +n or -n? **2.b**. This is a function. For all integers n, SQRT(n^2 + 1) is a well-defined real number. 2.c. This is not a function, since for n = +2 or n = -2, the value 1/(n^2 - 4) requires division by zero.

6.a. Domain is $\mathbf{Z}^+ \times \mathbf{Z}^+$ and range is $\mathbf{Z}^+$. **6.b**. Since the largest digit of a positive integer cannot be zero, we have domain $\mathbf{Z}^+$ and range {1, 2, 3, 4, 5, 6, 7, 8, 9). **6.d**. The domain and range are both $\mathbf{Z}^+$.

18. If we can find an inverse then the function is a bijection. Otherwise we have to explain why the function is not 1-1 or not onto. **18.a**. This is a bijection since the inverse is $f^{-1}(x) = (4 - x)/3$.
18.b. This is not 1-1 since f(17) = f(-17); it is also not onto since the range is the interval (-∞, 7].
18.c. This function is not defined on the range element -2 (divide by zero); it also misses 1 in the range (though this is not obvious at first), so it is NOT a bijection and this is a correct answer; however it ***is*** a bijection from R - {-2} to R - {1), with inverse $f^{-1}(x) = (1 - 2x)/(1 + x)$. **18.d**. This is a function that is continuous through its entire domain and takes on very small (negative) range values and very large ones so it is onto; it is also 1-1 with $f^{-1}(x)$ = fifthroot(x - 1). Note that not all high-order polynomial functions would be 1-1.

30. To clarify, suppose that g: A → B and f: B → C, so that $f \circ g$: A → C. We will show that if $f \circ g$ is 1-1 then g is also 1-1, so not only is the answer to the question "yes", but part of the hypothesis is not even needed. Suppose that g were not 1-1. By definition this means there are distinct elements a_1 and a_2 in A such that $g(a_1) = g(a_2)$. Then certainly $f(g(a_1)) = f(g(a_2))$, i.e., $f \circ g(a_1) = f \circ g(a_2)$, which means $f \circ g$ is not 1-1, a contradiction. Therefore g must be 1-1.

2.4. You can skip Special Integer Sequences pp 151-153, but start reading again at Summations, p. 153. Then can skip Ex. 16 and 17, but start again at Cardinality, and read the rest except the proof details of Ex. 21.

Exercises With Solutions in 2.4: pg. 160: 3.a-d, 9.c-f, 19.

Exercises For You To Solve in 2.4: pg. 160: 4.a-d, 10.a-c, 20. (Note: the identity given for use in Exercise 20 has a right-hand side of 1/k minus 1/(k+1), NOT a double division: 1/(k-1)/(k+1).)

Solutions for 2.4.

4.a. $a_0 = (-2)^0 = 1$, $a_1 = (-2)^1 = -2$, $a_2 = (-2)^2 = 4$, $a_3 = (-2)^3 = -8$. **4.b**. $a_0 = a_1 = a_2 = a_3 = 3$. **4.c**. $a_0 = 7 + 4^0 = 8$, $a_1 = 7 + 4^1 = 11$, $a_2 = 7 + 4^2 = 23$, $a_3 = 7 + 4^3 = 71$. **4.d**. $a_0 = 2^0 + (-2)^0 = 2$, $a_1 = 2^1 + (-2)^1 = 0$, $a_2 = 2^2 + (-2)^2 = 8$, $a_3 = 2^3 + (-2)^3 = 0$.

10.a. The first term is 3 and the nth term is obtained by adding 2n -1 to the prior term: add 1, then 3, then 5... (One good way to guess a sequence is to consider the sequence of differences between terms.) Alternatively we can note that the nth term is $n^2 + 2$. The next three terms are 123, 146, 171. **10.b**. This is an arithmetic progression whose first term is 7 and difference is 4. The next three terms are 47, 51, 55.**10.c**. The nth term is the binary expansion of n. The next three terms are 1100, 1101, 1110.

20. We use the suggestion and note that all the terms in the summation cancel out except for the 1/k when k = 1 and the 1/(k+1) when k = n. Thus Sum(k = 1 to n) 1/k(k+1) = Sum(k=1 to n) (1/k - (1/k+1)) = 1/1 - 1/(n+1) = n/(n + 1). Try it for small n for a few summations.

4.1. You can skip Examples 7, 10, and 13 and everything following.

Exercises With Solutions in 4.1: pg. 279: 5, 21, 65. (Uses gossip problem description just prior.)

Exercises For You To Solve in 4.1: pg. 279: 6, 20, 66.

Solutions for 4.1.

6. The basic step is clear, since 1*1! = 2! - 1. Assuming the inductive hypothesis, we have 1*1! + 2*2! + …+ k*k! + (k+1)*(k+1)! = (k+1)! - 1 + (k+1)*(k+1)! = (k+1)!(1+k+1) - 1 = (k+2)! -1, as desired.

20. The basic step is n = 7, and indeed $3^7 < 7!$ since 2187 < 5040. Assuming the statement for k, then $3^{k+1} = 3*3^k < (k+1)*3^k < (k+1)k! = (k+1)!$, the statement for k+1.

66. We prove this by mathematical induction. The basis step, g(4) = 2*4 - 4 = 4 was proved in Exercise 65. For the inductive step, assume that when there are k callers, 2k - 4 calls suffice. We must show that when there are k + 1 callers, 2(k + 1) -4 calls suffice, that is two more calls. The hint tells us how to proceed. In the first extra call, let the person k+1 exchange information with person k. Then persons 1 through k have all the information and can share it in 2k - 4 calls. At the end let person k+1 exchange information with person k again and pick up all information of everyone else.