CS450 - Structure of Higher Level Languages

Streams

October 21, 2020

Introduction

- Streams are abstract sequences.
- They are potentially infinite – we will see that their most interesting and powerful uses come in handling infinite sequences.
- For now let us think of them as finite in length.
- Finite streams are entirely equivalent to lists.
- Nevertheless, they have their own initializers and access routines:

```
the-empty-stream
; a data object -- a stream with no elements
(stream-null? x)
(stream-car x)
(stream-cdr x)
(cons-stream a x)
```

Implementing standard processes using streams

- To start out, we can think of streams as lists.
- Later, we will see why this is not a good idea in general, even for finite lists.
- Here are two computations we might want to perform:
    1. Given a binary tree whose leaves are integers, find the sum of the squares of the leaves that are odd.
    2. Construct a list of all the odd Fibonacci numbers

Sum of squares of the leaves that are odd

Given a binary tree whose leaves are integers, find the sum of the squares of the leaves that are odd.

```
(define (sum-odd-squares tree)
  (if (not (pair? tree))
      (if (odd? tree)
        (square tree)
        0)
      (+ (sum-odd-squares (left-branch tree))
       (sum-odd-squares (right-branch tree)) )))
```

List of Odd Fibonacci Numbers

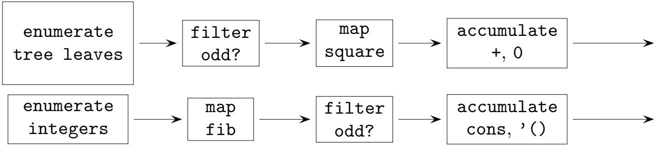
`fib(k)` with $k \leq n$

```
;;; Assume we have already defined the
;;; procedure (fib k) which evaluates
;;; to the k'th Fibonacci number --
;;; we have seen previously how to do this.

(define (odd-fibs n)
  (define (next k)
    (if (> k n)
      '()
      (let ((f (fib k)))
        (if (odd? f)
            (cons f (next (+ k 1)))
            (next (+ k 1)) ))))
  (next 1) )
```

List of Odd Fibonacci Numbers

Conceptually, what is going on in these two processes is this:



`enumerate tree leaves` → `filter odd?` → `map square` → `accumulate +, 0` →

`enumerate integers` → `map fib` → `filter odd?` → `accumulate cons, '()` →

The Enumerate Procedure

- We would like to write our procedures so that these processes become explicit. So first let's do (1).
- First we need a procedure to take a tree and create a stream consisting of the leaves of the tree:

```
(define (stream-enumerate-tree tree)
  (if (not (pair? tree))
      (cons-stream tree the-empty-stream)
      (stream-append
            (stream-enumerate-tree (left-branch tree))
            (stream-enumerate-tree (right-branch tree)) )))
```

The Append and Filter Procedures

Next, we need some general-purpose higher-order procedures that act on streams:

```
(define (stream-append s1 s2)
  (if (stream-null? s1)
      s2
      (cons-stream (stream-car s1)
                   (stream-append (stream-cdr s1) s2) )))

(define (stream-filter pred stream)
  (cond ((stream-null? stream) the-empty-stream)
        ((pred (stream-car stream))
         (cons-stream (stream-car stream)
                      (stream-filter pred (stream-cdr stream))))
        (else (stream-filter pred (stream-cdr stream)))))
```

The Map and Accumulate Procedures

```
(define (stream-map proc stream)
  (if (stream-null? stream)
     the-empty-stream
     (cons-stream (proc (stream-car stream))
                  (stream-map proc (stream-cdr stream)))))

(define (stream-accumulate proc init stream)
  (if (stream-null? stream)
     init
     (proc (stream-car stream)
      (stream-accumulate proc init (stream-cdr stream)))))
```

Putting it Together

Now in terms of these definitions, we have simply

```
(define (sum-odd-squares tree)
   (stream-accumulate + 0
          (stream-map square
                (stream-filter odd?
                      (stream-enumerate-tree
                             tree) ))))
```

Putting it Together

- In our original code, the set of leaves of the tree was implicit in the code.
- Here, however, we have made it an explicit object – a “stream”.
- Doing that makes it possible to write our code much more clearly, in terms of procedures that produce or consume such streams.
- It’s quite useful to note that these higher-order procedures are pretty general, and so they can be reused.
- For instance, let us now handle (2). We only need one new procedure.

Putting it Together

```
(define (stream-enumerate-interval low high)
  (if (> low high)
      the-empty-stream
      (cons-stream low
          (stream-enumerate-interval (+ low 1) high) )))
```

And now we can represent (2):

```
(define (odd-fibs n)
  (stream-accumulate cons '()
       (stream-filter odd?
             (stream-map fib
                 (stream-enumerate-interval 1 n) ))))
```

Putting it Together

- Here in our original code the set of numbers from 1 to n was implicit.
- In our new code, we have made it an explicit stream and as before, our procedures either produce or consume such streams.
- We can put these stream tools together in different ways. For instance, suppose we want to construct a list of the squares of the first `n` Fibonacci numbers.

Putting it Together

```
(define (list-square-fibs n)
  (stream-accumulate cons '()
          (stream-map square
              (stream-map fib
                  (stream-enumerate-interval
                             1 n) ))))
```

Here is another example:

```
(define (product-of-squares-of-odd-elements stream)
  (stream-accumulate * 1
         (stream-map square
                (stream-filter odd? stream) )))
```

Another Example

- Suppose we have a stream of records containing information about employees.
- We have a selector `salary` which extracts the employee's salary from that employee's record – `(salary record)` evaluates to the employee's salary.
- Suppose we want to find the salary of the highest-paid employee who is a programmer.

```
(define (salary-of-highest-paid-programmer record-stream)
    (stream-accumulate max 0
         (stream-map salary
              (stream-filter programmer? record-stream) )))
```

Stream-for-each

This is another higher-order procedure:

```
(define (stream-for-each proc stream)
  (if (stream-null? stream)
      'done ; or anything else
      (begin (proc (stream-car stream))
             (stream-for-each proc (stream-cdr stream)) )))
```

This is useful, for instance, for viewing a stream:

```
(define (display-stream stream)
  (stream-for-each display-line stream) )

(define (display-line x)
  (newline)
  (display x) )
```

Delayed evaluation

- Up to now, we have been regarding streams as the same as lists.
- However, thinking of streams – even finite streams – as lists leads to severe inefficiencies.
- For instance, suppose we want to compute the sum of the primes from `a` to `b`.
- Here is a straightforward way to do this:

```
(define (sum-primes a b)
 (define (iter count accum)
   (cond ((> count b) accum)
     ((prime? count) (iter (+ count 1) (+ accum count)))
     (else (iter (+ count 1) accum)) ))
 (iter a 0) )
```

A Stream Way to Write it

It would be nicer to write it like this:

```
(define (sum-primes a b)
  (stream-accumulate + 0
      (stream-filter prime?
           (stream-enumerate-interval a b) )))
```

But here we have to first create the whole list of integers from a to b, then create the whole list of primes from a to b, and then sum them.

A Stream Way to Write it

- Even worse: suppose we want to find the second prime in the interval $[10,000 \ldots 100,000]$.
- We could write this:

```
(stream-car (stream-cdr (stream-filter prime?
       (stream-enumerate-interval 10000
                                  100000))))
```

But this would first create a list of 90,000 numbers, checking each of them for primality, and then throwing away all but the first two.

A Stream On Demand

- The solution is to only create elements of a stream on demand.
- Specifically, we make `cons-stream` a special form which does not evaluate its second argument.
- `stream-cdr` performs the actual evaluation.
- In order to implement this, we use a new special form `delay` and and a new primitive procedure `force`.
- Both of these are built into Scheme.

`(cons-stream a b)` is equivalent to `(cons a (delay b))`

```
(define (stream-car stream) (car stream))
(define (stream-cdr stream) (force (cdr stream)))
```

A Stream On Demand

Let us see how to evaluate

```
(stream-car (stream-cdr (stream-filter prime?
                    (stream-enumerate-interval 10000
                         100000))))
```

where

```
(define (stream-enumerate-interval low high)
  (if (> low high)
     the-empty-stream
     (cons-stream low
        (stream-enumerate-interval (+ low 1) high) )))
```

Delayed Evaluation

We do this in four steps:

Step 1. Produce `(stream-enumerate-interval 10000 100000)`

Step 2. Pass this to `(stream-filter prime? ...)`

Step 3. Pass this to `(stream-cdr ...)`

Step 4. Pass this to `(stream-car ...)`

Delayed Evaluation

Step 1. We first produce

```
(stream-enumerate-interval 10000 100000);
```

this is

```
(cons 10000
   (delay
      (stream-enumerate-interval 10001 100000))
```

Step 2. We next want to evaluate

```
(stream-filter prime?
    (stream-enumerate-interval 10000 100000))
```

That is, we want to evaluate

```
(stream-filter prime? (cons 10000
   (delay
      (stream-enumerate-interval 10001 100000))
```

Delayed Evaluation

Remember:

```
(define (stream-filter pred stream)
  (cond ((stream-null? stream) the-empty-stream)
        ((pred (stream-car stream))
         (cons-stream (stream-car stream)
               (stream-filter pred (stream-cdr stream))))
        (else (stream-filter pred (stream-cdr stream))) ))
```

we have, since `(prime? 10000)` is `#f`,

```
(stream-filter prime? (stream-cdr stream))
```

where `stream` is

```
(cons 10000
   (delay (stream-enumerate-interval 10001 100000)))
```

Delayed Evaluation

Now `stream-cdr` forces the `delay`, like this:

```
(stream-filter prime? (force (delay
(stream-enumerate-interval 10001 100000))))
```

so we get

```
(stream-filter prime?
(stream-enumerate-interval 10001 1000000))
```

which is

```
(stream-filter prime?
    (cons 10001
       (delay (stream-enumerate-interval 10002 100000))))
```

etc.

Delayed Evaluation

This continues until we get to

```
(stream-filter prime?
    (cons 10007
       (delay (stream-enumerate-interval 10008 100000))))
```

Since 10007 *is* prime, this becomes

```
(cons 10007
      (delay (stream-filter prime?
             (stream-cdr (cons 10007
                 (delay
                    (stream-enumerate-interval 10008 10000
```

Delayed Evaluation

Step 3. This result is now passed to `stream-cdr` in our original expression. This forces the first `delay`, and evaluates to

```
(stream-filter prime?
    (stream-cdr (cons 10007
       (delay
         (stream-enumerate-interval
          10008 100000))))))
```

Both arguments to stream-filter have to be evaluated. Evaluating the second argument causes stream-filter to force the `delay` in the `cdr` of its argument yielding...

Delayed Evaluation

```
(stream-filter prime?
   (cons 10008
      (delay
        (stream-enumerate-interval 10009 100000))))
```

We keep going until it finds the next prime, which is 10009, where we get

Delayed Evaluation

```
(cons 10009
    (delay (stream-filter prime?
        (stream-cdr
            (cons 10009
                (delay (stream-enumerate-interval
                     10010 100000)))))))
```

Step 4. This is now passed to the `stream-car` in our original expression. This is just car, and so we get 10009.

Implementing `delay` and `force`

- Here is one way that one could implement `delay` and `force`:
- `delay` is a special form, such that `(delay <exp>)` is equivalent to `(lambda () <exp>)` (remember the environment model! A procedure is created but not evaluated)
- `force` is not a special form: it is a procedure which just evaluates its argument by calling it as a procedure:

```
(define (force delayed-object)
  (delayed-object))
```

Implementing `delay` and `force`

- In reality it's a little more complicated.
- Delayed objects are also tagged so that they print out as a PROMISE:

```
==> (define a (delay b))

a
==> a

(PROMISE
  b)
```

But that's a minor point.

Warning

- Note that we could `define` the variable `force`, because it is the name of a procedure, and procedures can be defined.
- However, we could not use `define` to specify what we mean by `delay`, because `delay` is a special form.
- Suppose we tried to do it, like this:

```
(define (pseudo-delay exp)    ;;; WRONG!!!
  (lambda () exp))    ;;; WRONG!!!
```

- Let us think – what would go wrong if you did this?
- Therefore `delay` has to be created in Scheme by some other technique.
- It can be specified as a macro – this is pretty typical, in fact.
- But we're not covering macros in this class, so just don't worry about it, it will be provided for you.