Here are solutions to some of the problems on the Fall 2011 final. The problem numbers refer to the versions I put into the book after the exam (they may be slightly reworded there).

Exercise 2.10.37

(a) A graphic next to the article said that standard mail in 2010 amounted to 30.3 pounds for every adult and child in the US. Verify this 30.3 pounds per person figure.

I'll use our standard estimate of 300 million people for the U.S. Population.

$$\frac{9.3 \text{ billion pounds}}{300 \text{ million people}} = 33 \frac{\text{pounds}}{\text{person}}.$$

That's close enough to the reported figure of 30.3 pounds. The actual population is a little larger. If I used 310 million I'd get a better match of 30 pounds per person.

Notes:

- You can also do the problem by multiplying 300 million people by 30.3 pounds per person and noting that the answer is close to 9.3 billion pounds.
- If you do the division upside down *and* put the decimal point in the wrong place the two errors cancel and you can come up with the number 32.2, which looks about right. But the units will be all wrong.

(b) How many pounds of first class mail did the Post Office deliver in 2009?

The 1+ trick for percentages does nicely here. $3.7/0.934 = 3.9614561$. Rounding to two significant figures, I'd say the Post Office delivered 4.0 billion pounds of first class mail in 2009.

Finding 6.6% of 3.7 and adding that to 3.7 gives an answer that's close to correct, but not right.

(c) First class mail mostly consists of bills, credit card statements, personal letters and greeting cards. First class postage is 44 cents for the first ounce and 20 cents for each additional ounce. Estimate the total cost of the postage on first class mail in 2010.

This is a little tricky. I estimate that *most* of the first class mail isn't overweight, so cost 44 cents per ounce to mail. To make the arithmetic easy, I'll assume that the *average* cost was a little larger, say 50 cents per ounce.

The Google calculator tells me

$$\boxed{\text{3.7 billion pounds} = \text{59 200 000 000 ounces}}$$

At half a dollar per ounce that comes to about 30 billion dollars.

Exercise 2.10.38

(a) Estimate the number of pounds per capita of beef, chicken, pork and turkey consumed in the year 2000.

From the graph I estimate the year 2000 per capita consumption in pounds of the various meats as

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beef: 64
chicken: 55
pork: 48
turkey: 13
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(This was a little tricky, because the graphs for chicken and pork cross where they meet.)

(b) The graph stops at about 2004. Use the trends it shows to estimate the number of pounds per capita of beef, chicken, pork and turkey that are being consumed this year.

"This year" when I am writing this answer is 2011. From the trends graph I estimate the 2011 per capita consumption in pounds of the various meats as

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beef: 64
chicken: 65
pork: 48
turkey: 13
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(Everything but chicken seems pretty much the same for the years just before 2004, so I'm guessing it stayed about the same since then.)

(c) The peak year for beef per capita beef consumption was about 1975. Was more *total* beef consumed that year than at any other time in the past century?

Thinking about the units tells me that I can find the total beef consumption by multiplying the per capita consumption by the population.

The U.S. population in 1975 was about 210 million (`http://www.bts.gov/publications/the_changing_face_of_transportation/html/figure_02_01.html`) so the total consumption in that year was about $90 \times 210 = 18,900$ million pounds, or about 19 billion pounds. In the years before 1975 the population was smaller and the per capita consumption was smaller, so I don't need to think about those years.

By 1980 the consumption fell to about 72 pounds per capita, while the population increased to about 225 million. Then total beef consumption was $72 \times 225 = 16,200$ million pounds, or just about 16 billion pounds. That's a lot less than in 1975. In 2005 the consumption was about 64 pounds per capita and the population was about 290 million, for a total beef consumption of $64 \times 290 = 18,560$ million pounds, or 18.5 billion pounds. That's still less than the 1975 figure.

I wonder what the story is now (2011)? With a population today of about 312 million and an estimated beef consumption of 64 pounds per person the total is almost 20 billion pounds – larger than the 19 billion pounds when at the *per capita* peak in 1975.

Exercise 3.8.51

(a) What percentage of the 1993 cost of gasoline was the federal tax in 1993?

The federal gasoline tax rate in 1993 was

$$\frac{0.184}{1.16} = 0.15862069 \approx 15.9\%.$$

(b) If the average cost of a gallon of gas in 2011 was \$3.40 per gallon, what percentage of the cost of gasoline then was the federal tax?

The federal gasoline tax rate at the end of 2011 was

$$\frac{0.184}{3.40} = 0.0541176471 \approx 5.4\%.$$

That's about one third what it was in 1993, since 2011 gasoline cost about three times what it did and the tax hadn't gone up.

(c) What would the cost of a gallon of gasoline have been at the end of 2011 if the *percentage* rather than the *amount* of federal tax were the same then as in 1993?

The cost of a gallon of gas before tax in 2011 was \$3.40 – \$0.184 = \$3.216. If the tax rate had stayed at its 1993 rate, that number would have been 84.1% of the total price (including tax). So the price per gallon at the pump would have been

$$\frac{\$3.216}{0.841} = \$3.82401902 \approx \$3.82$$

instead of \$3.40.

Note. Getting the before and after tax costs right is quite tricky. On an exam, more students than I expected got it right. I gave nearly full credit to computations based on the full price at the pump.

(d) Gasoline consumption in the United States was estimated to be about 175 million gallons per day in 2011. How much revenue was generated by the federal gas tax at its then current rate? How much would have been generated if tax were computed using your answer to part c?

2011:

$$175\frac{\text{million gallons}}{\text{day}} \times 0.184\frac{\$}{\text{gallon}} = 32.2\frac{\text{million }\$}{\text{day}}$$

In the previous answer I computed the tax on a gallon of gas to be \$3.824 − \$3.216 = \$0.608. Using that rate, revenue would be

$$175\frac{\text{million gallons}}{\text{day}} \times 0.608\frac{\$}{\text{gallon}} = 106.4\frac{\text{million }\$}{\text{day}}$$

That's about three times as much, which makes sense.

Note: You can also do this problem by multiplying the amount consumed by the total price at the pump and then computing the tax as a percentage of that.

Exercise 9.6.30

(a) Is the article correct in stating that an annual growth rate of 1.6% means India's population will double in 44 years? Back up your answer with appropriate calculations.

The rule of 70 says that a growth rate of 1.6% per year has a doubling time of about $70/1.6 = 43.75$ or about 44 years.

(b) Assuming that India's growth rate remains1.6% annually, what will its population be in 2030 when it surpasses China's?

2030 is 19 years from 2011. Then India's population will be $1.21 \times 1.016^{19} = 1.64$ billion.

That's *a lot* larger than the predicted peak for China in that year, so I suspect the prediction is based on an assumption that India's growth rate will slow down, although not enough to keep it from passing China.

(c) Assuming that India's growth rate remains 1.6% annually from 2011 on, what will its population be in the year 2100? Compare that figure to the current population of the world. Do you think India's growth rate can in fact continue at 1.6% for the 89 years from 2011 to 2100?

At the 2011 growth rate, India's population in 89 years would be $1.21 \times 1.016^{89} = 4.97$ billion, or just about 5 billion. The 2011 world population is 7 billion. I don't believe that India could hold nearly as many people then as the whole world held in 2011, so I don't think the growth rate can be 1.6% per year for the rest of the century.

Here's another interesting way to do this problem. The doubling time is about 44 years. Twice that is 88 years, so almost to the end of the century. At the 2011 rate of increase, by then the population will have quadrupled, to aboout 4.8 billion. Katie Corey,

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Exercise 13.5.8

(a) Estimate how many papers are submitted by students at your school each semester.

In the spring of 2011 there were about 13,000 students at UMass Boston. If each one wrote six papers a semester that would come to about 80,000 papers – a nice round number in the right ballpark.

(b) Suppose that most students are honest. Estimate how many students will be falsely accused of cheating.

Since most of the 80,000 papers are honest, the false positive rate applies – one percent of them, or 800 papers, will be falsely tagged as plagiarised. That might not be quite 800 students, since some students might be unjustly accused twice, but the order of magnitude is right.

(c) What are the advantages and disadvantages of using the software? (There are several arguments on both sides of the question. Think of as many as you can.)

An advantage is that some plagiarists will be caught who might otherwise get away with it. Another is that students might be less likely to cheat knowing that this software was being used.

I can think of several disadvantages. One is the anxiety caused by the false accusations. Another is the cost.