

HOMEWORK 1  
due: October 3, 2011

1. Write a JAVA class to convert a comma-separated file into an arff formatted table (the WEKA format).
2. Suppose that  $d : S \times S \rightarrow \mathbb{R}$  is a metric. Determine the correct answer to the following questions:
  - Is  $d^2(x, y)$  is a distance on  $S$ ?
  - Is  $\sqrt{d(x, y)}$  a distance on  $S$ ?

Give *full justification* of your answers to receive credit. Yes/No answers are not sufficient.

3. Let  $d$  be an ultrametric on a set  $S$  and let  $x, y, z, u$  be four members of  $S$  and let  $d(x, y), d(x, z), d(x, u), d(y, z), d(y, u), d(z, u)$  be the distances existent between these points. How many distinct values can these 6 numbers have?
4. Write a JAVA program that produces a number of 10 pairs of  $n$ -dimensional random vectors  $u$  whose components are uniformly located in the interval  $[0, 1]$ . Run the program for  $n = 10, 100, 1000$ . In each case compute the angle between vectors. What happens when the dimensionality increases?
5. Let  $u$  be a fixed element of a metric space  $(S, d)$ . Define the function  $d_u : S \times S \rightarrow \mathbb{R}$  as

$$d_u(x, y) = \begin{cases} 0 & \text{if } x = y \\ d(x, u) + d(u, y) & \text{if } x \neq y \end{cases}$$

for  $x, y \in S$ . Prove that  $d_u$  is a metric on  $S$ .