Homework 1

Posted: Monday, February 5, 2024
Due: Wednesday, February 21, 2024 at or before 4:00pm

1. List all partitions of the set $\{a, b, c, d\}$ and draw the Hesse diagram of the poset of these partitions.
2. Each of the following sets is not a subspace of the specified linear space. For each set explain why it is not a subspace.

Set	Lin. Sp.
$\left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mid x_1 \geqslant 0, x_2 \geqslant 0 \right\}$	$\mathbb{R}^2$
$\left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mid x_1 + 4x_2 = 5 \right\}$	$\mathbb{R}^2$
the set P_k of polynomials of degree exactly k	*lin.sp.of polynomials*

3. Let S be a subspace of a linear space L. Define the relation $\sim_S$ on L as containing those pairs $(\boldsymbol{x}, \boldsymbol{y})$ of elements of L such that $\boldsymbol{x} - \boldsymbol{y} \in S$. Prove that $\sim_S$ is an equivalence on L. Describe, the best you can the equivalence classes of $\sim_S$.
4. Prove that the set of vectors in $\mathbb{R}^3$

$$U = \left\{ \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ -8 \\ 1 \end{pmatrix}, \right\}$$

   is not a basis in $\mathbb{R}^3$.
5. Let P be the linear space $\mathbb{R}[x]$ of polynomials. Is the set $\{1+x+x^2, 1-x, 1-x^3\}$ linearly independent? Jusify your answer.