Homework 1

due October 5^{rd}, 2016

1. True or false: If $\mathbf{x}, \mathbf{y}$ are two orthogonal vectors and P is an idempotent matrix (that is, $P^2 = P$), then $P\mathbf{x}$ is orthogonal on $P\mathbf{y}$?
2. Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$. If $\| \mathbf{u} + \mathbf{v} \| = \| \mathbf{u} \| + \| \mathbf{v} \|$, prove that $a\mathbf{u} + b\mathbf{v} = \mathbf{0}$ for some $a, b \in \mathbb{R}$.
3. Let d be the distance between two vectors $\mathbf{y}$ and $\mathbf{z}$ in the Euclidean space $\mathbb{R}^n$. Suppose that $\mathbf{x}_0, \mathbf{u}, \mathbf{v}, \mathbf{x} \in \mathbb{R}^n$ such that $\mathbf{x} = (1-a)\mathbf{u} + a\mathbf{v}$ for some $a \in [0, 1]$ and $d(\mathbf{x}_0, \mathbf{u}) = d(\mathbf{x}_0, \mathbf{v}) = d(\mathbf{x}_0, \mathbf{x})$. Prove that $\mathbf{u} = \mathbf{v} = \mathbf{x}$.
4. Let U be a subset of the linear space L. Prove that if $U \cap U^{\perp}$ is either $\emptyset$ or $\{0_L\}$.
5. Prove that the set of vectors

$$B = \left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \right\}$$

   is a basis in $\mathbb{R}^3$. Apply the Gram-Schmidt algorithm to produce an orthonormal basis starting from B.
6. Let $f_0(x) = 1$, $f_1(x) = x$ and $f_2(x) = x^2$ be a basis for the linear space of polynomials defined on $[0, 1]$. Compute the angles between each of the pairs (f_i, f_j). Apply Gram-Schmidt algorithm to produce an orthonormal basis for this linear space.

`Please write neatly, preferrably using LaTeX.`