

## Homework 1

Due date: Monday, October 8, 2012, at 5:30pm

1. Let  $A = \{a, b, c\}$ . Determine the words  $x \in A^*$  that satisfy the equality  $ax = xx$ .
2. Let  $x, y, u, v$  be words in  $A^+$  such that  $xy = uv$  and  $|x| \leq |u|$ . Show that there exists  $z \in A^*$  such that  $u = xz$  and  $y = zv$ .
3. Let  $A$  be an alphabet and let  $a \in A$ .
  - (a) Prove that  $a^{-1}A^* = a^{-1}A^+ = A^*$ .
  - (b) Prove that  $a^{-1}A^n = A^{n-1}$ .

Let  $A$  be an alphabet. A language  $L \subseteq A^*$  is *k-bounded* if there exist  $k$  nonempty words,  $w_0, \dots, w_{k-1}$  in  $A^+$  such that  $L \subseteq \{w_0\}^* \cdots \{w_{k-1}\}^*$ .  $L$  is *bounded* if it is  $k$ -bounded for some  $k \in \mathbb{N}$ . If  $L$  is not bounded, we say that  $L$  is *unbounded*.

4. Show that the product and the union of two bounded languages is a bounded language.
5. Prove that  $A^*$  is bounded if and only if  $|A| = 1$ .