# CS671 - Machine Learning
## Homework 1
**Posted March 13, 2015**
**Due March 30, 2015**

1. A rhombus $R_{x_0,y_0,c,d}$ is a quadrilateral which has the vertices $(x_0-c, y_0), (x_0, y_0-d), (x_0+c, y_0), (x_0, y_0+d)$ (see Figure 1). Prove that the class of rhombi in $\mathbb{R}^2$ for which the ratio $c/d$ is a constant $k$ is PAC-learnable.

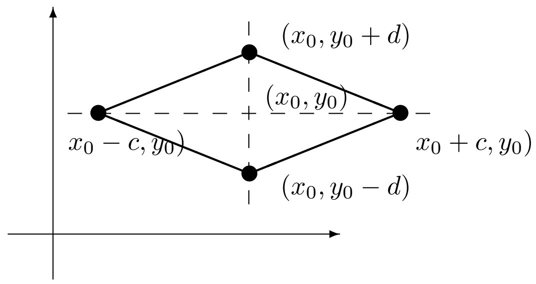


Figure 1: Rhombus having vertices $(x_0-c, y_0), (x_0, y_0-d), (x_0+c, y_0), (x_0, y_0+d)$

2. What is the Vapnik-Chervonenkis dimension of the class of rhombi defined above?

3. Consider the hypothesis family of sin functions of the form $f_\omega(x) = \sin \omega x$. These functions can be used to classify the points in $\mathbb{R}$ as follows. A point is labeled as positive if it is above the curve, and negative otherwise.

   (a) For $m > 0$, consider the set of points $S = \{x_1, \ldots, x_m\}$ with arbitrary labels $y_1, \ldots, y_m \in \{-1, 1\}$. A subset of $S$ is defined by a choice of the parameters $y_i$ and it consists of those $x_i$ such that $y_i = 1$. Define

   $$\omega = \pi \left(1 + \sum_{i=1}^{m} 2^i y_i'\right),$$

   where $y_i' = \frac{1-y_i}{2}$. Prove that with this choice of $\omega$ the set $S$ is shattered, that is, for every subset $T$ of $S$ there would be an $\omega$ such the $T$ equals the set of positive examples.

   (b) What is the Vapnik-Chervonenkis dimension of this classifier?

4. Let $\mathcal{C}_1, \mathcal{C}_2$ be two collections of sets. Define $\mathcal{C}_1 \wedge \mathcal{C}_2 = \{C_1 \cap C_2 \mid C_1 \in \mathcal{C}_1, C_2 \in \mathcal{C}_2\}$. Show that $\Pi_{\mathcal{C}}(m) \leqslant \Pi_{\mathcal{C}_1}(m)\Pi_{\mathcal{C}_2}(m)$.