Homework 4

Posted: April 25, 2017
Due: May 9, 2017

1. Let $A = \{0, 1\}$ be an alphabet. Define the language L as follows:

   - $\lambda \in L$.
   - if $x \in L$, then $0x1 \in L$.
   - if $x \in L$ and $y \in L$, then $xy \in L$.

   (a) Show that if $x \in L$, then x is a word that has exactly as many 0's as 1's.

   (b) Show that L is not regular.

2. Prove or disprove the following statements:

   (a) If L is a context-free language and $K \subseteq L$, then K is a context-free language.

   (b) If both L and L' are context-free languages on an alphabet A, then $L - L'$ is a context-free language.

   (c) If L is a context-free language, then LL^R is a context-free language.

   (d) If LL^R is a context-free language, then L is a context-free language.

3. Find an equivalent context-free grammar G' in Chomsky normal form for the following context-free grammars:

   (a) $G = (\{S\}, \{a, b\}, S, \{S \to a, S \to aS, S \to aSbS\})$;

(b) $G = (\{S, X, Y\}, \{a, b\}, S, \{S \to XYX, S \to ab, X \to SYS, X \to ba, Y \to XSX, Y \to b\})$;

(c) $G = (\{S\}, \{+, *, (,), a\}, S, \{S \to S + S, S \to S * S, S \to a, S \to (S)\})$.

4. Prove that the following languages are not context-free:

   (a) $\{a^p \mid p \text{ is prime }\}$;

   (b) $\{a^n b^{n^2} \mid n \in \mathbb{N}\}$;

   (c) $\{a^n b^{2n} c^n \mid n \in \mathbb{N}\}$.

5. Determine whether the following languages are

   (i) regular,

   (ii) context-free, but not regular,

   (iii) not context-free.

   (a) $\{a^m b^n \mid m, n \in \mathbb{N} \text{ and } \gcd(m, n) = 1\}$;

   (b) $\{xyx \mid x, y \in \{a, b\}^*\}$;

   (c) $\{xyx \mid x, y \in \{a, b\}^* \text{ and } |x| = 5\}$;

   (d) $\{xyx^R \mid x, y \in \{a, b\}^*\}$;

   (e) $\{xyx^R \mid x, y \in \{a, b\}^* \text{ and } |x| = 5\}$.