

CS/MA320 HW6 Solutions, FALL 2009

Your solutions were due under my door by Tuesday, November 24 by 2:00 PM. There is no class on Wednesday, November 25 (the day before Thanksgiving) nor for the rest of the week, and Quiz 5 on Hw6 will be on Monday, November 30. LOOK AT HW7 and start doing it as soon as you can, since Exam 2 will occur after turn-in.

Reading Assignments for these questions are what was covered in class notes. In particular, there was no coverage of Section 5.6.

6.1 Exercises With Solutions: pg. 398 & ff: 13, 15, 25.b,

Exercises For You To Solve: pg. 98 & ff: 14, 16, 24.d, 26.b, 38.a-c.

14 We saw in Example 7 of Section 5.3 that there are $C(53,5) = 2,598,960$ equally likely hands. We need to count the number of hands that have 5 different kinds (ranks) of cards, and since there are 13 kinds (2,3,...10,J,Q,K) we can choose them in $C(13,5)$ ways, then choose each card's suit in 4 ways, giving a total of $C(13,5)*4^5 = 1287*1024 = 1,317,888$. Thus the probability is $1,317,888/2,598,960 \cong 0.507$. **Note that the formula $C(13,5)*4^5$ is good enough for your solution. IN ANY QUESTION ON AN EXAM YOU CAN GIVE A FORMULA AS A SOLUTION UNLESS I ASK FOR A NUMERICAL CALCULATION (OF COURSE YOU CAN USE A CALCULATOR OR BORROW ONE IF YOURS ISN'T WORKING).**

16 Of our $C(52,5) = 2,598,960$ hands we have $C(13,5)*4$ that can make a flush by choosing a suit (4 ways) and then choosing 5 cards from that suit in $C(13,5)$ ways. If we want to say a flush BUT NOT A STRAIGHT FLUSH, we will have to rule out 5 cards in a row (A2345, through 10JQKA, or 10 ways) of a single suit (4 ways), so that answer would be $C(13,5)*4 - 40$. I would accept the answer that included straight flushes. The probability of a flush is $C(13,5)*4/C(52,5)$.

24.d As you can see from reading the 25.b solution, all of the numbers chosen are different so the answer is $1/C(48,6) = (1*2*3*4*5*6)/(48*47*46*45*44*43) = 1/(47*46*3*44*43)$ [because $2*4*6 = 48$ and $3*5 = 45/3$] = $1/12271512 = 8.149 \times 10^{-8}$.

26.b Here there are 48 possible numbers and 6 (distinct) correct ones, so to miss all of them we must make our choice of 6 from the remaining 42 (incorrect) numbers: $C(42,6)$. The number of ways of choosing 6 possible numbers all told is $C(48,6)$, so the likely hood of choosing all 6 wrong is $C(42,6)/C(48,6) = 42*41*40*39*38*37/(48*47*46*45*44*43) \cong 0.427$ [because the 6! Terms above and below cancel out].

38.a We assume that all the paired tosses of coins {HH, HT, TH, TT} are equally likely. (I know this is kind of begging the question, but why would the paired coin tosses with fair coins act any differently?) Thus $E_1 \cap E_2$ is the case TH with probability 1/4 while $p(E_1) = p(E_2) = 1/2$, so the two events are independent. $P(E_1 \cap E_2) = 1/4 = p(E_1) p(E_2) = (1/2)(1/2)$

38.b All triples of coin flips are: {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}. Then $p(E_1) = 1/2$, while $p(E_2)$, the probability that two and not three heads come up in a row gives us two elements of the sample space, HHT and THH, gives us $p(E_2) = 1/4$, and $p(E_1 \cap E_2) = 1/8$, so the two events are independent. $P(E_1 \cap E_2) = 1/8 = p(E_1) p(E_2) = (1/2)(1/4)$

38.c The value of $p(E_1) = 1/2$, and $p(E_2) = 1/4$, as before. However $p(E_1 \cap E_2) = 0$ since a T in the second of three flips disallows two heads in a row, so these two events are not independent.

6.2 In reading, skip Example 14 to end of Section.

Exercises With Solutions, pg. 414 & ff: 7 (Hint: you can write down all permutations -- or ask why one before the other would be preferred), 19, 23.

Exercises For You To Solve, pg. 414 & ff: 6, 8.a-c (In 8.c, think of 1&2 glued together, then permuted), 18, 24 (count them all), 34 (probability of k successes in n trials is $C(n,k)p^k(1-p)^{n-k}$).

6.a&b Since 3 must either precede 1 or follow it, there is no reason one would be more likely than another, so we assign the probability $1/2$ to both events. We could also write down all permutations of 1, 2, and 3 and count the cases of each event: 123, 132, 213, 231, 312, 321. Note 1 precedes 3 in the first three cases and the reverse order holds in the last three.

6.c Note that the two events 3 precedes 1 and 3 precedes 2 requires that 3 come first, the last two cases in the permutations above, with probability $1/3$.

8.a&b As in 6.a&b, since 1 must either precede 2 or follow it there is no reason to think one would be fore likely than another so both have probability $1/2$.

8.c For 1 to immediately precede 2, we can think of these numbers as glued together in forming the permutation of the n integers, giving $(n-1)!$ Permutations. With 1 and 2 independent, the number of permutations is $n!$ Thus the probability that 1 immediately precedes 2 in permutations of $\{1,2,\dots,n\}$ is $(n-1)!/n! = 1/n$.

8.d Half of the permutations have n preceding 1 and this has no effect on $n-1$ preceding 2, which is half again, so the probability of both is $1/4$.

8.e As in 6.c, n precedes 1 and n precedes 2 requires that n comes first in the subpermutation of 1, 2, and n , so the probability is $1/3$.

18.a Given that the first person is born on day m out of 1-7, the probability that the second is born on day m is $1/7$.

18.b Let p_n be the probability that n people are all born on different days of the week. It is easy to see that $p_1 = 7/7 = 1$, $p_2 = (7/7)(6/7)$ (the probability that the second is not born on the same day, the complement of 18.a), $p_3 = (7/7)(6/7)(5/7)$, and so on, $p_7 = (7/7)(6/7)(5/7)\dots(1/7)$, with 8 people, two people must be born on the same day by the pigeonhole principle so $p_7 = 0$. The formula can be stated as: $p_n = (7/7)(6/7)(5/7)\dots((8-n)/7)$. The probability that at least 2 people out of n are born on the same day of the week is calculated as $s_n = 1 - (7/7)(6/7)(5/7)\dots((8-n)/7)$.

18.c Calculating the products in formulas above, we get: $p_2 = (7/7)(6/7) = 0.857\dots$, $p_3 = (7/7)(6/7)(5/7) = 0.612\dots$, $p_4 = (7/7)(6/7)(5/7)(4/7) = 0.349854\dots$, so for 4 people the probability $s_4 = 1 - (7/7)(6/7)(5/7)(4/7) = 0.650\dots$ that two are born on the same day of the week is for the first time greater than $1/2$.

24 The conditional probability that exactly four heads appear in five flips of a fair coin given that the first flip came up tails is exactly the probability that the other four flips come up heads, or $1/16$.

34 Recall that the probability of k successes in n independent Bernoulli trials is $C(n,k)p^k(1-p)^{n-k}$.

34.a For 0 successes, we get $C(n,0)p^0(1-p)^{n-0} = (1-p)^n$, since $C(n,0) = p^0 = 1$.

34.b The event that there is at least 1 success is the complement of the event there are zero successes, so the answer is $1 - (1-p)^n$.

34.c There are two ways there can be at most one success: no successes (probability $(1-p)^n$) and one success, which from the formula is $C(n,1)p^1(1-p)^{n-1} = np(1-p)^{n-1}$, so the probability of at most one success is: $(1-p)^n + np(1-p)^{n-1}$.

34.d Since the probability of the event of at least two successes is just the complement of the event of 34.c that there is at most one success, the probability is $1 - [(1-p)^n + np(1-p)^{n-1}]$.

6.3. Exercises With Solutions: pg. 424 & ff: 1, 3 [in notes]

Exercises For You To Solve: pg. 424 & ff: 2, 4, 6.

2 This conceptually duplicates Exercise 1 with F and E reversed. We know from Bayes' Theorem, reversing F and E, that $p(E | F) = (p(F | E) p(E))/p(F)$. We wish to find $p(E | F)$, and we know that $p(E) = 2/3$, $p(F) = 3/4$, and $p(F | E) = 5/8$. Therefore, substituting in $(p(F | E) p(E))/p(F)$, we get $p(E | F) = ((5/8) (2/3))/(3/4) = (10/24)/(3/4) = (10/24)(4/3) = 10/18 = 5/9$.

4 This conceptually duplicates Exercise 3, with slightly different box and color labeling. Let F be the event that Ann picks the second box and E be the event that Frieda picks an orange ball. Then $p(F) = p(F^c) = 1/2$, and $p(E | F) = 5/11$ (5 of 11 balls are orange) while $p(E | F^c) = 3/7$. By Bayes' Theorem, $P(F | E) = (p(E | F)p(F))/(p(E | F) p(F) + p(E | F^c) p(F^c)) = ((5/11)(1/2))/((5/11)(1/2) + (3/7)(1/2)) = (5/22)/(5/22 + 3/14) = (5/22)/(35+33/154) = (5/22)(154/68) = 35/68$.

6 Let F be the event that the randomly chosen soccer player uses steroids. Since $p(F) = 0.05$, $p(F^c) = 0.95$. Let E be the event that the randomly chosen player test positive for steroid use. We are told that $p(E | F) = 0.98$ and $p(E | F^c) = 0.12$ (this is a false positive test result). We are asked to find $p(F | E)$, the probability that a soccer player uses steroids given that he tests positive. By Bayes' Theorem, $P(F | E) = (p(E | F)p(F))/(p(E | F) p(F) + p(E | F^c) p(F^c)) = (0.98*0.05)/(0.98*0.05 + 0.12*0.95) = 0.049/(0.049 + 0.114) = 0.049/0.163 \cong 0.301$.

6.4. Exercises With Solutions, pg. 439 & ff: 5, 13, 23.

Exercises For You To Solve, pg. 439 & ff: 6, 12 (this is a geometric progression), 24.

6 There are $C(50,6)$ equally likely possible choices of six (distinct) integers from among 50. We win nothing if we fail to guess correctly and \$10,000,000 if we guess correctly, so the expected value is $\$10,000,000 * 1/C(50,6) = \$10,000,000/15,890,700 \cong \0.63 .

12 If X is the number of times we roll the die, then X has a geometric distribution with $p = 1/6$.

a) $p(X = n) = (1-p)^{n-1}p = (5/6)^{n-1}(1/6) = 5^{n-1}/6^n$.

24 In Example 18 (and in the notes) we saw that the variance of the number of successes in n Bernoulli trials is npq. Here $n = 10$, $p = 1/6$, and $q = 5/6$. Therefore the variance is $50/36 = 25/18$.