CS 624: Analysis of Algorithms

Fall 2025 Assignment 1

Due: September 17, 2025, on Gradescope

1. I have read and understood the syllabus, the document titled "Acknowledging Intellectual Debts" and the course policy about academic honesty, unauthorized collaborations and the use of AI, and I agree to those terms. **Answer with your full name**.

2. Show that $\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$ for $0 < k \leq n$

3. Show that the generating function for $1, 3, 5, 7, 9 \cdots = A = 1+3x+5x^2+7x^3+9x^4 \cdots = \frac{1+x}{(1-x)^2}$.
   Hint: Use a similar technique to the Fibonacci series and the identity:
   $2x + 2x^2 + 2x^3 + 2x^4 + \cdots = \frac{2x}{1-x}$

4. Every permutation of n elements determines a set of inversions. Prove the converse: each permutation is uniquely determined by its set of inversions. **Hint:** Try to prove by contradiction. In other words – assume this is not the case and prove that it cannot be true.

5. Decide whether each of the following statements is true or false, and prove that your conclusion is correct.
   (a) $2^{n+1} = O(2^n)$
   (b) $f(n) = O\big(g(n)\big)$ implies $2^{f(n)} = O\big(2^{g(n)}\big)$

6. Prove that $\log_a x = O(\log_b x)$ for any $a > 0$ and $b > 0$.

7. Prove that if $f = O(g)$ and $g = O(h)$ then $f = O(h)$.

8. Give asymptotic tight bounds for T(n) for each of the recurrences. Justify your answers.
   (a) $T(n) = 2T(n/2) + n^3$
   (b) $T(n) = T(8n/11) + n$
   (c) $T(n) = 16T(n/4) + n^2$
   (d) $T(n) = 7T(n/2) + n^2 \log n$
   (e) $T(n) = 2T(n/4) + \sqrt{n}$

9. Problem 4.2 in Lecture notes 1 (page 7).