## Homework 4

Posted: March 20, 2024
Due: April 8, 2024

1. Let $A \in \mathbb{C}^{n \times n}$ be a matrix and let $a, b \in \mathbb{C}$ such that $a \neq 0$. Prove that $\operatorname{spec}\left(a A+b I_{n}\right)=\{a \lambda+b \mid \lambda \in \operatorname{spec}(A)\}$.
2. Let

$$
A=\left(\begin{array}{cc}
\sin \alpha & \cos \alpha \\
-\cos \alpha & \sin \alpha
\end{array}\right) \text { and } B=\left(\begin{array}{cc}
\sinh t & \cosh t \\
\cosh t & \sinh t
\end{array}\right)
$$

Prove that $\operatorname{spec}(A)=\{\sin \alpha+i \cos \alpha, \sin \alpha-i \cos \alpha\}$ and $\operatorname{spec}(B)=$ $\{\sinh t+\cosh t, \sinh t-\cosh t\}$.
3. Let $A=\left(a_{i}^{j}\right) \in \mathbb{C}^{n \times n}$ be an upper (lower) triangular matrix. Prove that $\operatorname{spec}(A)=\left\{a_{i}^{i} \mid 1 \leqslant i \leqslant n\right\}$.
4. An idemponent matrix is a matrix $A \in \mathbb{C}^{n \times n}$ such that $A^{2}=A$. Prove that $\operatorname{spec}(A) \subseteq\{0,1\}$.
5. Let $A$ be the matrix

$$
A=\left(\begin{array}{lll}
2 & a & 3 \\
a & 4 & 2 \\
3 & 2 & 1
\end{array}\right)
$$

For at least eight values of $a$ in the interval $[1,8]$ compute the eigenvalues of $A$ (using MATLAB ) and graph the absolute largest value of an eigenvalue as a function of $a$. Note that the eigenvalues of $A$ are real numbers. Why?

