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Math 114 Exam 2  
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General guidelines

- Both the questions on this exam call for Excel work. Some of your answers belong there. Some belong on these pages. Write complete sentences where that's appropriate.
- Don't use a calculator for arithmetic when you have Excel open on your desktop! Use some of the extra cells, and leave your work so I can see it.
- You have access to anything on your computer or the internet, class notes and other material and the text. I don't think I've asked questions that a web search will help you answer. You're free to try, of course, but don't waste time!
- After class you may improve your answers, and send me updated spreadsheets by midnight tomorrow. *Work independently. Don't consult with friends or classmates or tutors.*

The exam is posted on the course web page at <http://www.cs.umb.edu/~eb/114/exam2/exam2.pdf>.

The spreadsheet is at <http://www.cs.umb.edu/~eb/114/exam2/marathon.xlsx>.

The answers are at <http://www.cs.umb.edu/~eb/114/exam2/marathonsolution.xlsx>.

1. (10 points) Turning in your work. (These are not free points. To earn them you have to follow the instructions.)
  - Read the general guidelines.
  - Turn in this paper.
  - Make sure your name is on this paper and on both worksheets in your spreadsheet!
  - If you need feedback before the end of the day Thursday in order to decide whether to take the course pass/fail or to withdraw, be sure to say that on this paper.
  - Send me your spreadsheet as an attachment *to my gmail address:*  

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with the subject line **Math 114 Exam 2 spreadsheet.**
  - Send it to yourself, for safekeeping and reworking.

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2. (50 points) The 2012summary worksheet in the marathon spreadsheet (link above) contains data for the numbers of men and women who finished the 2012 Boston marathon, grouped by finishing times. For example, 26 men and one woman finished with a time between two and two and a half hours. (That one woman was a wheelchair racer.)

Here are the data:

Finishing time	Men	Women
2:00-2:30	26	1
2:30-3:00	444	27
3:00-3:30	1844	260
3:30-4:00	3389	1714
4:00-4:30	2819	2833
4:30-5:00	1861	1966
5:00-5:30	1068	1013
5:30-6:00	607	609
6:00-6:30	323	339
6:30-7:00	160	162

Answer the following questions. Do as much of the arithmetic in Excel as possible, and leave the formulas there for me to look at. Write your answers here.

- (a) Sketch a neat histogram for this data *here*.  
(not done)
- (b) Draw your histogram with Excel. Does it match your sketch?  
See spreadsheet.
- (c) How many men finished the marathon? How many women?  
Men: 12541, women: 8924, computed with Excel =SUM().
- (d) Use the data to estimate the mode, median and mean for the mens' finishing times.

Hint/warning: 2:30 is 2 hours and 30 minutes. That's 2.5 hours, not 2.3 hours.

- (a) Mode:  
The men's mode finishing time is three to three and a half hours (the highest bar), or about 3:15.
- (b) Median:  
I used Excel to discover that half the men finished in just over four hours (45% were faster than 4:00 and 67% faster than 4:30). I'll estimate 4:10 as the time for the middle runner.
- (c) Mean:

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Using the midpoints of the intervals and doing the computations in Excel, I estimate the mean as about 250 minutes, which is 4 hours and 10 minutes – about the same as the median.

(d) Show the mens' mode, median and mean on your histogram sketch on the previous page.

I showed them in the chart in the spreadsheet.

(e) Suppose my friend ran the marathon and finished ahead of half the men. What was his finishing time (approximately).

His finishing time must have been the median time: about 4:10.

(f) About what percentage of the women finished ahead of half the men?

I need to find the fraction of women who finished faster than 4:10. Only 22% finished in less than four hours, and about 54% finished in less than 4:30, so I'll estimate that 40% finished ahead of half the men.

This is really interesting – the women's median is about 4:20, which isn't much larger than the men's.

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3. (40 points) Faster than a speeding bullet.

The `marathonhistory` worksheet (the second tab in the spreadsheet) shows the history of the winning time for men and women from 1966 (when women first ran) through last year (2013). The tragedy then didn't happen until after the winners crossed the finish line.

I have graphed this information and asked Excel to put in the trend lines. You should see the chart in the spreadsheet. I've drawn the chart for you, and put in the trend lines.

- (a) Add a title and axis labels to the chart.

Done.

- (b) What is the average rate at which the men's finishing time changed from year to year?

The slope of the men's trendline is -0.16 minutes per year, or about 10 seconds per year.

- (c) Use the trendline to predict when the men's winner will finish in two hours. How confident are you in that prediction?

Extending the trendline shows that it crosses the 120 minute line in about 2055, so about 40 years from now.

The current record is 2:03 (123 minutes). I think the top runner will crack two hours a lot sooner than 40 years from now. If the *record* drops at an average rate of 10 seconds per year we'll see 120 minutes in only 18-20 years.

- (d) Use the trendline to predict when the men's winner will finish in one hour. How confident are you in that prediction?

Another hour off the time would take 360 years at the rate of one minute every six years. That's nonsense. No one will ever run that fast.

- (e) The trendlines suggest that in about six years the fastest woman will be as fast as the fastest man, and will be faster thereafter. Explain why the lines say that, and why it's nonsense.

The trendlines cross at about 2018 (four years, not six), at a time of about 127 minutes, or 2:07. I don't believe it. The women's trendline drops unrealistically fast, because of the really steep drops in times when women first started running the marathon.

- (f) Make a better prediction about the long run relation between men's and women's winner finishing times.

Hint: look at the data starting in about 1980.

I used Excel to find the women's times relative to the men's, by dividing. The graph of the values starting in about 1981 (yellow cells) show that the women's fastest times are steadily about 10%-20% larger than the men's. I suspect that will continue to be the case, as both records drop.