Chapter 10

Borrowing and Saving

When you borrow money – on your credit card, for tuition, for a mortgage – you pay it back in installments. Otherwise what you owe would grow exponentially. In this chapter we explore the mathematics that describes paying off your debt.

Chapter goals:

Goal 10.1. Examine how credit cards work.

Goal 10.2. Study balance and interest paying off a loan periodically.

Goal 10.3. Calculate monthly mortgage payments and examine the costs and benefits of home ownership.

Goal 10.4. Understand periodic compounding, APR and other interest terms.

Goal 10.5. Understand the basics of saving money with a long-term goal like retirement.

10.1 Credit card interest

We’ve learned how to read an electricity bill – now we’ll tackle a credit card bill. Figure[10.1] shows a sample from [http://www.practicalmoneyskills.com/flash/bank_tutor/index.html](http://www.practicalmoneyskills.com/flash/bank_tutor/index.html).

If you have a credit card you get a bill like this once a month. John Doe (the owner of this card) charged $125.24 in merchandise during January. He’s decided not to use this card any longer, and will settle his debt by paying $20 each month. When will he be debt free, and how much interest will he have paid?

He makes a minimum payment of $20 in February for his January purchases, so his balance is $125.24 − $20 = $105.24. He’s paid no interest so far. But now that changes. The credit card company charges interest on the balance he carries in February. The FINANCE CHARGE SUMMARY shows a periodic (that is, monthly) rate of 1.65% so he will pay $105.24 × 0.0165 = $1.74 in interest. The 1+ trick tell us that at the beginning of March he owes $105.24 × 1.0165 = $106.98.

After his $20 payment on March 1 his balance is $105.24 × 1.0165 − $20 = $86.98.

---

1 The billing cycle seems to end on the 13th of the month, but we’ll assume it’s the first.
The credit card company used $1.74 of the payment for the February interest. The rest they subtracted from his balance.

Table 10.2 tells the rest of the story. Figure 10.3 shows the Excel formulas we used in http://www.cs.umb.edu/~eb/qrbook/PayoffDebt/PayOffDebt.xlsx to calculate the values in that table, along with a graph showing how the balance decreases each month at a slightly faster rate, until it reaches 0.

<table>
<thead>
<tr>
<th>Month</th>
<th>Balance</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>125.24</td>
<td>0.00</td>
</tr>
<tr>
<td>Feb</td>
<td>105.24</td>
<td>1.74</td>
</tr>
<tr>
<td>Mar</td>
<td>86.98</td>
<td>1.44</td>
</tr>
<tr>
<td>Apr</td>
<td>68.41</td>
<td>1.13</td>
</tr>
<tr>
<td>May</td>
<td>49.54</td>
<td>0.82</td>
</tr>
<tr>
<td>Jun</td>
<td>30.36</td>
<td>0.50</td>
</tr>
<tr>
<td>Jul</td>
<td>10.86</td>
<td>0.00</td>
</tr>
<tr>
<td>total</td>
<td></td>
<td>5.80</td>
</tr>
</tbody>
</table>

John’s last payment is for the $10.86 balance.

In seven months he’s paid $5.80 in interest. That doesn’t seem too terrible. It’s $5.80/$125.24 = 0.046311 ≈ 4.6%. But don’t be fooled. This is a monthly bill, so that 4.6% isn’t the annual interest rate. John didn’t borrow that money for a year. Some of it he had for seven months, some for just one.

The law requires the credit card company to tell you the APR (annual percentage rate) somewhere on the monthly bill. You can find it on this one in the FINANCE CHARGE SUMMARY section: it’s 19.80%. You can check their arithmetic: 1.65% × 12 = 19.80%.

That large interest rate is why the credit card company wants you to pay just the small minimum each month. The full balance appears at the top of the bill labelled NEW BALANCE — but the only payment shown is the MINIMUM PAYMENT DUE. You have to know that it’s best for you to pay the full balance at once.

---

2 We will have more to say about the APR in Section 10.4.
10.1. CREDIT CARD INTEREST

There are other ways credit card companies make you pay for the convenience of borrowing their money. One kicks in if you miss a payment by even a day or so. Then they charge a substantial late payment fee and may also increase the already large interest rate. The law requires credit card companies to print a warning on your bill. Here’s what one says:

**Late Payment Warning:** If we do not receive your minimum payment by the date listed above you may have to pay up to a $39.00 late fee and your APRs will be subject to increase to a maximum Penalty APR of 29.99%.

Moreover, the late payment will show up on your credit report, so when you go to a bank later to take out a mortgage on the condo you want to buy, they may charge you a higher interest rate too.

Does all this mean that using a credit card is a bad idea? No, as long as you’re careful. Then you can take advantage of some of the good things credit can do for you:

- The bill above shows that John Doe paid his last balance of $168.80 on time in full, avoiding all finance charges. So, in fact, he borrowed that money for a month from the credit card company at no cost. If he kept it in a savings account until it was time to pay his bill he’d even have made a few pennies in the meanwhile.

- Some credit cards give you back a reward at the end of the year – perhaps 1% of your total purchase dollars or some frequent flyer airline miles.

- The credit card companies don’t rely only on interest charges and penalties to make money. They also collect a fee from the businesses where you use your card – perhaps 2%. So when making a large purchase you may be able to negotiate a discount for paying by cash or check since the merchant will save the credit card company fee. Even if you can’t get a better bargain you can help keep a small local merchant in business by paying with cash or check.

- We’ve seen that if you miss a payment your credit rating may suffer. But just avoiding credit errors won’t get you a good credit rating. For that you have to prove you can manage credit – by having a credit card and paying the balance in full when due. Then when it’s time to borrow money for a car or a condo your good credit rating may get you a lower interest rate.

Figure 10.3: Paying off a credit card debt.
10.2. CAN YOU AFFORD A MORTGAGE?

- If you have a balance on an existing card that you can’t afford to pay off immediately, consider opening a second card and transferring the balance. The new card company may offer you 0% interest for a while to encourage you to switch. If you do that and then don’t use the new card you can pay off the old balance over time without any further interest charges. Be sure to read the small print before you do this – the transferred debt may be interest free, but sometimes there’s a charge (perhaps three or four percent) to make the transfer.

- Finally, you may find a credit card issued by one of your favorite charities. Then the charity collects a small fraction of the fees the merchants pay.

Do remember:

Pay your full bill on time every month.

You can even arrange to have that happen automatically from your bank account, so you don’t have to remember and you save the cost of a stamp. Just make sure there’s enough money in the bank.

10.2 Can you afford a mortgage?

There’s a $250,000 condominium you want to buy. You’ve managed to scrape together $50,000 for the down payment (savings, your parents, . . . ) but will have to take out a mortgage for the $200,000 balance. Can you afford it? There are many websites that provide a place to start. Figure 10.4 is a screen shot from one. It shows that on August 6, 2012, in Denver, Colorado you could get a 30 year fixed rate mortgage at 3.5% annual interest with a monthly payment of $898.09 or a 15 year fixed rate mortgage at 2.5% with a monthly payment of $1,333.58. In this section we’ll look at what those numbers mean, see how they are calculated, and discuss a few important issues (some quantitative, some not) that you should think about when making a decision like this one.

Paying off a mortgage is like paying off a credit card balance when you make no new purchases. There’s an annual rate. Your balance at the end of a month includes interest computed at one twelfth of the annual rate. Each month you pay all the current interest and some of the principal. Since the principal is decreasing, there’s less interest each month so more of the payment goes toward the principal. One difference is that the credit card company sets the minimum payment; then it takes as long as it takes to pay off the balance, while the mortgage payment is figured out in advance so that everything is paid off at a particular time - usually 15 or 30 years.

The mortgage company uses this complicated formula to calculate the monthly payment:

\[ P \times \frac{r/12}{1 - (1 + \frac{r}{12})^{-12y}} \]  

\[ (10.1) \]

\( ^3 \)Federal legislation passed in response to the 2009 financial crisis forced credit card companies to change their policies so that “payments above the Minimum Payment due will be applied first to higher interest rate balances.” That notification appeared on one of the authors’ bills, along with the kind thought that “This may help you to pay off your highest interest rate balances more quickly and reduce your interest charges.” They did not reveal how much money they spent lobbying in Washington against the regulation.

\( ^4 \)These interest rates are near or at historical lows. When you read this they will probably be higher. For some history, see http://mortgage-x.com/trends.htm

And remember that these monthly payment numbers do not include other (required) fees for insurance and real estate taxes.
where $P$ is the principal (the amount of your mortgage), $r$ is the annual interest rate, and $y$ is the length of the mortgage, in years.

This is probably the most complicated formula in *Common Sense Mathematics*. We won’t explain where it comes from, and you need not memorize it. But you can understand some parts of it. It has the form

$$P \times (\text{complex expression involving } r \text{ and } y)$$

which tells you that your monthly payment is proportional to $P$. The complex part is the expression in parentheses – the *effective monthly rate*. That’s the number of dollars in your payment for each dollar you borrow. There the $r/12$ finds the monthly rate from the annual rate. The product 12$y$ is the number of months in $y$ years.

You can use that formula to check that the web site illustrated in Figure 10.4 has the right minimum payment of $898.09 for a 30 year $200,000 loan at 3.5% interest. We did the arithmetic in Excel, with the formula

$$(\text{STARTBALANCE}\times\text{INTERESTRATE}/12)/(1-(1+\text{INTERESTRATE}/12)^{(-12\times\text{YEARS})})$$

in cell C11 on the *mortgage* worksheet in [http://www.cs.umb.edu/~eb/qrbook/PayoffDebt/PayOffDebt.xlsx](http://www.cs.umb.edu/~eb/qrbook/PayoffDebt/PayOffDebt.xlsx). There you can see the principal balance at the end of each year and the total interest paid. On that 30 year mortgage it’s a little more than $123,000.

When you borrow you always pay back more than the amount you borrowed. In this case, lots more. Should that frighten you? Maybe or maybe not. Is it worth it? Perhaps, for several reasons.

- It would take a long time to save up the full purchase price (to avoid borrowing). Saving would be difficult because you would be paying rent the whole time. So you can think of the mortgage payments as money spent instead of rent.
10.3. SAVING FOR COLLEGE OR RETIREMENT

- The condo may well be worth more after 15 or 30 years than the total you paid for it – even including the interest on the mortgage.

- Inflation is pretty nearly inevitable over the years. These computations are all made in dollars computed in the year you make the purchase, but the actual value of that money when you pay it to the bank in later years will be less, in then current dollars. Think of it this way: your salary is likely to increase at least as fast as inflation, so the fixed monthly mortgage payments will be a smaller and smaller percentage of your take home pay.

- That said, you do want to minimize the amount of interest you pay, by paying attention to the significant difference between a 15 and a 30 year mortgage. The short one has a lower interest rate (2.5% instead of 3.5%) and a much lower total interest cost: about $40,000 instead of $123,000. So you should choose it if you can manage the extra $430 per month in payments.

- You will also get a lower rate if you have established a good credit rating in the years before you apply for your mortgage. So start now to use a credit card wisely.

Words of warning. The purpose of this discussion is to show you how, in principle, you pay off a loan by paying some interest and some principal\(^5\) periodically. That’s just one of the financial things you’ll need to understand when you think about buying a house or condo. Just asking the bank or shopping online for an interest rate isn’t sufficient. As with most other topics in this book (or in this course) we are giving you a foundation to ask further questions using your common sense.

- There are other up front costs: legal fees, title searches, inspections, points.

- The cost of owning is more than just the cost of the mortgage. You must be prepared for expenses that your landlord would cover if you were renting – things like real estate tax, insurance, repairs.

- Variable rate mortgages generally start out with lower rates than fixed rate mortgages – but payments can balloon when the initial rate expires.

There are many books and web pages that may help – here’s just one we found with a simple search:  
http://www.ourfamilyplace.com/homebuyer/checklist.html

10.3 Saving for college or retirement

In Chapter\(^9\) we studied how money accumulates when you invest a big chunk and let the interest compound. But you rarely save a big chunk of money all at once. A more realistic way to save, for college (for your children) or for retirement is to put away a fixed amount on a regular basis.

The kind of calculations we made above to study paying off a debt will help now to study how money saved regularly accumulates. Suppose you can invest $1200 a year and make your payment at the end of the year\(^6\). You think you can get a 6% return on your investment, since you’re willing to take some short term risk for the sake of long term return. At the end of the second year you will have

\[
$1200 \times 1.06 +$1200 = $2472
\]

\(^{5}\)This is a good place to learn the difference between “principle” and “principal”.

\(^{6}\)We’ll see soon why it’s better do deposit $100 every month instead.

© 2014 Ethan Bolker, Maura Mast 238
and after the third

\[\$2472 \times 1.06 + \$1200 = \$3820.32.\]

These calculations look just like the ones we made for paying down credit card debt, except that in this case we *add* the periodic payment to the balance rather than subtracting it. That means we can use the debt payment spreadsheet to see how our money accumulates by entering a *negative* “payment” to be added to the growing balance. Figure 10.5 shows the result (we haven’t changed the labels).

![Figure 10.5: Saving for Retirement](image)

In 25 years you will have accumulated nearly $66,000. Of that amount, you contributed just $1,200 \times 25 = $30,000. The rest, more than $35,000, is interest.\footnote{Making precise sense of the total accumulation in cell C40 and the total interest in cell D41 is a little tricky. You have to think carefully about whether these values are computed at the beginning or the end of the year, before or after interest is credited. You don’t need to do this careful analysis to understand the principle.}

### 10.4 Annual and effective percentage rates

We saw in Chapter 9 that compounding is a good thing for your investments. When you borrow money it’s a good thing for the credit card company. Suppose in the example in Section 10.1 John Doe carried a balance of $100 for a year, making no payments. Recall that the monthly interest rate is 1.65%, one twelfth of the 19.80% APR. At the end of the year interest will have been compounded twelve times and he would owe

\[\$100 \times 1.0165^{12} = \$121.70,\]

which corresponds to an *effective annual rate* (EAR) of 21.7% – almost 2 full percentage points higher than the already outrageous advertised APR of 19.8%.

The effective rate works *for you* rather than against you when you invest rather than borrow. If you make monthly deposits to your retirement account earning 6% annual interest the computation

\[
\left(1 + \frac{0.06}{12}\right)^{12} = 0.0616778119 \approx 1.0617. \tag{10.2}
\]
10.5. INSTANTANEOUS COMPOUNDING

...tells you your effective interest rate is about 6.17%.

In the example in Section 10.2 Figure 10.4 says that the 30 year 3.5% mortgage has an APR of 3.663%. Since payments are made monthly, we can check

\[
\left( 1 + \frac{0.035}{12} \right)^{12} = 1.03557
\]

which is an effective interest rate of 3.557%. To begin to investigate the discrepancy you have to read the small print. On February 13, 2011 The New York Times reported in an article headlined Calculating the Annual Percentage Rate that

THE lending industry has tried to make it easier for borrowers to understand the true cost of a mortgage by disclosing both its interest rate and its annual percentage rate, or A.P.R. But consumers may often wonder which figure they should focus on when buying or refinancing a property.

The answer, many mortgage experts say, may seem counterintuitive: while the A.P.R. is popularly seen as providing a more complete picture of what you are actually paying each month, it often omits some costs.

In the example in Section 10.2 the bank is trying to be helpful. Clicking on the APR link displays the window in Figure 10.6 which explains the meaning of the displayed APR.

![Figure 10.6: The small print](https://www.wellsfargo.com/homebody/mortgage/glossary)

10.5 Instantaneous compounding

Many qr courses spend a lot of time working with Equation 10.2. That may not be time well spent. What is important is understanding the idea of frequent compounding. But if you’re curious and adventurous, read the rest of this section.

---

8 It’s not a coincidence that the left side of this equation matches part of the complicated Formula 10.1.

If monthly compounding is good then daily compounding must be even better. To see what six percent annual interest compounded daily leads to, compute

\[
\left( 1 + \frac{0.06}{365} \right)^{365} = 1.06183131.
\]

That corresponds to an effective annual rate of \(6.183\%\). *Hourly* compounding gives

\[
\left( 1 + \frac{0.06}{8765} \right)^{8765} = 1.06183633
\]

which is a just a tiny bit better. Compounding *every minute* results in \(1.06183654\), which differs just in the seventh decimal place. These computations suggest that as you compound more and more often you do better and better, but by less and less. There seems to be a limit. In fact there is. You can find it with the magic number \(e\) and the Excel function \text{EXP} we discussed in Section 9.5. If you compound 6\% annual interest *every instant* the computation

\[
e^{0.06} = \text{EXP}(0.06) = 1.061836547
\]

tells you the effective interest rate to nine decimal places.

To compare monthly and instantaneous compounding in terms that are easier to understand, suppose you invested a thousand dollars. Then the Bing calculator tells you

\[
1000 \times \left( (e^{0.06}) - (\left(1 + \frac{0.06}{12}\right)^{12}) \right) = 0.158734681
\]

so the difference is about 16 cents. That’s chump change for a thousand dollar investment.

Finally, suppose you could find someone to pay you 100\% interest annually. Then without compounding, one dollar would double, and become two. If you compounded instantaneously your dollar would turn into \(e = \text{EXP}(1) = 2.72\) dollars in a year. \(^{10}\)

### 10.6 Exercises

**Exercise 10.6.1.** [W][Section 10.1] [Goal 10.1] Your credit report

The Fair Credit Reporting Act (FCRA) requires each of the nationwide credit reporting companies Equifax, Experian, and TransUnion to provide you with a free copy of your credit report, at your request, once every 12 months. The FCRA promotes the accuracy and privacy of information in the files of the nation’s credit reporting companies. The Federal Trade Commission (FTC), the nation’s consumer protection agency, enforces the FCRA with respect to credit reporting companies. \(^{11}\)

To get your reports, visit [www.annualcreditreport.com/index.action](http://www.annualcreditreport.com/index.action).

That’s not a clickable link. You have to type it into your browser. Here’s why, from [www.annualcreditreport.com/aboutThisSite.action](http://www.annualcreditreport.com/aboutThisSite.action):

\(^{10}\)See comment in instructor’s manual.

\(^{11}\) [http://www.consumer.ftc.gov/articles/0155-free-credit-reports](http://www.consumer.ftc.gov/articles/0155-free-credit-reports)
AnnualCreditReport.com is the official site to get your free annual credit reports. This right is guaranteed by Federal law. To verify that this is the official site, visit www.consumerfinance.gov/askcfpb/311/how-do-i-get-a-copy-of-my-credit-report.html. Don’t be fooled by look-alike sites. You can be sure that you are on the right site if you type www.annualcreditreport.com in your browser address line. Don’t come to this site by clicking on a link in another site or in an email.

Now you have your credit report. That’s not the same as credit score.

Based on the information in your credit report, lenders calculate your credit score so they can assess the risk you pose to them before they decide whether they will give you credit. The higher your score, the less risk you pose to creditors.

The information in your credit report is used to calculate your FICO (the acronym stands for Fair, Isaac and Company) score. Your score can range anywhere from 300-850. Aiming for a score in the 700s will put you in good standing. A high score, for example, makes it easier for you to obtain a loan, rent an apartment, or lower your insurance rate. 

Look for a place on the web that will give you an estimate of your credit score. Wherever you go, be sure to read the fine print, and don’t pay for anything.

**Exercise 10.6.2.** [S][Section 10.1] [Goal 10.1] How long to pay it off?

Starting in 2010 credit card companies were required to provide the information in Table 10.7 each month. The numbers there are for a bill with a balance of $2,020.37, a minimum payment amount of $40.00 and an annual percentage rate of 12.24%.

<table>
<thead>
<tr>
<th>If you make no additional charges using this card and each month you pay...</th>
<th>You will pay off the balance shown on this statement in about...</th>
<th>And you will end up paying an estimated total of...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Only the minimum payment</td>
<td>18 years</td>
<td>$3,843</td>
</tr>
<tr>
<td>$67</td>
<td>3 years</td>
<td>$2,426 (Savings=$1,417)</td>
</tr>
</tbody>
</table>

Table 10.7: Paying off a credit card balance

If you use the [http://www.cs.umb.edu/~eb/qrbook/PayoffDebt/PayOffDebt.xlsx](http://www.cs.umb.edu/~eb/qrbook/PayoffDebt/PayOffDebt.xlsx) spreadsheet to answer the following questions you will need the mortgage worksheet, since the plain worksheet only covers 25 payment periods.

(a) Verify the three year time to pay off the balance at a rate of $67 per month.

(b) Show that a constant monthly payment of $40 is much more than is needed to pay off the balance in 18 years. How can the 18 year claim be correct?

---

(c) 2014 Ethan Bolker, Maura Mast 242
The 2010 Consumer Credit Law allows banks to raise the minimum payment on an account to a constant amount sufficient to pay off the balance in five years. What would that minimum payment be for this bill?

**Exercise 10.6.3.** [Section 10.1] [Goal 10.1] Average household credit card debt.

On May 5, 2011 the Boston Metro (a free daily paper available at subway stops) carried an advertorial that read (in part)

> Every three minutes another person falls behind on credit card debt. . . . [T]he growing mountain of consumer debt [now] stands at $2.17 trillion.

along with the graph in Figure 10.8.

![Figure 10.8: Household Credit Card Debt History](http://chartingtheeconomy.com/?p=1174=1)

(a) What’s an advertorial?

(b) Is the claim that “every three minutes another person falls behind . . .” reasonable?

(c) Is the $2.17 trillion figure in the quotation consistent with the numbers in the graph?

(d) Which average household credit card debt is plotted on the vertical axis – the mean, the median or the mode? (Explain why you think so.)

(e) The data points since 1989 show approximately linear growth. Estimate the slope of the regression line (trendline), in dollars per household per year. (If you had the numbers in a spreadsheet you could ask Excel for that slope, but you can get a reasonable estimate from the picture.)

(f) Use your answer to the previous part of the problem to predict average household credit card debt for the years 2011 and 2019. How reliable do you think your predictions are?

**Exercise 10.6.4.** [Section 10.2] [Goal 10.3] Build your own mortgage.

Redo the computations in Section 10.2 for a house or condo of your choice in your town. Start with a reasonable cost and down payment. Find rates from at least two separate on line sites; check them with the formula and the spreadsheet.

---

13That graph comes from [http://chartingtheeconomy.com/?p=1174=1](http://chartingtheeconomy.com/?p=1174=1) where the U.S. Federal Reserve is indicated as the data source. The actual Metro article used a redrawing of the same graph.
10.6. EXERCISES

Exercise 10.6.5. [U][Section 10.2] [Goal 10.3] [Goal 10.2] The debt payoff spreadsheet can reproduce some of the computations from the exponential growth spreadsheet we introduced in Chapter 9. Test that by setting the monthly payment to 0 and the annual interest rate to 12 times the growth rate you want to study.

In particular, what happens if the annual growth rate is 1200%, the starting balance is 1, and the monthly payment is 0?

Exercise 10.6.6. [U][Section 10.2] [Goal 10.2] Use the debt payoff spreadsheet to examine your student loans.

Exercise 10.6.7. [R][S][Section 10.2] [Goal 10.2] [Goal 10.3] Rates for big loans tumble

The Boston Globe headlined that story on November 20, 2010. The text says

Over the past year, the average interest rate for so-called jumbo loans – $523,750 and up in the Boston area – has fallen from 6 percent to about 5 percent for a 30-year, fixed-rate mortgage. That translates into a monthly savings of about $375 on a $600,000 loan. 14

(a) What monthly payment will retire the loan when the interest rate is 6%?
(b) What monthly payment will retire the loan when the interest rate is 5%?
(c) Is the newspaper’s claim of a $375 monthly saving correct?

Exercise 10.6.8. [R][S][Section 10.2] [Goal 10.2] [Goal 10.3] [Goal 10.4] Mortgages in the news.

March 4, 2011.

From The New York Times article headlined Without Loan Giants, 30-Year Mortgage May Fade Away:

A person who borrows $100,000 at 6 percent interest will pay $600 each month for 30 years, compared to $716 each month for 20 years. 15

From The Boston Globe article headlined The end of 30-year fixed-rate mortgage?

The difference between a 15- and 30-year mortgage amounts to well over $600 per month on a $300,000 loan, a substantial amount that may prevent wide swaths of the middle class from buying homes. 16

Verify the calculations in each of these quotations.

Exercise 10.6.9. [U][W][Section 10.3] [Goal 10.5] [Goal 10.4] Retirement planning.

Find an online retirement income calculator. Use it with data you imagine for yourself. Write down what you do as you proceed. (Screen shots would be nice.) Record what it tells you at the end.

Their calculator is much more sophisticated than the simple one in Excel we introduced in this chapter. See if you can use ours to get answers that match what it told you.

16 http://www.boston.com/bostonglobe/editorial_opinion/oped/articles/2011/03/04/the_end_of_30_year_fixed_rate_mortgage/
Exercise 10.6.10. [S][Section 10.4] [Goal 10.4] Payday loans

The Boston Globe on New Year’s Day 2009 reported that

New Hampshire is giving payday lenders the gong as it rings in the new year. A law that takes effect today caps the interest rate on small loans at 36 percent a year, which the industry has said will put it out of business.

Payday lenders typically charge $20 per $100 for two-week loans backed by the borrower’s car title or next paycheck. That amounts to 1.43 percent interest per day, an annual rate of 521 percent.

The cap translates to a daily interest rate of about 0.1 percent, or total interest charges of $1.38 – a dime a day – on a $100, two-week loan.  

(a) What is a “payday loan”?

(b) Verify the computation that 1.43% interest per day is 521% interest annually.

(c) If the 1.43% daily interest is compounded daily then the true annual rate of interest is in fact much more than 521%. How much is it?

[See the back of the book for a hint.]

(d) Verify that paying interest of $1.38 on a two week loan of $100 is just about a “dime a day” and corresponds to a daily interest rate of about a tenth of a percent. What annual rate does that represent?

(e) Visit a payday loan website and report on what you discover there about interest rates.

Exercise 10.6.11. [S][C][Section 10.4] [Goal 10.5] [Goal 4.2] Charity sues R.I. hospital over donation in 1912

That headline accompanied an article in the City and Region section of The Boston Globe on February 23, 2008. The accompanying article describes a gift intended to provide a free bed in perpetuity for needy patients.

There you can read that

Mark E. Swirbalus, a Boston lawyer representing Children’s Friend, said that “as far as we know, the hospital never set aside a bed and never set aside the money.” The $4,000, if conservatively invested by the hospital in 1912, would be worth about $1.5 million today, he said.

(a) Is Swirbalus’ claim about a “conservative investment” correct?

[See the back of the book for a hint.]

---

17 http://www.boston.com/business/articles/2009/01/01/nh_caps_rates_on_payday_loans/
(b) What the hospital should have done was invest the money and use just the interest each year to fund the bed. That would work – if only there were no inflation that made the cost of the bed increase.

Suppose they got a 6.5% percent yearly return on investment and annual inflation was 3.5%. Explain why they would be able to spend about $120 on the bed in 1912, and could keep spending at that rate as the years went on.

(c) If the cost of providing a hospital bed in 1912 was $120, what would it be in 2008 if all you had to do was adjust for inflation?

Exercise 10.6.12. [S][Section 10.2] [Goal 10.3] Half the time, more than double the benefit.

Show that taking out a 15 year mortgage instead of a 30 year mortgage (for the same loan amount at the same annual rate) doesn’t double your monthly payment and more than halves the total interest you pay on your loan.

(The advantages are usually even greater since you can usually negotiate a lower interest rate for a shorter mortgage.)

Exercise 10.6.13. [S][Section 10.3] [Goal 10.5] Saving $50,000.

(a) Use the spreadsheet http://www.cs.umb.edu/~eb/qrbook/PayoffDebt/PayOffDebt.xlsx to figure out how much you’d have to save per year at your account at the end of 25 years.

(b) Then use the mortgage tab in that spreadsheet to answer the same question if you save a fixed amount each month rather than each year.


The Nilson Report

Spending for goods and services in 2013 by general purpose-type credit, debit, and prepaid cards issued in the United States reached $4.530 trillion. Market share is shown for credit, debit, and prepaid.
1. Credit Purchase Volume $2.399 Tril.
2. Debit Purchase Volume $1.949 Tril.
3. Prepaid Purchase Volume $0.182 Tril.

The Boston Globe reported that “merchants in the United States spent $71.7 billion on fees [for these transaction] last year.”

(a) What is the average merchant fee, as a percentage?

(b) Make sense of the $4.530 trillion total: think about it in units like dollars per person per day, dollars per transaction, ...

Exercise 10.6.15. [U] Excel templates from the internet


© 2014 Ethan Bolker, Maura Mast 246
(a) Do the calculations there match those in http://www.cs.umb.edu/~eb/qrbook/PayoffDebt/PayOffDebt.xlsx?

(b) Compare the formulas in that template with those in http://www.cs.umb.edu/~eb/qrbook/PayoffDebt/PayOffDebt.xlsx.

Review exercises

Sample routine review questions. When Common Sense Mathematics is published these and others may be available in an online homework system/