CS 624: Analysis of Algorithms

Assignment 5

Solution

1. The inductive hypothesis is: Let us assume by induction that at stage v_i, node v_i is reachable by a chain of tree edges from s (notice that it could be s itself). The inductive step is: When v_i is taken out of the queue - that is, it is in the head of the queue, and it puts another node in the queue, v_j, then v_j is connected by an edge to v_i which, by inductive hypothesis, is connected by a chain of tree edges, which extends the induction one step.

2. The d's and π's are as follows: (I followed alphabetical order when more than one option is available, but any correct answer is acceptable).

Vertex	d	π
u	0	null
s	1	u
t	1	u
y	1	u
r	2	s
v	2	s
x	2	y
w	3	r
z	3	x

3. Exercise 22.2-6 (page 602).

   See here: `http://mypathtothe4.blogspot.com/2013/04/graphs-222-6-clrs.html`

4. **Base case:** A tree with one node has one leaf and zero nodes of degree 2.

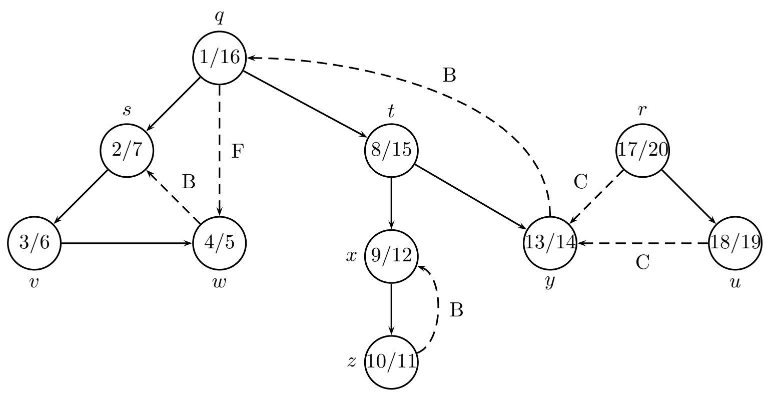
   Inductive hypothesis: Assume that the number of degree-2 nodes in any binary tree of size $1 \geq k < n$ is 1 fewer than the number of leaves, that is – if the tree has m leaves, it has $m - 1$ nodes of degree 2.

   Proof for n: Let us insert a new node to a binary tree of $n - 1$ nodes. The new node is added as a leaf to an existing tree, which by inductive hypothesis has m leaves, it has $m - 1$ nodes of degree 2 for some m. The newly inserted leaf is either the only child of a former leaf (in which case there are still m leaves and $m - 1$ nodes of degree 2), or the second child of a node that formerly had only one child. In this case there are now $m + 1$ leaves and m nodes of degree 2.

   To conclude – since this is true for every binary tree, in a full tree there are only either leaves or nodes of degree 2, and so as shown above there are one more leaves than internal nodes.

5. Exercise 22.3-2 (page 610).

   See figure, starting from q. Edges are marked by B, F and C for back, forward and cross respectively. Solid edges are tree edges.



6. Exercise 22.3-8 (page 611). The notation u.d in that problem refers to the "discovery time" or (as it is called in my notes) the "start time" for vertex u in the depth-first walk.

   This can happen if there is indeed a path in the graph but not in the DFS tree. In this case the start and finish times are disjoint.

7. Exercise 22.3-9 (page 612). (And here, u.f is the "finish time" for node u.)

   Answer: For the two questions 6 and 7 above – see for example Fig. 22.5 (a), page 607. The vertices x and w are an example of both cases (please accept any other reasonable solution)