Math 114 Final

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Name: _____

General guidelines

- Open everything – book, notes, internet. That's the only way we can ask really interesting questions, and see how you would attack questions you encounter in the real world, not just those on a quantitative reasoning final exam. But be careful not to spend time looking for answers on the internet when the best way to do the problem is just to think it through.

- This is a three hour exam. Even so, there may be too many questions. Use your time wisely – spend it on the ones you think you understand.

- Some of the questions call for printing a spreadsheet or a Word document. If you use Word, put all your work in one document, carefully labelled. Try not to save all the printing for the last minute. For safety's sake, put your spreadsheets on a flash drive or email them to yourself, and email them to your instructor.

- Please remember how much *common sense* matters in quantitative reasoning.

The first two questions you did at home. Turn in your answers with your exam:

1. (10%) Discussion of a recent media item.

2. (10%) Wrapup.

3. (10%) Malthus

   Thomas Malthus was an English economist and clergyman. He explored the issues of population growth and the food supply in a famous essay in 1798. He claimed that the food supply grows in a linear fashion but that the population grows exponentially.

   Malthus used the amount of food needed per person for one year as his basic unit of food supply. He estimated that in 1798, the food production in Britain was 7 million food units. He predicted that food production would grow at a rate of 280,000 units per year. In 1798, the population in England was about 7 million people. Malthus estimated that it would grow at a rate of 2.8% each year.

   (a) Write an equation for food production over time, using 1798 as year zero. Specify variables and units.

   (b) Write an equation for population over time, using 1798 as year zero. Specify variables and units.

   (c) Set up an Excel table that displays the amount of food and the population over a time period of 30 years since 1798. Use your functions from parts a) and b) to fill in the table. Construct a properly formatted scatterplot of the data values. Put your name on your spreadsheet, format it for printing, then print it.

   (d) Using these models, when would the population be greater than the food supply?

4. (10%) Iodine 131.

   An article in *The New York Times* on April 6, 2011 discussed levels of radioactive Iodine (Iodine 131) in fish caught near Japan. The article noted that Japan recently revised the safety limit for Iodine 131 in fish to 2,000 becquerels per kilogram. (A becquerel is a measure of radiation.)

   (a) What is the safe level of Iodine 131 in a fish that weighs 3.5 kilograms?

   (b) Radioactive iodine has a half-life of about 8 days. If a fish contained 10,000 becquerels of Iodine 131 per kilogram, how long would it take for the Iodine to decay to a "safe" level?

5. (10%) UMass enrollments.

UMass Boston's Vice Chancellor for Enrollment Services is responsible for all aspects of student admissions and enrollment. She has hired you as a data analyst to help her keep track of enrollment for Fall 2011. The May 2011 report on fall enrollment is now available. It compares the enrollment through the end of the first week of May 2011 with the same period for 2010.

(a) The report noted that UMass Boston's total undergraduate enrollment on May 1, 2010 was 4,631. Of these, 4,402 were continuing students out of the total undergraduate population. The remainder were new students enrolling for the first time. What percentage of undergraduates were continuing students?

(b) The undergraduate enrollment on May 1, 2011 was 5,162. Of these, 4,883 are continuing students. What percentage of undergraduates are continuing students?

(c) The final total fall 2010 undergraduate enrollment was 11,568. The Vice Chancellor has a target for fall 2011 of a 5% increase over last year. How many additional undergraduates, both continuing and new students combined, will need to be enrolled in September 2011 to meet this target?

(d) Can you assure the Vice Chancellor that her target will be met or exceeded? Using measures of change or other quantitative measures, write a quantitative statement addressed to her that compares fall 2010 enrollments with fall 2011 enrollments. Indicate whether her staff needs to place more attention on enrolling continuing students or new students, or both to meet the target.

6. (10%) College costs.

The College Board published the chart below with data about increases in tuition and fees from 2009-2010 to 2010-2011 in public four year colleges and universities. The right-hand column gives the percentage of colleges and universities that increased tuition and fees by the dollar range in the left-hand column (file `http://www.cs.umb.edu/~eb/114/final/CollegCostIncrease.xlsx`, also on the desktop in the lab.)

Dollar Increase	Percent of Colleges
Under $200	10.8%
$200 to $399	26.8%
$400 to $599	23.4%
$600 to $799	15.4%
$800 to $999	7.9%
$1,000 to $1,199	5.0%
$1,200 to $1,399	2.5%
$1,400 to $1,599	1.4%
$1,600 to $1,799	2.4%
$1,800 to $1,999	0.8%
$2,000 to $2199	3.6%

(a) Use Excel to determine the average (mean) increase in tuition and fees.

(b) Use Excel to determine the median increase in tuition and fees.

(c) Use Excel to determine the mode increase in tuition and fees.

(d) Draw a histogram, print it.

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7. (10%) Conventional or hybrid?

The chart below gives price and fuel efficiency for Ford Escape SUVs with conventional or hybrid engines.

Car Model	Price	City Mileage (mpg)	Highway Mileage (mpg)
Ford Escape (conventional)	$24,000	21	28
Ford Escape (hybrid)	$30,000	34	31

Suppose gas costs $3.95/gallon.

   (a) Compare the cost to drive the conventional Ford Escape versus Ford Escape hybrid for 10,000 city miles.

   (b) Compare the cost to drive the Ford Escape conventional versus Ford Escape hybrid for 10,000 highway miles.

   (c) Compare the cost to drive the conventional Ford Escape versus Ford Escape hybrid for 10,000 miles if 60% of the driving is city miles and 40% highway miles.

   (d) Write formulas for the cost to drive the conventional Ford Escape and the cost to drive the hybrid Ford Escape M miles .(For gas mileage (mpg = miles per gallon), use a combined city/highway mileage of 24 mpg for the conventional Ford Escape and 33 mpg for the hybrid Ford Escape).

   (e) Use your formulas in part (d) to determine how many combined city/highway miles you would have to drive a hybrid Ford Escape to make up for the initial higher purchase price. You may do this algebraically or using Excel. (Use 10,000 mile intervals up to 200,000 miles in Excel.) Give a clear answer if your answer is an estimate.

   (f) Your friend has asked you as a quantitative reasoning student to suggest which Ford Escape is more cost effective. Write your friend a letter with your recommendation. Be sure to include the necessary quantitative information.

8. (10%) Too many plastic bags.

An article from October 3, 2007 in *The Washington Post* about how paper and plastic bags impact the environment, states that: "Worldwide, an estimated 4 billion plastic bags end up as litter each year. Tied end to end, the bags could circle the Earth 63 times."

   (a) Verify this estimate. Note any assumptions you make.

   (b) The article also notes that it takes 594 BTUs of energy to produce a plastic bag. Estimate the number of plastic bags used in the United States each year, then calculate the energy needed to produce these bags. Make sure you note any assumptions you make. Use the correct prefix (million, billion, trillion, etc.) and round appropriately.

   (c) Convert your answer in part (b) to barrels of oil.

9. (10%) Breast cancer screening.

In his *Chances Are* blog in *The New York Times* on April 25, 2010 Steven Strogatz wrote about a diagnostic puzzle presented to several doctors:

> The probability that [a woman in this cohort] has breast cancer is 0.8 percent. If a woman has breast cancer, the probability is 90 percent that she will have a positive mammogram. If a woman does not have breast cancer, the probability is 7 percent that she will still have a positive mammogram. Imagine a woman who has a positive mammogram. What is the probability that she actually has breast cancer?
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> . . .
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> [When 24 doctors were asked this question], their estimates whipsawed from 1 percent to 90 percent. Eight of them thought the chances were 10 percent or less, 8 more said 90 percent, and the remaining 8 guessed somewhere between 50 and 80 percent. Imagine how upsetting it would be as a patient to hear such divergent opinions.

(a) What is the correct answer?

Hint: build the contingency table, based on a population of 1,000 women tested.

(b) What percentage of the 24 doctors got the correct answer?

10. (10%) Professor Bolker's wife is writing a book on the social psychology of deafness; her agent wants to convince a publisher to print it.

   (a) Estimate the number of households in the United States with at least one adult who's deaf, or just hard of hearing.

   (b) If there were a good book that could help these households, estimate how many copies might sell in the first year.

   (c) Put your answers to parts (a) and (b) in a paragraph that will convince a publisher to offer Dr. Bolker a contract for her book. (You'll have to explain your reasoning and the sources for your data.)