Homework 4

Posted: November 5, 2025
Due: November 19, 2025

1. Let $A \in \mathbb{C}^{n\times n}$ be a matrix and let $a, b \in \mathbb{C}$ such that $a \neq 0$. Prove that $\mathsf{spec}(aA + bI_n) = \{a\lambda + b \mid \lambda \in \mathsf{spec}(A)\}$.
2. Let
$$A = \begin{pmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{pmatrix} \text{ and } B = \begin{pmatrix} \sinh t & \cosh t \\ \cosh t & \sinh t \end{pmatrix}$$
Prove that $\mathsf{spec}(A) = \{\sin\alpha + i\cos\alpha, \sin\alpha - i\cos\alpha\}$ and $\mathsf{spec}(B) = \{\sinh t + \cosh t, \sinh t - \cosh t\}$.
3. Let $A = (a_i^j) \in \mathbb{C}^{n\times n}$ be an upper (lower) triangular matrix. Prove that $\mathsf{spec}(A) = \{a_i^i \mid 1 \leqslant i \leqslant n\}$.
4. An idemponent matrix is a matrix $A \in \mathbb{C}^{n\times n}$ such that $A^2 = A$. Prove that $\mathsf{spec}(A) \subseteq \{0, 1\}$.
5. Let A be the matrix
$$A = \begin{pmatrix} 2 & a & 3 \\ a & 4 & 2 \\ 3 & 2 & 1 \end{pmatrix}$$
For at least eight values of a in the interval $[1, 8]$ compute the eigenvalues of A (using MATLAB) and graph the absolute largest value of an eigenvalue as a function of a. Note that the eigenvalues of A are real numbers. Why?