Homework 4

Posted: November 7, 2018
Due: November 21, 2018

1. Consider the context-free grammar

$$G = (\{S\}, \{a, b\}, S, \{S \to aS, S \to aSbS, S \to \lambda\})$$

   Prove that $L(G)$ is the set of all words $x \in \{a, b\}^*$ such that for every prefix y of x, $n_a(y) \geq n_b(y)$.

2. Let $G = (A_N, A_T, S, P)$ be a context-free grammar and let $\alpha, \beta \in (A_N \cup A_T)^*$ be two words such that $|\alpha| > |\beta|$ and $\alpha \overset{*}{\underset{G}{\Rightarrow}} \beta$. Prove that α contains at least one nonterminal symbol X such that $X \overset{*}{\underset{G}{\Rightarrow}} \lambda$.

3. Using the algorithm discussed in class construct a λ-free context-free grammar G' such that $L(G') = L(G) - \{\lambda\}$, where G is one of the following context-free grammars:

   (a) $G = (\{S, X, Y, Z\}, \{a\}, S, \{S \to XYZ, X \to a, Y \to a, Z \to a, X \to \lambda, Y \to \lambda, Z \to \lambda\})$;

   (b) $G = (\{S, X\}, \{a, b\}, S, \{S \to aX, S \to bX, S \to a, X \to aX, X \to bXb, X \to \lambda\})$.

4. Let $G = (\{S, X, Y, Z\}, \{a, b\}, S, P)$ be a context-free grammar, where P consists of the following productions:

$$\begin{aligned} P = \{ & S \to SXYZ, S \to SX, S \to YS, X \to XS, X \to YS, \\ & Y \to YaZ, Z \to XbY, X \to a, S \to b\}. \end{aligned}$$

   Compute all productive non-terminal symbols. Which productions you may remove from P without affecting the language generated by the grammar?

5. Find an equivalent context-free grammar G' in Chomsky normal form for the following context-free grammars:

   (a) $G = (\{S, X, Y\}, \{a, b\}, S, \{S \to XYX, S \to ab, X \to SYS, X \to ba, Y \to XSX, Y \to b\})$;

   (b) $G = (\{S\}, \{+, *, (,), a\}, S, \{S \to S + S, S \to S * S, S \to a, S \to (S)\})$.