Homework 2

posted October 11, 2017
due October 25, 2017

1. Let $\mathcal{C}$ be a collection of sets such that $\mathsf{VCD}(\mathcal{C}) \geq 2$. Prove that:
   (a) $\mathcal{C}$ contains two sets A, B such that $|A \oplus B| = 1$;
   (b) both $\mathcal{P}'(\mathcal{C}, s)$ and $\mathcal{P}''(\mathcal{C}, s)$ are non-empty for some $s \in S$.
2. Let $\mathcal{C}, \mathcal{D}$ be two collections of subsets of a set S. Prove that for every $m \in \mathbb{N}$ we have $\Pi_{\mathcal{C}\cup\mathcal{D}}[m] = \max\{\Pi_{\mathcal{C}}[m], \Pi_{\mathcal{D}}[m]\}$.
3. Let S be a nonempty set and let $\mathcal{C} = \{\{x\} \mid x \in S\}$. Prove that $\mathsf{VCD}(\mathcal{C}) = 1$.
4. Let S be a nonempty set. Prove that if $\mathcal{C}$ is a collection of subsets of S such that $|\mathcal{C}| \geqslant 2$, then $\mathsf{VCD}(\mathcal{C}) \geqslant 1$.
5. Prove that if $\mathcal{C}$ is a chain of subsets of a set S, then $\mathsf{VCD}(\mathcal{C}) = 1$.
6. Let $\mathcal{C}, \mathcal{D}$ be two collections of subsets of a set S. Prove that for every $m \in \mathbb{N}$ we have $\Pi_{\mathcal{C}\cup\mathcal{D}}[m] = \max\{\Pi_{\mathcal{C}}[m], \Pi_{\mathcal{D}}[m]\}$.