Preface to the Dover Edition

I wrote this book to show how the first theorems in abstract algebra from the standard undergraduate curriculum illuminate the timeless number theory wonders we owe to Fermat, Euler and Gauss. Today I might follow the advice of several adopters and use those wonders to teach algebra and number theory in parallel. So if you're a student in an abstract algebra class, open this book from time to time to see the new concepts you're learning in a different context. If you've studied abstract algebra and find yourself in a traditional number theory course, see how the proofs here capture the essence of the proofs there. If you're teaching a course in number theory or abstract algebra consider recommending this book as a supplementary text for some students.

What would change if I were writing now?

I'd try for less brisk pedantry and more informality in style (though not in rigor). I'm a different kind of teacher today. I find *Theorem … Proof … Remark … Theorem … Proof … Remark* a lot less appealing than I used to.

Computers, rare then, are ubiquitous now. If you can program (in any language, or in a spreadsheet) you might occasionally want to write a program to generate data from which to generate conjectures. But don't throw away your pencil and paper. You will always need to work out the first few cases by hand to test your program – and for me, most of the learning happens at that stage.

Rereading the book I found myself tempted to update references. The internet provides information (mostly accurate, as far as mathematics goes) with an ease unimaginable in 1969. A Google search will tell you more about almost any topic covered here. But URLs age faster than Dover books, so search for yourself. Better yet, take out your pencil and paper and solve the problems.

I'd change the content in just a few places.

Chapter 4 would end with the discussion of public key cryptography obligatory in every contemporary number theory text. The Queen of Mathematics has condescended to offer her subjects utility as well as beauty. (Hardy might not be pleased.)

The standard proof of quadratic reciprocity in Chapter 5 (Gauss' and Eisenstein's Lemmas) magically tells me *that* the theorem is true, but not *why*. I wish I'd known then the proof using cyclotomic polynomials at the end of Friedberg's charming informal *An Adventurer's Guide to Number Theory* (published by Dover, so affordable).

I now know a nice way to treat some of the elementary theory of continued fractions in the spirit of the book; I would add that material to Chapter 6.

In 1995 Fermat's conjecture became Wiles' theorem, so the historical discussion in Chapter 7 needs updating. But the content remains worthwhile, treating the special cases $n = 4$ and $n = 3$ settled by Fermat and Euler.

I thought I was a good proofreader. I apologize for the length of the list of errata (and thank readers for finding them). I hope there are no errata in the errata.

I remember reading Dover's dollar edition of Abbott's *Flatland* when I was in high school. I'm delighted that my number theory book can join that company (even though dollar books are history).

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