

# CS/Math 320: Quiz 4 Solutions: November 16, 2009

NAME \_\_\_\_\_

**Show work for partial credit in questions below. You can use back side of any sheet.**

1. (15 pts) Given 4-cent stamps and 9-cent stamps, find the first sequence of 4 postage amounts in a row of the form  $N$ ,  $N+1$ ,  $N+2$  and  $N+3$  using sums of only these stamps. Note: one amount will use 3 9-cent stamps, one will use 2 9-cent stamps, 1 will use 1 9-cent, and one will use NO 9-cent stamps.

**Answer:**  $6*4 = 24$ ,  $4*4 + 9 = 25$ ,  $2*9 + 2*4 = 26$ , and  $3*9 = 27$

2. (10 pts) Based on problem 1, use strong induction to show every postage amount greater than  $n$  can be represented by sums of 4-cent and 9-cent stamps.

**Answer:** We showed in Problem 1 that 24 can be represented by what we will call  $R(24)$ , and we also have representations  $R(25)$ ,  $R(26)$  and  $R(27)$ . Note that as long as we have the prior 4 numbers represented in this way, we can always get the next 4 number represented by adding a 4-cent stamp:  $R(28) = R(24)+4$ ;  $R(29) = R(25)+4$ ;  $R(30) = R(26)+4$ ;  $R(31) = R(27)+4$ . In general:  $R(N+4) = R(N)+4$ .

3. (12 pts) Calculate how many positive integers **less than 1000** are divisible by 5 (label this quantity  $D(5)$ ), how many are divisible by 11 (labeled  $D(11)$ ), how many are divisible by 55 (labeled  $D(55)$ ), and how many are divisible by 5 or 11 or both (labeled  $D(11 \vee 5)$ ).

**Answer:**  $D(5) = \text{FLOOR}(999/5) = \text{FLOOR}(199.8) = 199$ ;  $D(11) = \text{FLOOR}(999/11) = \text{FLOOR}(90.8) = 90$ .  $D(11 \wedge 5)$  (both) =  $\text{FLOOR}(999/55) = \text{FLOOR}(18.16) = 18$ .  $D(11 \vee 5) = 199 + 90 - 18 = 271$ .

NOTE that  $199 + 90 = 289$ , in the Venn diagram of intersecting sets, counts the intersection ( $D(11 \wedge 5) = 18$ ) twice, so to get  $D(11 \vee 5)$  we need to subtract 18.

4. (13 pts) Continuing with problem 3, calculate how many positive integers less than 1000 are divisible by exactly one of 5 and 11 (labeled  $D(\text{one\_of}(5,11))$ ), and how many are divisible by neither 5 or 11 (labeled  $D(\text{neither}(5,11))$ ).

**Answer.**  $D(\text{one\_of}(5,11)) = D(5) + D(11) - 2*D(11 \wedge 5) = 199 + 90 - 36 = 253$ . In this case we want to subtract out ALL of the intersection  $D(11 \wedge 5)$ , so we have to subtract it twice.

$D(\text{neither}(5,11)) = \text{complement}(D(5 \vee 11)) = 999 - 271 = 728$ , i.e. we are subtracting out the number of integers divisible by 5 or 11 or both. **If this is difficult to understand, work it out again for positive integers less than 60, and actually write out all the integers calculated in each case.**

