Homework 4

Posted: March 20, 2024 Due: April 8, 2024

- 1. Let $A \in \mathbb{C}^{n \times n}$ be a matrix and let $a, b \in \mathbb{C}$ such that $a \neq 0$. Prove that $\operatorname{spec}(aA + bI_n) = \{a\lambda + b \mid \lambda \in \operatorname{spec}(A)\}.$
- 2. Let

$$A = \begin{pmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{pmatrix} \text{ and } B = \begin{pmatrix} \sinh t & \cosh t \\ \cosh t & \sinh t \end{pmatrix}$$

Prove that $\operatorname{spec}(A) = \{\sin \alpha + i \cos \alpha, \sin \alpha - i \cos \alpha\}$ and $\operatorname{spec}(B) = \{\sinh t + \cosh t, \sinh t - \cosh t\}.$

- 3. Let $A = (a_i^j) \in \mathbb{C}^{n \times n}$ be an upper (lower) triangular matrix. Prove that $\operatorname{spec}(A) = \{a_i^i \mid 1 \leq i \leq n\}.$
- 4. An idemponent matrix is a matrix $A \in \mathbb{C}^{n \times n}$ such that $A^2 = A$. Prove that $\text{spec}(A) \subseteq \{0, 1\}$.
- 5. Let A be the matrix

$$A = \begin{pmatrix} 2 & a & 3 \\ a & 4 & 2 \\ 3 & 2 & 1 \end{pmatrix}$$

For at least eight values of a in the interval [1,8] compute the eigenvalues of A (using MATLAB) and graph the absolute largest value of an eigenvalue as a function of a. Note that the eigenvalues of A are real numbers. Why?