CS 450

Lecture 3: Data abstraction and pairs

Carl D. Offner

1 What to read in Chapter 1

We have already covered sections 1.1.1–1.2.4 and section 1.3.2 (lambda).

You should also read

1.3.1 Procedures as arguments. (Read this carefully.)
1.3.2 Lambda (We already did this.)
1.3.3 Processes as general methods.
1.3.4 Procedures as returned values.

These last two sections you can read quickly and not so carefully.

2 The `let` special form

Suppose we want to evaluate an expression like this:

$$f(x, y) = (x + y + xy)^2 + (x + y - xy)^2$$

It would be nice to be able to compute $x + y$ and xy only once, rather than twice. ($x + y$ and xy are what compiler-writers call *common subexpressions*.) We can do this like this:

```
(define (f x y)
  (let ((a (+ x y))
        (b (* x y)))
    (+ (square (+ a b))
       (square (- a b))) ))
```

Actually, the **let** special form is really "syntactic sugar"—it is transformed internally as follows:

```
(let ((var1 exp1)                ((lambda (var1 var2 ... varn)
      (var2 exp2)     ==>           body)
         ...                       exp1 exp2 ... expn)
      (varn expn))
  body)
```

3 Data abstraction

Suppose our version of Scheme did not contain support for fractions (i.e., for rational numbers represented as fractions). We could create a rational number package by writing procedures of the following form:

- A *constructor*

  ```
  (make-rat n d)
  ```

- Two *selectors*

  ```
  (numer x)
  (denom x)
  ```

These procedures must be related in the following ways:

```
(numer (make-rat n d)) ==> n
(denom (make-rat n d) ==> d
(make-rat (numer x) (denom x)) ==> x
```

Assuming we have these procedures, we can construct procedures to implement the usual operators. For instance, we know that

$$\frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_1 d_2 + n_2 d_1}{d_1 d_2}$$

with the other usual formulas for the other arithmetic operations.

So we define

```
(define (add-rat x y)
  (make-rat (+ (* (numer x)(denom y))
               (* (numer y)(denom x)))
            (* (denom x)(denom y)) ))

(define (sub-rat x y)
  (make-rat (- (* (numer x)(denom y))
               (* (numer y)(denom x)))
            (* (denom x)(denom y)) ))

(define (mul-rat x y)
  (make-rat (* (numer x)(numer y))
            (* (denom x)(denom y)) ))

(define (div-rat x y)
  (make-rat (* (numer x)(denom y))
            (* (denom x)(numer y)) ))
```

and also

```
(define (equal-rat? x y)
  (= (* (numer x)(denom y))
     (* (denom x)(numer y)) ))

(define (print-rat x)
  (display* (numer x) "/" (denom x)) )
```

So everything works out fine, as long as we can implement the constructor and the selectors.

4 Pairs

The basic constructors and selectors for data structures in Scheme are these:

constructor: cons

selectors: car and cdr

```
(define x (cons 1 2))
```

We say that `cons` creates a *pair*. With this definition,

```
(car x) ==> 1
(cdr x) ==> 2
```

Suppose we also define

```
(define y (cons 3 4))
(define z (cons x y))
```

Then

```
(car (car z)) ==> 1 ;; this is also produced by (caar z)
(car (cdr z)) ==> 3 ;; this is also produced by (cadr z)
```

So now we can implement our rational number package as follows:

```
(define (make-rat n d)
  (cons n d))

(define (numer x)
  (car x))

(define (denom x)
  (cdr x))
```

(Incidentally, we could just as well have written

```
(define make-rat cons)
```

Of course we have not dealt at all with questions of efficiency, representations, and lowest terms. For instance, should rational numbers be stored internally in lowest terms, or should they only be put into lowest terms when they are output? There are tradeoffs to be made here.

But in any case, we have the following hierarchy of objects and procedures:

1. What the user sees is rational numbers, together with arithmetic operations on them (`add-rat`, etc.).
2. Those arithmetic operations are implemented in terms of the constructors and selectors (`make-rat`, etc.) and in terms of Scheme's primitive arithmetic operations on integers (`+`, etc.).
3. The constructors and selectors are implemented in terms of Scheme's primitive constructors and selectors (`cons`, `car`, and `cdr`).
4. `cons`, `car`, and `cdr` are implemented in some fashion. In fact, we have said nothing about how pairs are implemented in Scheme.

5 What is Data?

What really is a pair? It could be implemented internally in various ways. It could be any data structure so long as `cons`, `car`, and `cdr` work together properly.

In fact, *it might not be a data structure at all.* It might be a function:

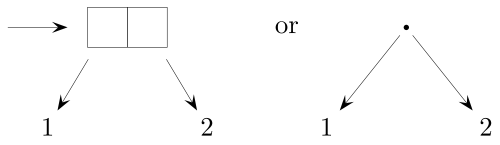
```
(define (cons x y)
  (define (dispatch m)
    (cond ((= m 0) x)
          ((= m 1) y)
          (else (error "Improper argument")) ))
  dispatch)

(define (car z) (z 0))
(define (cdr z) (z 1))
```

Another way to write the definition of `cons` is like this:

```
(define (cons x y)
  (lambda (m)
    (cond ((= m 0) x)
          ((= m 1) y)
          (else (error "Improper argument")) )))
```

Thus in Scheme, the difference between data and procedures fades away. This will be very important in this course.

The conventional way to represent a pair (say, `(cons 1 2)`) is like this:



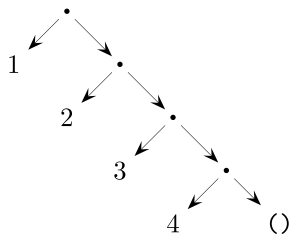
6 Lists (or sequences)

The *empty list* is represented in a Scheme expression as `'()`. (We'll talk about that single quote mark later.)

The list consisting of the numbers 1, 2, 3, 4 in that order is created by the constructor `(list 1 2 3 4)`. This is syntactic sugar for

```
(cons 1
      (cons 2
            (cons 3
                  (cons 4
                        '() ))))
```

That is, it is turned internally into the following data structure:



So a list is actually constructed internally out of pairs. That is, it's really a special kind of binary tree. Now on output, pairs are represented with dots. For instance,

```
==> (cons 1 2)

(1 . 2)
==>
```

However, when the data structure is actually a list, it is represented as such. For instance, the above list could be represented on output as

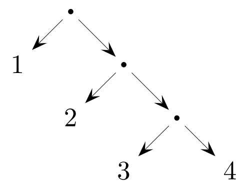
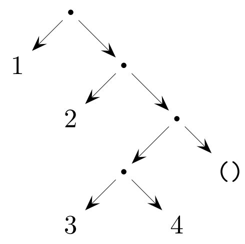
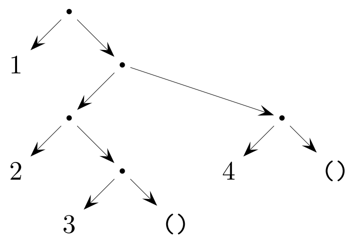
```
  (1 . (2 . (3 . (4 . ()))))
```

But that's not what actually happens:

```
==> (list 1 2 3 4)

(1 2 3 4)
```

```
==> (cons 1
          (cons 2
                (cons 3
                      (cons 4
                            '() ))))

(1 2 3 4)
==>
```

Note the following:

Internal Data Structure	Output Representation	Input Representation
	`(1 2 3 .  4)`	`(cons 1 (cons 2 (cons 3 4)))`
	`(1 2 (3 .  4))`	`(list 1 2 (cons 3 4))`
	`(1 (2 3) 4)`	`(list 1 (list 2 3) 4)`

Note that if

```
(define x (list 1 2 3 4))
```

Then

`(car x)`	is	1	an element of the list
`(cdr x)`	is	`(2 3 4)`	the rest of the list

and further,

`(cons 10 x)`	is	`(10 1 2 3 4)`
`(caddr x)`	is	3
`(cddddr x)`	is	`()`

7 Procedures that manipulate lists

```
(define (count-down-from n)
  (cond ((= n 0) (display 0) (newline))
        (else (display n) (newline) (count-down-from (- n 1))) ))
```

Question: Why can't we just use "if" here?

```
(define (count-up-to n)
  (cond ((= n 0) (display 0)(newline))
        (else (count-up-to (- n 1)) (display n)(newline)) ))

(define (add-elements items)
  (if (null? items)
      0
      (+ (car items) (add-elements (cdr items))) ))

(define (add-squares items)
  (if (null? items)
      0
      (+ (square (car items)) (add-squares (cdr items))) ))

(define (add-transformed items func)
  (if (null? items)
      0
      (+ (func (car items)) (add-transformed (cdr items) func)) ))

(define (list-ref items n)
  (if (= n 0)
      (car items)
      (list-ref (cdr items) (- n 1)) ))

(define (length items)
  (if (null? items)
      0
      (+ 1 (length (cdr items))) ))
```

Here is an iterative form of `length`:

```
(define (length items)
  (length-iter items 0))
```

```
(define (length-iter items result)
  (if (null? items)
      result
      (length-iter (cdr items) (+ result 1)) ))


==> (define squares (list 1 4 9 16 25))

squares
==> (define odds (list 1 3 5 7 9))

odds
==> (append squares odds)

(1 4 9 16 25 1 3 5 7 9)
==>

(define (append x y)
  (if (null? x)
      y
      (cons (car x) (append (cdr x) y)) ))

(define (map func sequence)
  (if (null? sequence)
      '()
      (cons (func (car sequence)) (map func (cdr sequence))) ))

==> (map abs (list -10 2.5 -11.6 17))

(10 2.5 11.6 17)
==> (map (lambda (x) (* x x))
         (list 1 2 3 4))

(1 4 9 16)
==>

(define (filter predicate sequence)
  (cond ((null? sequence) '())
        ((predicate (car sequence))
         (cons (car sequence) (filter predicate (cdr sequence))))
        (else (filter predicate (cdr sequence))) ))
```