CS 320 Practice Midterm Exam Solutions

Here are problems that might occur on Exam 1. I would be less likely to ask you to provide definitions since it will be an Open Book Exam. 10 points per question.

1. **Simplify $(\neg q \wedge (q \rightarrow p)) \rightarrow \neg p$ down to use only one operator**.

Answer: Using $(q \rightarrow p) \Leftrightarrow \neg q \vee p$:

$(\neg q \wedge (\neg q \vee p)) \rightarrow \neg p \Leftrightarrow (\neg q) \rightarrow \neg p$ by absorption (p. 24), $\Leftrightarrow p \rightarrow q$

Answer: $p \rightarrow q$

Note: you also could use a truth table with 4 rows and recognize the T-F pattern for $p \rightarrow q$.

2. **Show that $\forall x(P(x)\rightarrow Q(x)) \rightarrow (\forall x\, P(x) \rightarrow \forall x\, Q(x))$ by using universal generalization, i.e., considering an element of the domain.**

Given $\forall x(P(x)\rightarrow Q(x))$, to show $(\forall x\, P(x) \rightarrow \forall x\, Q(x))$.

$\forall x(P(x)\rightarrow Q(x))$ means for any a, $P(a) \rightarrow Q(a)$. $\forall x\, P(x)$ means for all a, P(a). But also for any a, $P(a) \rightarrow Q(a)$. Thus for any a, Q(a), i.e., $\forall x\, Q(x)$, as required.

3. a) **If A and B are sets define the Cartesian product $A \times B$.**

Answer: $A \times B$ is the set of all pairs (a, b) for all $a \in A$ and $b \in B$.

b) **If $|A| = n$ and $|B| = m$, what is $|A \times B|$?**

Answer: nm

4. a) **Define: $a \mid b$, where a and b are integers.**

Answer: $b = ka$ for some integer k.

b) **From the definition in a) prove that if $a \mid u$ and $a \mid v$ then $a \mid w \quad \forall w \in \{ un + vm \mid n \text{ and } m \text{ are integers}\}$.**

$a \mid u$ means $u = xa$ for some x. $a \mid v$ means $v = ya$ for some y.

Consider any w in the set. Then $w = un + vm$ where n and m are integers.

Substituting, $w = xan + yam = (xn + ym)a$, a multiple of a, so $a|w$.

5. **Consider the function $h(k) = k \bmod 13$, where k is any integer. (a) What is the domain of h and (b) what is the range of h?**

Answer: (a) all integers, (b) integers k in [0,12], i.e., 0 <= k <= 12.

6. **Given f1(x) = 3 x + 2 and f2(x) = $4x^2 + 3x - 5$, f3(x) = $3^x + 2^x$, give the big-O expression for f(x) for large x.**

Answer: f1(x) is O(x), f2(x) is I(x^2), f3(x) = O(3^x).

7. **Use the Euclidean algorithm (p. 229) to find gcd(78, 105). Show all steps, following Example 12, p. 229, in format.**

Answer.
105 = 1*78 + 27
78 = 2*27 + 24
24 = 1.24 + 3
24 = 8*3

So gcd(78, 105) = 3.

8. **Given integers x, a, b with x | ab and gcd(x, a) = 1, what can you conclude?**

Answer: x|b. See Lemma 1, p. 233.

9. **Solve for x: 4x ≡ 6 (mod 25). Check that your solution works**.

We need the inverse of 4 mod 25 so we can knock out the 4 on the left hand side.

(We know that an inverse exists because 4 and 25 are relatively prime.)

4x = 1 mod 25 = 1, 26, 51, 76,… and 76 = 4*19, so 19 is the inverse of 4 mod 25.

Multiplying both sides by 19, and using the fact that 4*19 = 1 (mod 25) we have:

x = 6*19 (mod 25) = 114 (mod 25) = 14 (mod 25), the answer.

Checking: 4*14 (mod 25) = 56 (mod 25) = 6 (mod 25) as needed.

10 **Explain in detail the difference between ordinary induction and strong induction.**

Both involve a proposition depending on n (or equivalent), P(n). Both have a basis step, but the induction step is different. For ordinary induction, we assume P(k) for just one k value (at or above the basis step) and prove P(k+1), For strong induction, we assume P(k) for all values of k from the basis step to say n, and prove P(n+1).