1. Prove that for every recursive enumerable set $S$ there exists a computable function $f$ such that $S = \text{Dom}(f) = \text{Ran}(f)$.

Hint: modify one of the $S$ programs presented in class in relation with this topic.

2. Let $f(x_1, \ldots, x_n)$ be a function computed by program $P$. Suppose that for some primitive recursive function $g(x_1, \ldots, x_n)$

$$\text{STP}(x_1, \ldots, x_n, \#(P), g(x_1, \ldots, x_n))$$

is true for all $x_1, \ldots, x_n$. Show that $f$ is primitive recursive.

3. Let $A, B$ be sets. Prove or disprove:

   (a) if $A \cup B$ is r.e. then both $A$ and $B$ are r.e;

   (b) if $A \subseteq B$ and $B$ is r.e., then $A$ is r.e.

4. Given a partially computable function $f(x, y)$ prove that there exists a primitive recursive function $g(u, v)$ such that

$$\Phi_{g(u,v)}(x) = f(\Phi_u(x), \Phi_v(x)).$$

5. Let $K_0 = \{ \langle x, y \rangle \mid x \in W_y \}$. Show that $K_0$ is recursively enumerable.