Double, double toil and trouble

Ethan D. Bolker
Maura B. Mast

December 6, 2007

Plan

Lecture notes

Double, double toil and trouble[1]

If you invest \$1000 at 7% interest (compounded annually) how long until your money doubles? That is, when will you have twice \$1000, or \$2000.

We have several ways at our disposal to answer this question. All depend on the formula

$$\begin{aligned} \text{Value after T years} &= \text{Initial investment} \times (1 + (\text{interest rate}))^T \\ &= 1000 \times (1.07)^T \end{aligned}$$

We can make the computations with a calculator, or with the exponential column in the LinearExponential spreadsheet we've used before. Either way, if we try out several values of T we soon discover

year	value
0	1000
1	1070
...	...
10	1967
11	2105

[1] Shakespeare's Macbeth, Act IV, Scene 1

so the answer to the question is "somewhere between ten and eleven years (probably nearer to ten)."

What if we invested \$1500? How long until we doubled our accumulation to \$3000? A second set of experiments leads to

year	value
0	1500
1	1605
...	...
10	2951
11	3157

and the same answer to our question: "somewhere between ten and eleven years."

A few more experiments should serve to convince you that the time to double at a 7% interest rate is just over ten years, however much you start with.

Suppose you can invest at a 14% interest rate[2] rather than at 7%. Clearly your money will double sooner since the interest rate is higher. How much sooner? If you do the experiments – easier in Excel than with a calculator – you will discover that the doubling time is about 5 years. It's not surprising that this is sooner than the 10 years it took at 7% interest. It may be just a little surprising that it's just half as long.

If you try the experiment assuming a 3.5% interest rate you find that the doubling time is 20 years. A pattern emerges. It's clearly visible in the following table, that contains the results of two more experiments

annual interest rate (%)	doubling time (years)
1.0	70
3.5	20
7.0	10
10.0	7
14.0	5

This is an example of *inverse variation* – values in the second column decrease as those in the first column increase. The relation is particularly simple: the product of the numbers in each row is just 70.

This particular instance of inverse variation has a name that makes it easy to remember. The *Rule of Seventy* says that

[2] Pretty good if you can find it somewhere!

Money invested at R% doubles in 70/R years.

If you prefer a formula, remember the rule of 70 this way:

$$\text{time to double} = \frac{70}{\text{interest rate}}$$

You can also use the rule backwards. If you want to double your money in 15 years you will have to find an investment that pays $70/15 = 4.7\%$ interest.

The rule of 70 is a convenient way to make quick estimates. It's not exact – our analysis doesn't deal with fractions of years, nor with the fact that the interest may be compounded more often than once a year.

Finally, where does the 70 come from? Why that particular number and no other? Unfortunately, the answer is too complex for an elementary text on quantitative reasoning. If some day you go on to study more advanced mathematics you may learn that *the natural logarithm of 2 is 0.6931.* The 70 comes from rounding 0.6931×100.

When values decrease . . .

Eventually, move this to another place.

Depreciation.

Negative slopes, relative change by factors less than 1.