Paying off a debt

Ethan D. Bolker Maura B. Mast

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Plan

Lecture notes

Can you afford a mortgage?

There's a \$250,000 condominium you want to buy. You've managed to scrape together \$50,000 for the down payment (savings, your parents, ...) but will have to take out a mortgage for the \$200,000 balance. Your credit is good, so you've convinced the bank to let you borrow that money at 7% interest each year.

Suppose you are now paying \$1,600 per month in rent, which comes to \$19,200 per year. You think you can afford to use \$17,000 of that for mortgage payments; you'll need the rest for condo fees.

How long will it take you to pay off your loan? How much total interest will you pay?

At the end of the first year you owe the bank

$$200,000 \times 1.07 = 214,000,$$

the \$200,000 principal and \$14,000 in interest. You pay them \$17,000, so your new balance is

$$200,000 \times 1.07 - 17,000 = 197,000$$

That's less than what you owed at the start of the year, but not nearly as much less as you'd like. You still owe most of the \$200,000 in spite of your \$17,000 payment, since most of that went for interest.

Things look a little better the next year, when your balance becomes

$$197,000 \times 1.07 - 17,000 = 193,790$$

This time you paid only \$13,790 instead of \$14,000 in interest, so you reduced the principal by \$3,210, not just \$3,000.

Each year a little less of your fixed payment covers the interest so a little more goes to reduce the principal. To find out when you'll be debt free, just repeat this calculation over and over again - that's a job for Excel. We've written the spreadsheet for you. Here's the chart showing how much you owe at the end of each year.



The picture suggests that it will take just about 25 years to pay off the mortage. The numbers in the spreadsheet tell you more precisely that after 25 years you'll owe just \$10,253, so the next year's payment will finish the job. The spreadsheet also shows you that you will have paid \$235,971 in interest in those 25 years. That's more than you borrowed in the first place. Your \$250,000 condo really cost you about \$490,000 – nearly double the price you paid!

Borrowing money always involves paying back more than you borrowed. In this case, lots more. Is it worth it? Perhaps, for several reasons.

• It would have taken you a long time to save up the full purchase price (to avoid borrowing). And you would have been paying rent the whole time, so saving would have been difficult. So you can think of the payments you made as money you spent instead of paying rent.

- The condo may well be worth more after 25 years than the total you paid for it even including the interest on the mortgage.
- Inflation is pretty nearly inevitable over the years. We've computed the total interest in 2007 dollars, but the actual value of that money when you pay it to the bank each year will be less, in then current dollars.

Words of warning. The purpose of this exercise is to show you how, in principle, you pay off a loan by paying some interest and some principal periodically. That's just one of the financial things you'll need to understand when you think about buying a house or condo. Just asking the bank for an interest rate and using it in this spreadsheet isn't sufficient. There are many books and web pages that may help – here's just one we found with a simple search: http://www.ourfamilyplace.com/homebuyer/checklist.html.

Credit card interest

In the previous section we saw why borrowing is sometimes wise. Now lets look at a time when it's not. We'll read this sample credit card bill, from http://www. practicalmoneyskills.com/english/resources/tutor/statements/credit_ state.php#

][Your First Bank			CREDIT CARD STATEMENT			SEND PAYMENT TO Box 1234 Anytown, USA		
ACCOUNT NUMBER NAME 4125-239-412 Johr CREDIT LINE CRED \$1200.00 \$107		AME ohn Doe	E STATES n Doe 2/13/		ENT DATE PAYMEN 01 3/09/0		T DUE DATE	
		REDIT AVA	LABLE	NEW BALANCE \$125.24		MINIMUM PAYMENT DUE \$20.00		
REFERENCE	SOLD	POSTED	A	TIVITY SINCE	LAST STAT	EMENT	AMOUNT	
483GE7382 32F349ER3 89102DIS2 NX34FJD32 84RT3293A 873DWS321	1/12 1/13 1/18 1/20 2/09	1/25 1/15 1/15 1/18 1/21 2/09	PAYMENT THARK YOU RECORD RECUCLER ANYTONN USA BEEFORAMA REST ANYTONN USA GREAT RXPECTORATIONS BIG CITY USA DINO-GIL PERCOLEMA ANYTONN USA SHIRTS 'N SUCH TINYVILLEUSA		TOWN USA TOWN USA CITY USA TOWN USA YVILLE USA	-168.80 14.83 30.55 27.50 12.26 40.10		
Pervicus Balance (*) 168 Purchases (*) 125 Dash Advances (*) 168 Credits (*) 168 Credits (*) 168 Late Charges (*) 168 Late Charges (*) 125 Late Charges (*) 125		80 24 80 24		Current Am Amount Pa Amount O Minimum P	iount Due st Due ver Credit Line Payment Due	20.00		
Periodic Rate 1.65 Annual Percentage Rate 19.8		HASES ADVANCES 1 0.54% 6.48%		For Customer Service Call: 1-800-XXX-XXXX For Lost or Stolen Card, Call: 1-800-XXX-XXXX 24-Hour Telephone Numbers				

If you have a credit card you get a bill like this once a month. John Doe (the owner of this card) charged \$125.24 in merchandise during this billing period. He's decided not to use this card any longer, and will settle his debt by paying as

little as possible each month. When will he be debt free, and how much interest will he have paid?

The arithmetic for this problem is exactly like the arithmetic for mortgage payments. After he makes the first payment he will owe

$$125.24 \times 1.0165 - 20 = 107.31$$

since $$125.24 \times 0.0165 = 2.07 of his \$20 payment is the interest charge. The rest is subtracted from \$125.24 to reduce the balance owed.

Here is the story for the first seven payments

Month	Balance	Interest
0	125.24	2.07
1	107.31	1.77
2	89.08	1.47
3	70.55	1.16
4	51.71	0.85
5	32.56	0.54
6	13.10	0.22

His last payment is for \$13.10 along with the $13.10 \times 0.0165 = 0.22$ interest. His total interest payments add up to \$8.08.

That doesn't seem too terrible. It's 8.08/125.24 = 0.065 = 6.5%. – less than the mortgage rate for that condo. But don't be fooled. This is a *monthly* bill.

The law requires the credit card company to tell you the annual rate somewhere on the monthly bill. In this case it's it's 19.80%. And you can check their arithmetic: $1.65\% \times 12 = 19.80$. But in fact there's a subtle computation here that hides the true interest rate. Suppose you carry a balance of \$100 for the whole year. Each month the credit card company adds the interest for the month, and then charges you interest on the interest. At the end of the year this will have happened twelve times and you will owe them

$$100 \times 1.0165^{12} = 121.70$$

which corresponds to an *effective* annual rate of 21.7% – almost 2 full percentage points higher than the advertised rate of 19.8%.

Applying interest to interest is called *compounding*. It's a good thing for your investments, since they grow exponentially. It's a bad thing for your credit card balance (but good for the credit card company). It's why the credit card company

sets a low minimum payment, and encourages you to pay just that. Sometimes the minimum payment appears prominently on the bill while the full balance owed is buried somewhere. They really would like you to pay just the minimum, so that they can collect interest at outrageous rates on the balance.

There are other ways credit card companies make you pay for the convenience of borrowing their money.

One way is to compute the interest and charge it to your account every day rather than at the end of the month, or, perhaps, to use an average daily balance for the whole month. Either of these may increase the cost to you.

Another is to make it very expensive if you miss a payment by even a day or so. If you do that they will charge you a late payment fee, and may also increase the already large interest rate. Moreover, a late payment is reported to the credit rating companies, and will show up on your credit report. So when you go to a bank later to take out a mortgage on the condo you want to buy, they may charge you a larger interest rate too.

Finally, each month Your First Bank suggests that you transfer balances from one of your other credit cards to theirs. They will lend you the money to pay off that balance and charge you just 0% interest. Sounds like a good deal, so take them up on their offer, for, say \$1000. Then at the end of the month you owe them \$1125.24 - \$1000 for the transferred balance and \$125.25 for purchases. You decide to pay the full amount of your purchases and make no new purchases, hoping to avoid paying any interest at all.

When you get your next bill you're surprised to see

unpaid balance at	19.80% annual rate	125.24
interest 125.24 *	0.0165	2.07
unpaid balance at	0% annual rate	874.76
interest 874.76 *	0.00	0.00

The bank applied your payment to reduce the balance from the transfer, leaving the balance from your purchases earning interest for them at a compounded effective annual rate of 21.7% (you know that the 19.80% is a fiction). If you call the listed 800 telephone number to complain, the person you speak to will tell you to read the fine print in the contract, where it says that the credit card company will apply any payments *first* to *the part of the outstanding balance that is charged the lowest interest rate*. So you cannot now avoid paying compounding interest on that \$125.24 without paying off the entire \$1000 transferred balance first.

Does all this mean that using a credit card is a bad idea? No, as long as you're careful. Note that in the bill above John Doe paid his last balance of \$168.80 bill on time in full. By doing that he has avoided all finance charges. So, in fact, he borrowed that money for a month from the credit card company at no cost. If he kept it in a savings account until it was time to pay his bill he'd even have made a few pennies in the meanwhile. Some credit cards even give you back 1% of your total purchase dollars at the end of the year.

The credit card companies also collect a fee from the businesses where you use your card – perhaps 2%. So when making a large purchase you may be able to negotiate a discount for paying by cash or check since the merchant will save the credit card company fee. Even if you can't get a better bargain you can help keep a small local merchant in business by paying with cash or check.

Finally, you may find a credit card issued by one of your favorite charities. Then the charity collects a small fraction of the fees the merchants pay.

Saving for college or retirement

Not covered in class this way in this lecture, but the discussion belongs here.

We've already discussed how difficult it is to arrange to have \$100,000 for college or \$??? for retirement by investing a fixed sum all at once and letting the compound interest accumulate, even at a good interest rate. It's easier to imagine saving something each month or year than finding that huge initial investment.

The same calculations we made above to study paying off a debt also serve to study accumulating money by saving regularly. Suppose you can manage to put away \$1200 on the first day of each year. You think you can get a 10% return on your investment, since you're willing to take a short term risk for the sake of long term return. Then at the start of the second year you will have

$$1200 \times 1.10 + 1200 = 2520$$

and at the start of the second year

$$2520 \times 1.10 + 1200 = 3972$$

These calculations look just like the ones we made for paying down a debt, except that in this case we *add* the periodic payments to the balance rather than subtracting them. Once we notice that we can use the debt payment spreadsheet to see how our money accumulates, by entering a *negative* "payment" to be added to our accumulating balance.

Here's the picture (we haven't changed the labels):



The actual figures show that in 25 years you will have accumulated \$118,016. Of that amount, you contibuted just $1200 \times 25 = 330,000$; \$88,016 is interest. If in fact you saved \$100 each month rather than \$1200 each year you'd have even more. How much more we leave as an exercise for the reader.