

Homework 3

Due date: Monday, December 10, 2012, at 5:30pm

1. Consider the context-free grammar

$$G = (\{S\}, \{a, b\}, S, \{S \rightarrow aS, S \rightarrow aSbS, S \rightarrow \lambda\})$$

Prove that $L(G)$ is the set of all words $x \in \{a, b\}^*$ such that for every prefix y of x , $n_a(y) \geq n_b(y)$.

2. Let $G = (\{S, X, Y, Z\}, \{a, b\}, S, P)$ be a context-free grammar. The set of productions P consists of the following:

$$\begin{array}{lll} S \rightarrow XYZa & S \rightarrow YZb & S \rightarrow ZX \\ X \rightarrow ab & X \rightarrow \lambda & X \rightarrow ZXZ \\ Y \rightarrow ba & Y \rightarrow ZYZ & Z \rightarrow \lambda \end{array}$$

Construct a context-free grammar G' such that $L(G') = L(G) - \{\lambda\}$.

3. Prove that the language $L = \{a^n b^{2n} c^{3n} \mid n \in \mathbb{N}\}$ is not context-free.
4. Let G be a λ -free context-free grammar. Prove that every derivation tree for a word $w \in L(G)$ has at most $2|w| - 1$ internal nodes.
5. Let $G = (A_N, A_T, S, P)$ be a grammar in Chomsky normal form. Prove that $L(G)$ is infinite if and only if there exists $w \in L(G)$ that is derived from S in more than $|A_N|$ steps.