Homework 3

Due December 5, 2011

1. Let $A = \{0, 1\}$ be an alphabet. Define the language L as follows:

   - $\lambda \in L$.
   - if $x \in L$, then $0x1 \in L$.
   - if $x \in L$ and $y \in L$, then $xy \in L$.

   (a) Show that if $x \in L$, then x is a word that has exactly as many 0's as 1's.

   (b) Show that L is not regular.

2. Prove that the language $\{x \in \{a, b\}^* \mid n_a(x) \leq n_b(x)\}$ is not regular.

3. Let $A = \{a, b\}$ and let L be the language $L = aA^*bA^*b$. Compute the minimal automaton that will accept L.

4. Let $G = (\{S, X, Y, Z\}, \{a, b\}, S, P)$ be a context-free grammar whose set of production is

$$\begin{aligned} P \;=\; & \{S \to XaYbZc, S \to abc, X \to \lambda, X \to Ybc, Y \to \lambda, \\ & Y \to Zac, Z \to \lambda, Z \to Xab\} \end{aligned}$$

   Construct an equivalent grammar without erasure rules.

5. Construct a grammar G_1 equivalent to G given in problem 4 such that G_1 is in Chomsky's normal form.