CS 624: Analysis of Algorithms

Spring 2026 Assignment 1

Due: Feb. 15, 2026, on Gradescope

1. This question is based on Appendix C in CLRS, 4^{th} edition, question C.1-11 (page 1183). Argue that for any integers $n \geq 0, j \geq 0, k \geq 0$ and $j + k \leq n$:

   $\binom{n}{j+k} \leq \binom{n}{j} * \binom{n-j}{k}$

   Provide both an algebraic proof and an argument based on a method for choosing j + k items out of n. Give an example in which equality does not hold.

2. Given the following function: $t_n = 3t_{n-1} + 4t_{n-2}$, where $t_0 = 0$ and $t_1 = 1$. Find an explicit term for t_n using generating functions.

3. Prove the correctness of the following algorithm for evaluating a polynomial

   $$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

   at a number x:

Algorithm 1 Horner(a,x)

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p = a_n
for i = n − 1 to 0 do
    p := p · x + a_i
end for
return p
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   This algorithm, as you probably know, is called *Horner's method*. You can use induction on the loop invariant using initiation, maintenance and termination.

4. Prove that if $f = O(g)$ and $g = O(h)$ then $f = O(h)$.

5. Give asymptotic tight bounds for T(n) for each of the recurrences. Justify your answers.

   (a) $T(n) = 2T(n/2) + n^3$

   (b) $T(n) = T(8n/11) + n$

   (c) $T(n) = 16T(n/4) + n^2$

   (d) $T(n) = 7T(n/2) + n^2 \log n$

   (e) $T(n) = 2T(n/4) + \sqrt{n}$

6. Problem 4.2 in Lecture notes 1 (page 7).

7. Problem 4.1 in Lecture notes 2 (page 13).

8. Let $\{f_n : n = 0, 1, \ldots\}$ be the Fibonacci sequence (where by convention $f_0 = 0$ and $f_1 = 1$).

(a) This question is based on material from lecture notes 2. Show that $\sum_{n=1}^{\infty} \frac{nf_n}{2^{n-1}} = 20$. Do this by using a generating function as shown in the last section of the Lecture 2 notes, and differentiating. **Hint:** The derivative of $\frac{x}{1-x-x^2}$ is $\frac{1+x^2}{(1-x-x^2)^2}$.

(b) Show why (in the same way as you proved the first part of this problem) you might think that $\sum_{n=1}^{\infty} nf_n = 2$. Then show why this could not possibly be true (it doesn't have to be a long answer, but it has to be convincing).