Homework 1

Posted: February 6, 2018
Due: February 21, 2018

1. Give an example of two languages L, K over the alphabet $A = \{a, b\}$ such that $LK = KL$; also, give an example of two such languages where $LK \neq KL$.

2. Let A be an alphabet such that $a, b, c \in A$. Prove that there are no words $x, y, z \in A^*$ such that $xaybz = ybxcz$.

3. Let $x, y \in A^*$ be two words such that $xyy = yxx$. Prove that $x = y$.

4. Let $L = \{a\}^*\{b\}^+$. Compute $x^{-1}L$ for $x \in \{a, b, ab, ba\}$.

5. A word on an alphabet A is *square-free* if it contains no infix of the form xx, where $x \in A^+$.

   (a) List all square-free words of length three over the alphabet $\{a, b\}$.

   (b) Show that for the alphabet $\{a, b\}$ there are no square-free words of length at least equal to 4.

   (c) Let $f : A \longrightarrow A$ be a one-to-one mapping. Prove that if x is square-free then so is $f(x)$, where f is extended to a morphism $f : A^* \longrightarrow A^*$.

6. Let u, v be words, $u, v \in A^*$. Prove that

$$(u^{-1}L)v^{-1} = u^{-1}(Lv^{-1}).$$