CS/MA320 HW3 Fall 2009

Solutions are due in class, Wednesday, Oct. 7 or under my door on Tuesday Oct. 13 by 2:00 PM. (NO LATER OR IT WON'T COUNT!). Solutions will be on-line by Tuesday evening and I will bring copies to class, and Quiz 3 will be on Wednesday, Oct. 14, in the first 20-25 minutes.

Reading Assignments for the following Sections are in the Notes.

3.1 Exercises With Solutions: pg. 177: 7, 55.

Exercises For You To Solve: pg. 177: 8, 56.

8. procedure largest even location($a_1, a_2, \ldots a_n$: integers)
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k := 0; largest := min_integer;
for i := 1 to n
    if (a_i is even and a_i > largest) then
    begin
        k := i
        largest := a_i
    end
end { k is the desired location or 0 if there are no even numbers }
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56. We need an example where using a 12-cent coin before dimes or nickels is inefficient. Consider making change to represent 15 cents, which can be done with one dime and one nickel. But with a 12-cent coin we would start with one of those and then use three pennies.

3.2 Exercises With Solutions: pg. 190: 1.a-c, 25.

Exercises For You To Solve: pg. 190: 22 (for a-c as in 1.), 26, 28.a (HARD).

22. The approach in these problems is to pick out the most rapidly growing term in each sum and discard the rest, and also discard the multiplicative term of the most rapidly growing term).
a) This function is $\Theta(1)$ so it is not $\Theta(x)$ since 1 grows more slowly than x. For the same reason the function $f(x) = 10$ is not $\Omega(x)$. **b**) This function is $\Theta(x)$; we can ignore the "+ 7" since it is a lower order term and we can ignore the constant coefficient 3. Since $f(x)$ is $\Theta(x)$ it is also $\Omega(x)$.
c) The function $f(x) = x^2 + x + 1$ grows faster than x so $f(x)$ is not $\Theta(x)$ but it is $\Omega(x)$.

26. We just need to look at the definitions. To say $f(x)$ is $O(g(x))$ means that there are constants C and k such that $|f(x)| \leq C|g(x)|$ for all $x > k$. Note that we are assuming that C and k are both positive. To say that $g(x)$ is $\Omega(f(x))$ is to say there are positive numbers C' and k' such that $|g(x) \geq C'|f(x)|$ for all $x > k$. These are saying exactly the same thing if we set $C' = 1/C$ and $k' = k$.

28.a. By Exercise 25 we need to show that $3x^2 + x + 1$ is $O(3x^2)$ and $3x^2$ is $(O(3x^2 + x + 1)$. The second fact is trivial since $3x^2 \leq 3x^2 + x + 1$ (i.e., $C_1 = 1$) for $x > 0$ ($k_1 = 0$); the first fact is also easy since for all $x > 1$ ($k_2 = 1$) $3x^2 + x + 1 \leq 3x^2 + 3x^2 = 6x^2$ for all $x > 1$, thus $C_2 = 2$ and $C_2 = 1$.

3.3 Exercises With Solutions: pg. 199: 3, , , ,

Exercises For You To Solve: pg. 199: 4, 6.a (a bit hard, but valuable).

4. If we start with x and square it then square successive results k-1 times, we get x^{2^k}. Thus we can compute x^{2^k} with only k multiplications rather than the x^{2^k} multiplications it would take by the naïve method of multiplying x multiple times.

6.a. By the way that S - 1 is defined, it should be clear that $S \wedge (S - 1)$ is the same as S except that the rightmost 1 bit has been changed to a zero. (This is because taking (S - 1) borrows from the rightmost 1 bit and leaves borrow bits all the way down to the rightmost-but-1 bit, and all of these borrow bits disappear when anded with S since there are no bits in S below the rightmost one (by definition).) Thus we add *1* to *count* for every 1-bit originally in S since we stop as soon as all 1-bits disappear from S.

3.4. Exercises With Solutions: pg. 208: 5, 7, 9.a-c, 11.

Exercises For You To Solve: pg. 208: 6, 8,10 a-c, 12, 24.

6. Under the hypothesis we have c = as and d = bt for some s and t. Multiplying we obtain cd = ab(st), which means ab | cd, as desired.

8. Not true. A simple counterexample is a = 4 and b = c = 2.

10. In each case, we can carry out the arithmetic on a calculator. YOU **ARE** ALLOWED TO BRING A CALCULATOR TO QUIZZES! **a**. When 44 is divided by 5, the quotient is 5 and remainder is 4. **b**. When 777 is divided by 21, the quotient is 37 and remainder is 0. (Trick: 21 = 3×7, 777/7 = 111 and the sum of digits is 3 so it is divisible by 3 also and the remainder is clearly 0.) **c**. When -123 divided by 19, find quotient and remainder. If it were +123 divided by 19 we would have a quotient of 6 and remainder of 9, but with -123 we would have -6 and -9, but the remainder must be positive so add 19 and get a remainder of 10 and a quotient of -7.

12. YOU ARE NOT RESPONSIBLE FOR THIS ONE. (It is too much dependent of parsing definitions and not dependent enough on intuition, which I prefer.) If m is a positive integer then a mod m = b mod m if a = b (mod m). Assuming a = b (mod m), we know that a - b = mc which implies that (1) a = b + mc. Now considering b mod m, we know that (2) b = qm + r (for some q and r. with non-negative r < m: i.e. (3) r = b mod m. Now harking back to (1) a = b + mc, we can substitute for b from (2) and rewrite (1) as (4) a = qm + r + mc = (q + c)m + r. Thus r = a mod m matches (3) r = a mod m and the two are equal,. QED.

24. Show that if n is an odd positive number then $n^2 = 1 \pmod 8$. Let n = 2k + 1 for some integer k. Then $n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 4k(k + 1) + 1$. Since either k or (k + 1) is even, 4k(k + 1) is a multiple of 8, say 4k(k + 1) = 8s, and $n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 4k(k + 1) + 1 = 8s + 1$ and $n^2 = 1 \pmod 8$.

3.5. Exercises With Solutions: pg. 217: 1.c-f (explain any Tricks you use), 3.

Exercises For You To Solve: pg. 217: 2.c-f (explain any Tricks), 4, 8.

2. **c**. Note that 93 us divisible by 3, and 93 = 3×31, and 31 is not divisible by 2, 3 0r 5 (next prime is 7 which squared = 49) so 31 is prime. It's easy to see that **d.-f**. 101 and 107 and 113 are all prime.

4. Perform trial divisions (with tricks). 39 = 3×13, $81 = 3^4$, 101 is prime, 143 = 11×13, $289 = 17^2$, 899 = 29×31.

8. Following the hint, consider the n numbers in the sequence: (n + 1)! + 2, (n + 1)! + 3, (n + 1)! + 4, . . . , (n + 1)! + (n + 1). The first number is divisible by 2, the second divisible by 3, and so on until the last is divisible by (n + 1), giving us the desired sequence of n composite numbers.