Common Sense Mathematics
Answers to some excersises, Spring, 2012

Ethan Bolker

February 8, 2012

Exercise 2.10.3

There are two different units of volume each called "gallon", the US gallon and the imperial gallon. I found that information in wikipedia. Since it isn't controversial, I trust that it's correct.

Google is making sure we know which one they are talking about.

When I ask Google for

one gallon in imperial gallons

it tells me

one US gallon = 0.83267384 Imperial gallons

Exercise 2.10.4

When I ask the Google calculator for

(2011 - 1776) years in seconds

it tells me

(2011 - 1776) * years = 7.4158776×10^9 seconds

That's about 7,400,000,000 seconds, or 7.4 gigaseconds. Reporting more than those two digits of precision would make no sense at all.

Exercise 2.10.16

(a) Verify the store's unit price calculation in the quotation.

$$\frac{6.69 \text{ \$}}{16.9 \text{ ounces}} \times \frac{32 \text{ ounces}}{\text{quart}} \times \frac{4 \text{ quarts}}{\text{gallon}} = \frac{50.67 \text{ \$}}{\text{gallon}}.$$

You can get the same answer from the Google calculator without having to do the work. If you ask for

6.69 dollars per 16.9 ounces in dollars per gallon

Google tells you

(6.69 U.S. dollars) per (16.9 US fluid ounces) = 50.6698225 U.S. dollars per US gallon

Amanda Miner discovered a different way to solve the problem. She noted that one gallon is 128 ounces. Since 128/16.9 = 7.57, a gallon jug of olive oil would be 7.57 times as large as the 16.9 ounce bottle. Then she checked the ratio of prices: $50.67/$6.69 = 7.57 too. That means the gallon jug is 7.57 times more expensive. The fact that those ratios are the same means the $50.67 per gallon unit price is right.

(b) Visit the blog. There you will find some comments in each of the categories

- "I've always looked at unit prices."
- "This is new to me, and cool. I'll do this from now on."
- "My husband/boyfriend told me about this."

The consensus of student answers is that about 25% have always looked at unit prices, 25% found the idea new, and 50% learned about it from a significant other.

(c) If you were to post a comment, what would it be? (Optional: post it.)

Amanda Miner wrote "If I posted a comment, I guess it would be 'This is completely new to me. If it wasn't for my QR class, I would probably be getting ripped off now.'"

Exercise 2.10.21

Imagine that the trip from Here to There is 60 miles. Then it takes two hours to get there and one hour to come back. That's three hours for the 120 mile round trip, so the average speed is (120 miles)/(3 hours) = 40 miles/hour.

This conclusion surprises many people. You can see the idea even more clearly if you imagine that it's possible to make the return trip at infinite speed – in no time at all. Then the average speed is (120 miles)/(2 hours) = 60 miles/hour, which is *not* the average of 30 and infinity.

Exercise 2.10.35

(a) Is her estimate of about 0.0024 seconds per species correct?

Since I am looking for an answer with units

$$\frac{\text{seconds}}{\text{species}}$$

and I know

$$\frac{50 \text{ minutes}}{1,255,000\text{species}}$$

I organize the computation this way:

$$\frac{50 \text{ minutes}}{1,255,000\text{species}} \times \frac{60 \text{ seconds}}{\text{minute}} = 0.00239043825\frac{\text{seconds}}{\text{species}} \approx= 0.0024\frac{\text{seconds}}{\text{species}}$$

so Professor Bolker is right.

Another way to do the problem – multiply rather than divide (most people like multiplication better than division):

$$\begin{aligned} 0.0024\frac{\text{seconds}}{\text{species}} \times 1,255,000 \text{ species} &= 3,012 \text{ seconds} \\ &\approx 3,000 \text{ seconds} \\ &= 50 \text{ minutes} \end{aligned}$$

which is just how much time she has for her lecture.

(b) Do her species counts by group add up to approximately 1,255,000?

I checked the sum. It's 1,254,500 which is close enough.

(c) Does the way she apportions lecture time match the apportionment of species?

I will multiply the number of species in each group by 0.0024 seconds per species and see if the alloted time is OK

Arthropods: $1,000,000 \times 0.0024 = 2,400$ seconds, which is 50 minutes.

Molluscs: $110,000 \times 0.0024 = 264$ seconds, which is 4.4 minutes. Professor Bolker rounds that to 4 minutes.

Other nonchordates: $94,000 \times 0.0024 = 225$ seconds, which is indeed 3.75 minutes.

Now for the chordates.

Fish: $31,000 \times 0.0024 = 74.4$ seconds – a minute and a quarter.

Reptiles, amphibians, mammals and birds check out too.

(d) There are in fact about 10,000 bird species. [1] not the 3K - 4.5K in Professor Bolker's letter. How would you rearrange the last two minutes of her class to take that new information into account.

She would need 24 seconds for birds, not 7 seconds. Since she's doing the groups in decreasing order of abundance she'd put them between the fish and the reptiles. That would leave the mammals for last.

(e) You probably know about reptiles, amphibians, mammals and birds. What are arthropods, molluscs, chordates? Don't just copy and paste a definition from the internet - provide one that shows that you understand what you've written. Did you learn anything doing this part of the problem?

(f) While you were answering the previous question you probably found web sites for each of those groups. Do the number of species of each kind match Professor Bolker's assumptions? (Full sentences, and citations, please, not just "yes" or "no".)

(g) How is Professor Bolker related to one of the authors of this book?

She's Professor Ethan Bolker's daughter.

Exercise 2.10.28

Waiting for student solutions to put here.

[1] http://www.birding.com/species.asp