

Homework 1

Posted: February 6, 2017

Due: February 20, 2017

1. Give an example of two languages L, K over the alphabet $A = \{a, b\}$ such that $LK = KL$; also, give an example of two such languages where $LK \neq KL$.
2. Let A be an alphabet such that $a, b, c \in A$. Prove that there are no words $x, y, z \in A^*$ such that $xaybz = ybxcz$.
3. Let $x, y \in A^*$ be two words such that $xyy = yxx$. Prove that $x = y$.
4. Let $L = \{a\}^*\{b\}^+$. Compute $x^{-1}L$ for $x \in \{a, b, ab, ba\}$.
5. A word on an alphabet A is *square-free* if it contains no infix of the form xx , where $x \in A^+$.
 - (a) List all square-free words of length three over the alphabet $\{a, b\}$.
 - (b) Show that for the alphabet $\{a, b\}$ there are no square-free words of length at least equal to 4.
 - (c) Let $f : A \rightarrow A$ be a one-to-one mapping. Prove that if x is square-free then so is $f(x)$, where f is extended to a morphism $f : A^* \rightarrow A^*$.
6. Let L be a language on the alphabet A . Prove that $(A^*)^{-1}L = \text{SUFF}(L)$ and $L(A^*)^{-1} = \text{PREF}(L)$.