Homework 1

Posted: February 4, 2019
Due: February 20, 2019

1. List all partitions of the set $\{a, b, c, d\}$ and draw the Hesse diagram of the poset of these partitions.
2. Let $\mathcal{C}$ be a non-empty collection of non-empty subsets of a set S. Prove that $\mathcal{C}$ is a partition of S if and only if every element a of S belongs to exactly one member of the collection $\mathcal{C}$.

Let π be a partition of a set S. A subset T of S is *π-saturated* if it equals a union of blocks of π.

3. Let π be a partition having m blocks defined on a set having n elements, where $m \leqslant n$. How many of the subsets of S are π-saturated?
4. Prove that if $\pi, \pi' \in PART(S)$ and $\pi' \leqslant \pi$, then every π-saturated set is a π'-saturated set.
5. Let S be a set and let U be a subset of S. For $a \in \{0, 1\}$, define the set

$$U^a = \begin{cases} U & \text{if } a = 1, \\ S - U & \text{if } a = 0. \end{cases}$$

   (a) Prove that if $\mathcal{D} = \{D_1, \ldots, D_r\}$ is a finite collection of subsets of S, then the collection $\pi_{\mathcal{D}}$ defined by

$$\pi_{\mathcal{D}} = \{D_1^{a_1} \cap D_2^{a_2} \cap \cdots \cap D_r^{a_r} \neq \emptyset \mid (a_1, a_2, \ldots, a_r) \in \{0, 1\}^r\},$$

   is a partition S.
   (b) Prove that each set of $\mathcal{D}$ is a $\pi_{\mathcal{D}}$-saturated set.