Homework 3

Posted: October 21, 2019
Due: November 6, 2019

1. Let $A = \{a, b\}$ be an alphabet. Compute the minimal dfa capable of recognizing the language A^*abA^+.

2. Prove that the language $\{a^n b^{n+10} c^{n+20} \mid n \in \mathbb{N}\}$ is not regular.

3. Let $G = (\{S, X, Y, Z\}, \{a, b\}, S, \{S \to XYZ, X \to SYZ, Y \to SXZ, X \to a, Y \to b, Z \to a\})$ be a context-free grammar. Prove that if $x \in L(G)$, the length of x has the form $3 + 2k$, where $k \geq 0$.

   Hint: Use induction on the length of the derivation $S \overset{*}{\underset{G}{\Rightarrow}} x$.

4. Give an example of a non-regular language such that $\mathsf{PREF}(L)$, $\mathsf{SUFF}(L)$, and $\mathsf{INFIX}(L)$ are all regular languages.

5. Consider the context-free grammar $G = (\{S, X, Y\}, \{a, b\}, S, P)$, where the set of productions P is given by

$$P = \{S \to aXb, S \to Yb, X \to YaS, X \to b, Y \to bX, Y \to a\}.$$

   (a) Prove that the word $x = abbaabb$ belongs to $L(G)$ by constructing a derivation d for x. Construct the derivation tree $\mathtt{T}$ that corresponds to d.

   (b) Give the leftmost and the rightmost derivations that corresponds to $\mathtt{T}$.