Chapter 3

Percentages, Sales Tax and Discounts

The focus of this chapter is the study of relative change, often expressed as a percent. We augment an often much needed review two ways – stressing quick paperless estimation for approximate answers and, for precision, a new technique: multiplying by 1+(percent change).

Chapter goals:

Goal 3.1. Understand absolute and relative change.

Goal 3.2. Work with relative change expressed as a percentage.

Goal 3.3. Master strategies for deciding how to arrange percentage calculations.

Goal 3.4. Learn (and appreciate) the “1+” trick for computing with percentages.

Goal 3.5. Calculate successive percentage changes.

Goal 3.6. Calculate percentage discounts.

Goal 3.7. Understand the difference between percentages and percentage points.

3.1 Don’t drive and text

On January 13, 2010 an article in The Washington Post said in part

Twenty-eight percent of all traffic accidents are caused when people talk on cellphones or send text messages while driving, according to a study released yesterday by the National Safety Council. The vast majority of those crashes – 1.4 million of them - are caused by cellphone conversations, while 200,000 are blamed on text messaging, the council report said. [1]

This quote suggests several questions. First, is the statement “the vast majority ...” reasonable? Second, and more interesting, given this information, can we estimate how many accidents there were altogether? Finally, can we check that estimate?

3.1. DON’T DRIVE AND TEXT

There are three numbers to work with: 28% of all accidents, 1.4 million cellphone accidents and 200,000 texting accidents.\(^2\)

To answer the first question we have to compare the number of crashes caused by cellphone conversations to the total number of “those crashes,” not to the number of “all traffic accidents”.

Since the first paragraph reports on crashes caused by cellphone talk or text messaging, we can find the total we need by adding up the number of each kind: 1.4 million and 200,000 respectively. So there 1.6 million of “those accidents”.

The fraction of the total due to cell phone conversations is \[
\frac{1.4 \text{ million}}{1.6 \text{ million}} = \frac{1.4}{1.6} = \frac{7}{8},
\]
which certainly qualifies as “the vast majority.” The convenient round numbers meant we didn’t need a calculator to find the answer.

To report the answer as a percentage, remember that \(\frac{3}{4}\) is 75\%. Since \(\frac{7}{8}\) is halfway from \(\frac{3}{4}\) to 1, it’s halfway from 75\% to 100\%, so it’s 87.5\% ≈ 85\%.

By the summer of 2013, 41 states in the U.S. banned texting while driving. These statistics suggest that a complete cell phone ban would prevent many more crashes.

The second question asks for the number of “all traffic accidents.” We’re told that 28\% of that unknown number is 1.6 million. To find the answer we need to do something with 28\% and 1.6 million. Multiply? Divide? Try to guess which? Guessing is not a good idea – you’ll be right only half the time. Thinking is better. Start by writing the equation in words. Then you can figure out whether to multiply or divide.

\[
\frac{\text{cellphone or texting accidents}}{\text{all accidents}} = \frac{1.6 \text{ million}}{\text{all accidents}} = 28\% = 0.28.
\]

That means

\[
1.6 \text{ million} = 0.28 \times \text{all accidents}
\]

so there were about

\[
\frac{1.6 \text{ million}}{0.28} \approx 5.7 \text{ million}
\]

accidents of all kinds.

We can check that answer with a ballpark estimate. Since 28\% is a just about 25\%, which is one fourth, the quotation says that just about one accident in four was caused by texting. Then

\[4 \times 1.6 \text{ million} = 6.4 \text{ million}.
\]

That’s a little larger than our precise answer, but in the right ballpark.

Does this answer make sense? Treat it as a Fermi problem. There are about 300 million people in the United States so 5.7 ≈ 6 million accidents means that about 2\% of the population, or one person out of every 50 was involved in one. If you have 50 acquaintances, on average one of them will have had an accident. Does that number seem to you to be in the right ballpark? You can confirm it with a web search – or work Exercise 2.9.29 (page 52).

This story has two morals. First, percentages are a good way to think usefully about numbers. Second, don’t use your cell phone (at all) while driving, and don’t let the driver do it either when you’re a passenger.

\(^2\)See comment in instructor’s manual.

\(^3\)Just read it aloud and think money: “three quarters is seventy-five [per]cent[s].”
3.2 Red Sox ticket prices

The front page of the *The Boston Globe* on September 9, 2008 provided the information on Red Sox ticket prices shown in Figure 3.1.

![Tickets at Fenway Park](image)

Figure 3.1: Red Sox Ticket Prices

Which ticket price increased the most?

The natural way to compare two numbers is to subtract. The *absolute* increases in ticket prices were $52.16 - $42.26 = $9.90 for ordinary seats and $325 - $275 = $50 for the most expensive seats. So the price of the most expensive seats increased much more in absolute terms.

But you often learn more when you *compare a change to its starting value*. A calculator will tell you that the *relative* change in the cost of an ordinary seat was

\[
\frac{9.90}{42.26} = 0.23426408 \approx 0.23 = 23\%.
\]

(3.1)

Most of the digits in the exact answer make no sense when talking about the ticket price, so we rounded that answer to two decimal places: 0.23. That’s 23/100, which is 23 percent in everyday language.

This answer makes sense. 23% is nearly 25%, which is 1/4. The $9.90 increase in the ticket price is indeed just a little less than one fourth of the original $42.26.

For the most expensive seats the relative change was $50/$275 \(\approx 0.18\), or about 18% – so the *percent change* for premium tickets was *less* than that for ordinary seats even though the *dollar change* was greater.

3.3 Patterns in percentage calculations

At the risk of encouraging you to mark parts of this text with a highlighter instead of thinking about and absorbing what they have to say, we’ll summarize the idea behind our study of Red Sox ticket price increases. Then we will show how to work with the same numbers from different points of view.

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4 Percentages are just fractions with 100 in the denominator. We use them instead of the fractions they stand for because fractions make people uncomfortable. It’s easier to think about the number 23 than the number 0.23.
3.3. PATTERNS IN PERCENTAGE CALCULATIONS

- Subtraction is the obvious way to compare two numbers. The difference tells you the *absolute change*.
- Often division is more informative, even though it’s less natural. It tells you the *relative change*. Relative change is frequently reported as a percentage.

\[
\text{absolute change} = \text{new value} - \text{reference value}
\]

\[
\text{relative change} = \frac{\text{absolute change}}{\text{reference value}} = \frac{\text{new value} - \text{reference value}}{\text{reference value}}
\]

and

\[
\text{relative percent change} = \text{relative change} \times 100 = \frac{\text{new value} - \text{reference value}}{\text{reference value}} \times 100
\]

When thinking about the price change for nonpremium Red Sox tickets there are four numbers that matter:

- The original value: $42.26. (Sometimes this is called the old value, or the reference value).
- The new value: $52.16.
- The absolute change: $9.90.
- The relative change: 0.23, or, written as a percent, 23%.

If you know any two of those four numbers you can find the other two. The job when reading a paragraph is to understand which two are given, and which others you might want to know.

So far we’ve seen how to find the absolute and relative changes if you know the original and new values. Those are the easiest problems.

It’s also easy to find the new value when you know the original value and the absolute change. Just add.

Suppose you’re told that tickets now sell for $52.16 and that they’ve gone up by $9.90 and you want to know what they used to cost. Then you have the new value and the absolute change, so subtract to find the original value $42.06⁵.

Now suppose instead that you you know the original value and the relative change ($42.06 and 23% in this example). It’s a little harder (but still straightforward) to find the new value. First find the absolute change, which is 23% of the original value:

\[0.23 \times 42.06 = 9.6738 \approx 9.67.\]

Then add:

\[42.06 + 9.67 = 51.73.\]

That’s not the known correct answer of $52.16 since the 23% we’re working with was rounded from the true percentage.

The hardest question is the one that asks you to find the original value when you know the new value and the relative change (as a percent). Here’s a made up version of the problem we’re working on that calls for that.

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⁵You can do the subtraction in your head if you notice that $9.90 is just a dime less than $10.00.
Prices for plain old ordinary Red Sox ticket have increased a whopping 23% to $52.16. That’s outrageous! We should go back to the good old days.

What did tickets cost in the good old days, before the increase? Reading the first sentence tells you the meaning of the two numbers in it. The 23% is the relative change; the $52.16 is the new value. We’d like to know the original value before the change.

Many people start by finding 23% of $52.16, hoping that will tell them the absolute increase, so they can subtract. The numbers tell us that’s not right: $0.23 \times $52.16 = $11.9968, which is $12, but we know the absolute increase is about $10!

You can’t undo a 23% increase with a 23% decrease!

Remember that every time you’re tempted. Now read on to find out a correct way to solve the problem.

3.4 The one-plus trick

We found the relative change in Red Sox ticket prices by subtracting to find the absolute change ($9.90), then dividing by the reference value ($9.90/$42.16). Here’s a another way – just divide:

\[
\frac{\text{new value}}{\text{reference value}} = \frac{$52.16}{\$42.26} = 1.23. \tag{3.2}
\]

The “.23” in “1.23” is the 23% relative increase. The “1” is 100%; it represents the original ticket price.

To understand Equation 3.2 better, we can rewrite it without fractions.

\[
$52.16 = 1.23 \times $42.26 = (1.00 + 0.23) \times $42.26 = 1.00 \times $42.26 + 0.23 \times $42.26 = \text{original value + absolute change}
\]

Note that the absolute change in ticket prices is a number of dollars while the relative change has no units: the dollars in Equation (3.2) cancel, leaving the 1.23 as a naked number.

We’ve now seen two ways to find the relative change. The straightforward way finds the absolute change first (by subtracting), then dividing by the original value. The new way divides the new value by the original value and then “throws away the 1”.

Which way is better? That depends. The first way is familiar, and easy to understand. Those are good reasons to rely on it. The second way is new, and confusing. Those are good reasons to reject it. But in this case there is are several reasons to take the time to learn the new way, which we call the “one plus trick.” When you get used to it you will have a technique that simplifies many percentage calculations.\[^6\]  

\[^6\]See comment in instructor’s manual.

\[^7\] The first time you see something like this it’s a trick. After a while it becomes a technique, or, in the mathematician and teacher George Pólya’s words, “What is the difference between method and device? A method is a device which you used twice.”

\[^8\]See comment in instructor’s manual.

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First, the using the one-plus trick to find the relative change when you start with the old and new values is faster than finding the absolute change first: one division, rather than a subtraction and a division. That saves time, and allows fewer places to make a mistake.

Second the trick points to an easy way to answer the question we left hanging at the end of the last section: if ticket prices increase by 23% to a (new) value of $52.16, what were they before? The question tells us that

\[ \text{new value} = 52.16 = 1.23 \times \text{original value} \]

so

\[ \text{original value} = \frac{52.16}{1.23} = 42.41. \]

This is pennies off the correct original value of $42.26 because the 23% is a rounded percentage increase.

Here’s another problem where we’re told the result of a percentage increase and want to find the starting value. Suppose the enrollment in your school has increased 25 percent over the past few years to 15,000 students.

What was the enrollment before the increase?

The ‘1+’ trick tells us that

\[ 1.25 \times \text{enrollment before increase} = 15,000 \text{ students}. \]

Dividing both sides of this equation by 1.25:

\[ \text{enrollment before increase} = \frac{15,000 \text{ students}}{1.25} = 12,000 \text{ students}. \]

We can check this answer. The absolute increase from 12,000 to 15,000 is 3,000, which is in fact 25% of 12,000.

If you try to the problem without the trick your intuition may lead you astray. You might be tempted to compute the enrollment before the 25% increase by finding 75% of the new 15,000. But 3,000 is only 20% of the new 15,000 enrollment. That means the old enrollment was 80% of the new, not 75%.

The 25 percent increase from the old to the new value corresponds to a 20 percent decrease from the new to the old – and you can know that without knowing either the new value (15,000 students) or the old (12,000 students).

\[ \frac{1}{1.25} = 0.80 = 1 - 0.20. \]

To fix this kind of calculation firmly in mind, imagine an extreme case. If some quantity increases by 100% it has doubled. To get it back to where it started, you don’t decrease it by 100% (which would make it vanish). You cut it in half. That’s a 50% decrease.

### 3.5 Two raises in a row

Lucky you. You’ve just received a 20% salary increase. Last year your raise was 10%. How far ahead of the game are you now?
Almost everyone’s first guess is 30% – they add the two percentage increases. We’ll use the 1+ trick to find the correct answer and see why the 30% guess is wrong.

If working with naked percentages makes you uncomfortable, here’s a general strategy that may help you through this kind of problem. Imagine a particular starting salary and work with that actual number to find out what happens. Suppose that two years ago you were making $100K. Then after the first raise the 1+ trick tells you your salary was

\[ 1.10 \times 100K = 110K. \]

Then the second raise kicks in, and you’re earning

\[ 1.20 \times 110K = 131K. \]

Wow! That’s a 31% increase overall – even better than the first (wrong) guess of 30%. That’s because the second raise was applied to your salary after the first raise took effect. Here’s how to combine the calculations:

\[
\text{new salary} = 1.20 \times (1.10 \times 100K) \\
= 1.20 \times 110K \\
= 131K.
\]

Even better, we can do the multiplication in the other order:

\[
\text{new salary} = 1.20 \times (1.10 \times \text{old salary}) \\
=(1.20 \times 1.10) \times \text{old salary} \\
= 1.31 \times \text{old salary}.
\]

Then the 1+ trick tells us the combined increase is 31%, and we didn’t even have to imagine actual dollar values to work with.

The 1+ trick tells you how to combine percentage increases. To work this kind of problem correctly you have to remember first what not to do:

\[ \text{Don’t combine percentage increases by adding them! Use the 1+ trick and multiply.} \]

### 3.6 Sales tax

You’ve decided on a new $149.99 cell phone. In Massachusetts (in 2011) the sales tax was 6.25%. What will you pay for the phone?

First a quick approximation. That $149.99 is really $150. The sales tax used to be 5%, not 6.25%. Then the approximation was easy. Five percent is half of ten percent – half of $15 is $7.50. Estimating the new tax is a little harder. 6.25% is more than half of 10%, so the tax will be more than half of $15. Call that about $9 and the phone should cost about $159.

You can find the cost exactly with the 1+ trick and your calculator. Recall that “percent” means “divide by 100” – because “per” means “for each” (or often, “divide by”) and “cent” is the Latin root for “hundred”. So 6.25% is \[ \frac{6.25}{100} = 0.0625. \] The total cost is 6.25% more than the list price of $149.99 so

\[ \text{total cost} = 1.0625 \times 149.99 = 159.364375 \approx 159.36. \]

\[ ^9 \text{Since it’s your salary, why not imagine a nice large one that’s easy to work with.} \]

\[ ^{10} \text{The leading zero in 0.0625 doesn’t change the value of the number. Writing .0625 would be just as good. But the zero before the decimal point helps the reader. Without it you might not see the decimal point. We will always use it. You should learn to.} \]
### 3.7 20 percent off

Suppose you have a newspaper coupon that offers you a 20% discount on that $149.99 phone. What is the discounted cost?

The mental arithmetic is easy: you save twice 10%, which is $30, so you will pay only $119.99.

With a calculator you can check the answer and see if the pennies change:

\[
149.99 \times 0.20 = 29.998;
\]

then

\[
149.99 - 29.998 = 119.99200 \approx 119.99.
\]

They don’t.

With a version of the 1+ trick you can do this computation in one step without computing the discount first and subtracting. After a 20% discount, you will pay 80% of the sticker price. Estimating mentally, 10% is $15 so the phone will cost \(8 \times 15 = 4 \times 30 = 120\) dollars.

Since \(100\% - 20\% = 80\%\) is the same as \(1 - 0.2 = 0.8\), this is really the “1+” trick in disguise. The “1” is invisible because the percent change is a decrease, so subtracted rather than added.

\[
\text{discounted price} = (1.00 - 0.20) \times \text{list price}
\]

\[
= 0.80 \times 149.99
\]

\[
= 119.99.
\]

With sales tax, your phone will cost

\[
1.0625 \times 0.80 \times 149.99 = 127.49.
\]

### 3.8 Successive discounts

Because you also get your cable TV and internet access from the telephone company, you have a second coupon that gives you an additional 30% off on your new cell phone. How should you calculate the result of combining your 20% and 30% discounts?

Don’t succumb to temptation. It’s easy to add the percentages and think you’ll get the phone at half price (50% off). You won’t, because the second discount applies only to the reduced cost from the first discount, so the final cost will be 70% of (80% of $149.99):

\[
0.70 \times (0.80 \times 149.99) = 0.70 \times 119.99 = 83.994 \approx 84.99
\]

(3.3)

so you will pay about $85. That’s more than half the sticker price. Your total discount is $149.99 – $83.99 = $65, so the savings as a percent (in other words, the effective percentage discount) is

\[
\frac{65}{150} = 0.44 \text{ or } 44\%.
\]

We could have discovered that right away by rearranging the multiplication in Equation 3.3

\[
0.70 \times (0.80 \times 149.99) = (0.70 \times 0.80) \times 149.99 = 0.56 \times 149.99.
\]
You pay 56% of the list price so the discount is $1.00 - 0.56 = 0.44 = 44\%$. You don’t have to compute the final cost to know that.

Would the answer be different if we took the 30% discount first, then the 20% discount? No, because $0.70 \times 0.80 = 0.80 \times 0.70$. The order of the discounts doesn’t matter.

### 3.9 Percentage points

There are a few occasions when adding or subtracting percentages is correct. Here is one.

The Federal Reserve Board sets the prime interest rate (in part to try to control inflation, but that’s a complicated story), which in turn determines interest rates for savings accounts and for loans such as mortgages. To describe a rise in the prime rate from 2.25% to 2.75% you say “The interest rate has risen half a \textit{percentage point}.”

The half comes from simple subtraction: $2.75 - 2.25 = 0.5$. That subtraction is correct provided we remember to express the answer using \textit{percentage points}, not as a percent. This is an absolute difference.

In this example the \textit{percent} increase in the prime rate is $0.5/2.25 = 0.22 = 22\%$.

### 3.10 Exercises

**Exercise 3.10.1.** \([S]\) \([\text{Goal 3.2]}\] \([\text{Section 3.2}]\) \textit{Ordinary vs. premium}

Figure 3.1 gives some data about prices of different levels of seats at Fenway Park.

(a) Compare the cost of an ordinary seat to the cost of a premium seat in 2003. You need to decide which comparison is the most informative. Consider absolute and relative change, perhaps with percentages.

(b) Compare the cost of an ordinary seat to the cost of a premium seat in 2008.

(c) Use your calculations to make a statement about the cost of ordinary seats compared to premium seats between 2003 and 2008.

**Exercise 3.10.2.** \([S][R]\)\([\text{Goal 3.2}]\] \([\text{Section 3.2}]\) \textit{The New York marathon}

46,795 runners finished the 2011 New York marathon. The 1981 marathon had 13,203 finishers. \[11\]

What was the percentage increase during this time period?

**Exercise 3.10.3.** \([R][A][S]\) \([\text{Section 3.1}]\] \([\text{Goal 3.4}]\] \([\text{Goal 3.2}]\) \textit{A raise at last!}

In a recent negotiation, the union negotiated a 1.5% raise for all staff. If a staff member’s annual salary is $35,000, what is her salary after the raise takes effect?

**Exercise 3.10.4.** \([S][R]\) \([\text{Section 3.1}]\] \([\text{Goal 3.3}]\) \textit{The cell phone market.}

In the \textit{The New Yorker} on March 29, 2010 you could read on page 45 that

\[11\] (Jacob Looney, Fall, 2012)
Apple has just 2.2 per cent of the world cell-phone market, but that means it sold twenty-five million iPhones last year.

(a) How large was the 2009 world cell-phone market? (That is, how many people bought a cell-phone in 2009?)

(b) Estimate the percentage of the world population that bought a cell-phone in 2009.

(c) Estimate the percentage of the United States population that bought a cell-phone in 2009.

Exercise 3.10.5. [S][Section 3.1][Goal 3.3] The 2010 oil spill

Citing a Reuters report on the April 2010 Gulf of Mexico oil spill, the website of the Louisiana Oil & Gas Association noted that

BP continues to siphon more oil from the blown-out deep-sea well. It said it collected or burned off 23,290 barrels (978,180 gallons/3.7 million liters) of crude on Sunday, still well below the 35,000 to 60,000 barrels a day that government scientists estimate are gushing from the well.

(a) Check that the reporters are describing the same amount of oil independent of the units (barrels, gallons or liters) used.

(b) Criticize the article for its inconsistent use of significant digits (precision) in reporting these numbers.

(c) What percentage of the oil is being collected or burned off? Your answer should be a range, not a number, expressed with just the appropriate amount of precision.

Exercise 3.10.6. [S][W][Goal 3.3] New taxes?

On January 14, 2013 The Boston Globe reported that

Boosting the income tax from the current rate of 5.25 percent to 5.66 percent would raise $1 billion annually.

Then calculate total taxable income and the tax revenue collected at the current rate of 5.25 percent. Do your answers make sense?

[See the back of the book for a hint.]

Exercise 3.10.7. [W][G][S][Section 3.1][Goal 3.3] Youth sports head injuries

The Massachusetts Department of Public Health collects annual data on brain injuries in school athletics. In 2011, the department reported that in their survey of middle and high school students, about 18 percent of students who played on a team in the previous 12 months reported symptoms of a traumatic brain injury while playing sports. These symptoms include losing consciousness, having memory problems, double or blurry vision, headaches or nausea. During that year, about 200,000 Massachusetts high school students participated in extracurricular sports.
(a) How many reported injuries were there?

(b) The survey reports the number injured as a percentage of the number of students participating in extracurricular sports. What is the number injured as a percentage of the population of Massachusetts?

(c) How many injured high school students would you expect in the town in which you live?

(d) Discuss the reliability of the reported 18% figure. What factors might make the true value greater? What factors might make it less?

Exercise 3.10.8. [S] Section 3.1 Goal 3.3  Listen up on public broadcasting

In 2011, the United States government was facing a $1.5 trillion federal budget deficit. Numerous cuts were proposed, including cutting federal funding of the Corporation for Public Broadcasting (CPB). The Western Reserve Public Media website argued against the cuts, saying the following:

It is clear that our federal government must find ways to tighten the country’s fiscal belt and control government spending, but legislation to eliminate funding for public broadcasting overlooks the critical value that . . . [these] stations provide to all Americans.

. . . Public television is America’s largest classroom, . . . all at the cost of about $1.35 per person per year . . .

Eliminating the . . . investment in CPB would only reduce the $1.5 trillion federal budget deficit by less than 3 ten-thousandths of one percent, but it would have a devastating impact on local communities nationwide.

(a) Use the information in the second paragraph to estimate annual federal spending on public broadcasting.

(b) Find evidence online showing that your answer to the previous question is in the right ballpark.

(c) Show that the information in the last paragraph of the quotation leads to an estimate of annual federal spending on public broadcasting that is wrong by two orders of magnitude.

(d) How would you rewrite the last paragraph so that it was correct?

(e) The article discusses the spending on public broadcasting as a percentage of the federal deficit. That’s unusual – most expenditures are reported as a percentage of the federal budget. What percentage of the federal budget is that spending?

(f) (Optional, no credit). Do you listen to public radio or public television? If so, do you contribute when they ask for money?

Exercise 3.10.9. [S] Section 3.3 Goal 3.2 Goal 3.3 How to confuse holiday shoppers

There are at least two typographical errors in Figure 3.2, which appeared in The Boston Globe on December 19, 2012.

(a) Check all the percentage calculations in the figure.

(b) Where you found mismatches, assume that the percentage is correct and use it to find out what the listed price should be.

(c) Why are there so many errors? Suggest how that might have happened.
Exercise 3.10.10. [R][S][Section 3.1] [Goal 3.3] *Coming as Tourists, Leaving With American Babies*

From *The Arizona Republic* on August 27, 2011:

In 2008, slightly more than 7,400 children were born in the U.S. to non-citizens who said they lived outside the country, according to the National Center for Health Statistics. The figure was the most recent available. …

Although up nearly 50 percent since 2000, the 7,462 children are still just a tiny fraction of the 4,255,156 babies born in the U.S. that year.  

(a) What percentage of the total births were those to foreign residents? Report your answer with the proper number of decimal places.

(b) Criticize the precision of the numbers in the article.

(c) About how many children were born in 2000 in the U.S. to non-citizens living elsewhere?

Exercise 3.10.11. [S][Section 3.1] [Goal 3.3] *Trucks test brakes.*

Figure 3.3 shows some road signs you might see at the top of a hill on a highway, warning that a steep grade lies ahead.

(a) If the grade is two miles long and the grade is 10%, how many feet lower is the bottom of the hill than the top?

(b) What could a grade of 100% mean?

Exercise 3.10.12. [S][Section 3.1] [Goal 3.3] *Who knew there was mercury in fog?*

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Figure 3.3: Steep grade

A researcher at the University of California at Santa Cruz studied the level of methyl mercury in coastal fog. The UCSC student newspaper described the findings, saying

According to the study . . . eight fog water samples were gathered during the summer months of 2011 from four different locations in the Monterey Bay area. In these samples, methyl mercury concentrations ranged from about 1.5 parts per trillion to 10 parts per trillion, average at 3.4 parts per trillion – five times higher than concentrations formerly observed in rainwater.  

The Green Blog at *The New York Times* notes that “[T]he threshold level in fish considered safe for consumption is 0.3 parts per million.”

(a) What are the highest levels of methylmercury recorded in rainwater?

(b) Express the methylmercury content of the fog in parts per million.

(c) Express the methylmercury content of the fog as a percent.

(d) Compare the methylmercury content of the fog to the safe threshold for methylmercury in fish.

[See the back of the book for a hint.]

**Exercise 3.10.13.** [S][Section 3.1] [Goal 3.3] *For-profit colleges could be banned from using taxpayer money for ads*

In April 2012, Senators Tom Harkin and Kay Hagan introduced a Senate bill to prohibit colleges from using federal education dollars for advertising or marketing. Referring to the for-profit colleges as “companies”, the bill stated:

The 15 companies that received 86 percent of their revenues from Federal student aid programs spent $3,700,000,000 (23 percent revenues) on advertising, marketing and recruitment in fiscal year 2009.

(a) What were the total revenues of those 15 for-profit colleges in fiscal year 2009?

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18 http://www.cityonahillpress.com/2012/04/21/mercury-in-the-fog/
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(b) How much did those colleges receive in Federal student aid in fiscal year 2009?

(c) How much of the money they spent on advertising, marketing and recruitment could be considered as coming from the Federal government?

(d) Could the colleges maintain the same level of advertising, marketing and recruiting expenses if they did not use any Federal dollars for those purposes?

(e) What was the fate of Harkin and Hagan’s bill?

And check out Doonesbury for February 19, 2014, at http://www.gocomics.com/doonesbury/2014/02/19

Exercise 3.10.14. [S] [R][Section 3.1] [Goal 3.3] Paper jams

A box of 20 reams of computer printer paper advertises “99.99% jam free.” How many paper jams would you expect from that box?

Exercise 3.10.15. [R][S][Section 3.2] [Goal 3.1] 300 million?

We regularly use 300 million as an estimate for the population of the United States.

[See the back of the book for a hint.]

(a) Find the absolute and relative errors when you use that figure instead of the correct one.

(b) Answer the previous question for the years 2000, 2005 and 2010.

Exercise 3.10.16. [R][S][Section 3.2] [Goal 3.1] The minimum wage

The federal minimum wage increased from $5.85 per hour to $7.25 per hour in 2009. Describe this increase in absolute and relative terms.

Exercise 3.10.17. [S][R][Section 3.2] [Goal 3.1] Health care spending.

The January 2010 National Health Expenditures report stated that overall health care spending in the United States rose from $7,421 per person in 2007 to $7,681 per person in 2008.

(a) Calculate the absolute change in health care spending per person from 2007 to 2008.

(b) Calculate the percentage change in health care spending per person from 2007 to 2008.

(c) Estimate the total amount spent on health care in the United States in 2007 and in 2008.

Exercise 3.10.18. [S][Section 3.2] [Goal 3.1] [Goal 3.2] AMG

The graph in Figure 3.4 appeared on October 6, 2010 on page B10 of The Boston Globe, along with the paragraph

Beverly-based Affiliated Management Group Inc. jumped 4.4 percent to its highest share price since May 4. . . .

(a) What were the absolute and relative changes in AMG share price between Monday and Tuesday of the week?
Exercise 3.10.19. [S][Section 3.2] [Goal 3.2] *Five nines: chasing the dream?*

The web posting with that title at [http://www.continuitycentral.com/feature0267.htm](http://www.continuitycentral.com/feature0267.htm) discusses the cost to industries when their computer systems are down. The “five nines” in the title refers to 99.999% availability.

Tables 3.5 and 3.6 appear there along with this quotation:

Let us examine the math of it, first. ... the maximum downtime permitted per year may be calculated as reflected in Table 3.5. Please do not debate with me leap years, lost seconds or even changes to Gregorian Calendar. Equally let us not debate time travel! To quote a Cypriot saying: “I am from a village: I know nothing.” The figures [in the table] are sufficiently accurate to make the points this article is trying to get across.

<table>
<thead>
<tr>
<th>Uptime</th>
<th>Uptime</th>
<th>Maximum Downtime per Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Six nines</td>
<td>99.999%</td>
<td>31.5 seconds</td>
</tr>
<tr>
<td>Five nines</td>
<td>99.99%</td>
<td>5 minutes 35 seconds</td>
</tr>
<tr>
<td>Four nines</td>
<td>99.9%</td>
<td>52 minutes 33 seconds</td>
</tr>
<tr>
<td>Three nines</td>
<td>99.9%</td>
<td>8 hours 46 minutes</td>
</tr>
<tr>
<td>Two nines</td>
<td>99.0%</td>
<td>87 hours 36 minutes</td>
</tr>
<tr>
<td>One nine</td>
<td>90.0%</td>
<td>36 days 12 hours</td>
</tr>
</tbody>
</table>

Table 3.5: Uptime and Maximum Downtime

(a) Check the arithmetic in Table 3.5

(b) Why do the numbers in the second column increase by a factor of 10 from each line to the next?
### Exercise 3.10.20. Section 3.3  Goal 3.2  Goal 3.3  How many Masters degrees?

The National Center for Educational Statistics reported that in 2008-2009, 178,564 masters degrees in education were awarded in the United States, and that this represented 27% of all master’s degrees awarded.

How many master’s degrees were awarded in 2008-2009?

(a) Make a quick back-of-an-envelope estimate for the answer.

(b) Calculate to find a more precise result, appropriately rounded.

(c) Compare your estimate and your calculation.

### Exercise 3.10.21. Section 3.3  Goal 3.3  Rent in Boston

According to a survey conducted in spring 2006 by Northeast Apartment Advisors, an Acton, MA research firm, the average monthly rent in Greater Boston was $1,355, an increase of 3.6 percent from 2005. What was the average monthly rent in 2005?

### Exercise 3.10.22.  Section 3.3  Goal 3.3  Counting birds

In *The Washington Post* on April 25, 2010 Juliet Eilperin wrote that birder Timothy Boucher said he had

... seen and identified 4,257 species of birds in his life. So his “life list,” as birders say, covers 43 percent of the bird species that exist.

(a) If this report is correct, about how many bird species are there?

(b) Can you find independent evidence that your answer is right?

---

21 The web posting discusses the estimated costs of these upgrades, and compares them to the savings, but that’s more complex quantitatively than what we can do in this book.


23 [http://www.nmhc.org/Content/ServeContent.cfm?ContentItemID=2475](http://www.nmhc.org/Content/ServeContent.cfm?ContentItemID=2475)

Exercise 3.10.23.  [S][Section 3.3]  [Goal 3.1]  [Goal 3.2]  60 years in seconds

In Chapter 2 we calculated the number of seconds in 60 years two ways, and found $1.89341556 \times 10^9$ and 1,893,415,558.

(a) What is the absolute difference between these two figures?

(b) What is the relative change, expressed as a decimal and as a percentage?

(c) Which way of describing the difference is likely to be more useful.

Exercise 3.10.24.  [S][Section 3.3]  [Goal 3.2]  How long is a year?

In Fermi problems we use 365 days/year, or perhaps even 360 or 400 days/year to make the arithmetic easier.

(a) What is the relative error when you use 400 days/year instead of 365 days/year?

(b) What does the Google calculator tell you when you ask it for one year in days?

(c) Why isn’t the answer above “365 days”?

(d) What are the relative and absolute errors when you use Google’s answer instead of 365 days?

(e) Why isn’t Google’s answer “365.25 days”, which would take leap years into account?

[See the back of the book for a hint.]

Exercise 3.10.25.  [C][W][S][Section 3.3]  [Goal 3.3]  Improving fuel economy.

If your car’s fuel economy increases by 20% will you use 20% less gas?

(a) Show that the answer to this question is “no” if you measure fuel economy in miles per gallon.

(b) Show that the answer to this question is “yes” if you measure fuel economy in gallons per mile.

[See the back of the book for a hint.]

Exercise 3.10.26.  [S][Section 3.3]  [Goal 3.2]  bytes

In Chapter 1 we saw that a kilobyte is 1024 bytes, not the 1000 bytes you expect.

(a) What is the percentage error if you use 1000 bytes instead of 1 kilobyte?

(b) How many bytes in a Megabyte (exact answer, please)? In a Gigabyte?

(c) What is the percentage error if you work with 1,000,000 bytes per Megabyte?
Exercise 3.10.27. [W][Goal 3.5] [Section 3.3] [Goal 3.4] [Goal 3.3] Hard times.

The boss says “Hard times. In order to avoid layoffs, everyone takes a 10% pay cut.” The next day he says “Things aren’t as bad as I thought. Everyone gets a 10% pay raise, so we’re all even.”

Is the boss right?

What if the 10% pay raise comes first, followed by a 10% cut?

Exercise 3.10.28. [S][Section 3.3] [Goal 3.3] [Goal 2.3] Gulf oil spill could lead to drop in global output

In a June 19, 2010 article on the Gulf of Mexico oil spill The Denver Post quoted the Dow Jones Newswire:

Global oil output could decline up to 900,000 barrels a day from projected levels for 2015 if oil-producing countries follow the U.S. lead and impose moratoriums on development of new offshore oil reserves. . . . [that] would represent a mere 1 percent or so of global oil output.

(a) Use the figures in the quotation to estimate the projected global output for 2015.

(b) Do enough research to verify the order of magnitude of your answer. (The kind of answer will depend on the year in which you do this problem.)

Exercise 3.10.29. [S][Section 3.3] [Goal 3.3] [Goal 3.2] Taking the fifth.

Several years ago the liquor industry in the United States started selling wine bottles containing 0.75 liters instead of bottles containing 1/5 of a gallon (“fifths”). They charged the same amount for the new bottle as the old. What was the percentage change in the cost of wine?

Exercise 3.10.30. [S][Section 3.3] [Goal 3.3] Tax Holiday

Occasionally a state designates one weekend in August as a tax-free holiday. That is, consumers can purchase some items without paying the sales tax. Many stores advertise the savings in the days before the weekend, to encourage customers to shop with them. The sales tax in South Carolina (in 2013) was 6%. Would it be right for an ad to read, “No sales tax this weekend - save 6% on your purchases!”

Exercise 3.10.31. [S] [W][Section 3.3] [Goal 3.3] [Goal 3.6] Measuring markups

The standard markup in the book industry is 50%: the retail price of a book is one and one half times the wholesale price. [27] [See the back of the book for a hint.]

(a) What percentage of the retail price of a book is the bookstore’s markup?

(b) The New England Mobile Book Fair advertises its bestsellers as “30% off retail.” What is their markup on bestsellers, as a percentage of the wholesale price?

(c) Answer the previous question if the Book Fair discounts bestsellers by 40%. Would they ever do that?

[See the back of the book for a hint.]

25 http://www.denverpost.com/nacchio/ci_15330002
26 We consider other numbers related to the Gulf oil spill in Exercise 3.10.5 (page 72).
27 See comment in instructor’s manual.
3.10. EXERCISES

Exercise 3.10.32. [S][C][Section 3.3] [Goal 3.2] [Goal 3.3] Goldman Sachs limits pay, earns $4.79 billion in fourth quarter

On January 21, 2010 the *Tampa Bay Times* carried an Associated Press report with that headline reporting that

The big bank [Goldman Sachs] said yesterday that it rewarded employees with $16.2 billion in salaries and bonuses for 2009. That’s up 47 percent from the previous year but much lower than many expected. In all, compensation accounted for 36 percent of Goldman’s $45.17 billion in 2009 revenue, the lowest annual ratio since the company went public in 1999. In 2008, Goldman set aside 48 percent of its revenue to pay employees.  

(a) Are the numbers $16.2 billion, 36% and $45.17 billion in the quotation consistent?

(b) How much did Goldman Sachs pay out in bonuses in 2008?

(c) What were Goldman Sachs’ revenues in 2008?

(d) What do you make of the discrepancy between the headline and the text in the article?

Exercise 3.10.33. [S][C][Section 3.3] [Goal 3.2] [Goal 3.3] [Goal 3.7] More than 40m now use food stamps

According to Bloomberg News, June 3, 2010, as reported in the Austin (TX) *American-Stateman*

WASHINGTON – The number of Americans receiving food stamps in March topped 40 million for the first time as the jobless rate hovered near a 26-year high.

Recipients of Supplemental Nutrition Assistance Program subsidies for food purchases totaled 40.2 million, up 21 percent from a year earlier and 1.2 percent more than in February, the Department of Agriculture said yesterday in a statement on its website.  

An average of 40.5 million people, more than an eighth of the population, will get food stamps each month this budget year.  

(a) About how many Americans were receiving food stamps in 2009 ("a year ago" when this article appeared)?

(b) How did the percentage of the population receiving food stamps change between 2009 and 2010?

Exercise 3.10.34. [S][C][Section 3.3] [Goal 3.3] [Goal 3.2] [Goal 3.6] Gamblers spending less time, money in AC casinos

From the Jefferson (Missouri) *News-Tribune* on December 7, 2010:

A new statistical study shows the amount of time gamblers spent inside casinos in the nation’s second-largest gambling market is down more than 22 percent, and the amount of money they spend is down almost 30 percent over the past four years.
And the hit to the casinos’ bottom line is substantial: gross operating profit per hour is down 61 percent. . . .

The study examined third-quarter figures from 2006 to 2010 in three areas: gross gaming revenue per visitor hour (the amount of money casinos take in for every hour a gambler is on their premises); total visitor hours; and gross operating profit per visitor hour. . . .

Gross gaming revenue fell from $9.13 per hour in 2006 to $6.42 for the city’s 11 casinos.

Gross operating profit per visitor hour went from $2.74 in 2006 to $1.05 in the third quarter of this year.

Atlantic City’s casino revenue fell from a high of $5.2 billion in 2006 to $3.9 billion in 2009.

(a) The units “per hour” in the third paragraph are wrong. What should they be?

(b) Calculate the percentage change in gross operating profit per visitor hour from 2006 to 2009. Does this match the assertion in the article?

(c) Calculate the percentage change in gaming revenue per visitor hour from 2006 to 2009. Does this match the assertion in the article?

(d) The article says the total number of hours gamblers spent in the third quarter is down 22% and the gross revenue per hour is down 30%. Calculate the percentage change in total third quarter gross revenue. Is your answer consistent with the numbers in the article?

Exercise 3.10.35. [S][Section 3.3] [Goal 3.1] [Goal 3.2] Tracking Harvard’s endowment.

On September 23, 2011 The Boston Globe published data from Harvard University on the performance of its endowment from 2007 to 2011. We’ve displayed that information in Figure 3.7,

The graph on the left shows the percentage change from year to year; the one on the right the actual value.

(a) Between 2007 and 2008 the graph on the left goes down while the one on the right goes up. How is that possible?

(b) For the years 2008, 2009, 2010 and 2011 use the second graph to compute the absolute and relative changes in endowment assets from the previous year

(c) Compare the relative changes you computed with the information in the first graph. What might explain the discrepancies?

Exercise 3.10.36. [S][Section 3.3] [Goal 3.2] [Goal 3.3] [Goal 3.7] [Goal 1.1] [Goal 1.3] [Goal 1.4] [Goal 1.5] Banks to Make Customers Pay Fee for Using Debit Cards

From The New York Times, Friday, September 30.

Until now, [debit card] fees have been 44 cents a transaction, on average. The Federal Reserve in June agreed to cut the fees to a maximum of about 24 cents. While the fee amounts to pennies per swipe, it rapidly adds up across millions of transactions. The new limit is expected to cost the banks about $6.6 billion in revenue a year.

(a) Use the data in the article to estimate the number of debit card transactions in a year.
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[See the back of the book for a hint.]

(b) What assumptions did you make in arriving at your estimate?

(c) Is the phrase “millions of transactions” a good way to describe the order of magnitude of your answer?

(d) What percentage of a $10 debit card transaction does a merchant have to pay to the bank that issued the card? How will that percentage change when the new rule takes effect?

(e) Rewrite your estimate in (a) in transactions per day.

(f) Estimate the number of people in the United States who use a debit card. Then rewrite your answer to the previous problem in transactions per person per day.

(g) (Optional) Do you use a debit card? Did you know how much your use cost the merchant? Will you change your behavior if your bank charges $5/month for debit card use?

Exercise 3.10.37. [S] [Section 3.3] [Goal 3.1] [Goal 3.3] Taxing gasoline.

The federal tax on gasoline at the end of 2011 was 18.4 cents per gallon. That hadn’t changed since 1993, when gasoline cost just $1.16 per gallon (tax included).

(a) What percentage of the 1993 cost of gasoline was the federal tax in 1993?

(b) If the average cost of gas in 2011 was $3.40 per gallon, what percentage of the cost of gasoline then was the federal tax?

(c) What would the cost of a gallon of gasoline have been at the end of 2011 if the percentage rather than the amount of federal tax were the same then as in 1993?

(d) Gasoline consumption in the United States was estimated to be about 175 million gallons per day in 2011. How much revenue was generated by the federal gas tax at its then current rate? How much would have been generated if tax were computed using your answer to part c?

Exercise 3.10.38. [S] [Section 3.3] [Goal 3.1] [Goal 3.3] How long is a microcentury?

The mathematician John von Neumann is often identified as the source of the fact that a 50 minute lecture lasts about as long as a microcentury.

(a) Check this fact by doing your own arithmetic.
(b) What are the absolute and relative errors in the “about”?  

(c) If you could listen to lectures one after another day in day out for a century about how many would you hear?  

(d) You can find several answers on the web to the question “how long is a microcentury?” Critique them.  

(e) Estimate how many years it would take for professors at your school to have delivered a century’s worth of classes. (Don’t assume the classes were back to back year ‘round.)

Exercise 3.10.39. [Section 3.4] [Goal 3.4] Count the keystrokes and measure the time it takes to do the sales tax computation in the text with and without the “add one” trick.

Exercise 3.10.40. [S][W] [Section 3.4] [Goal 3.2] [Goal 3.3] [Goal 3.4] Currency conversion.

When you read this question you will see that you need at least two days to answer it.

(a) Suppose your company is sending you to France for business, and asks you to convert $1250 (US dollars) to euros in preparation for your trip. Find a currency conversion calculator on the web and use it to find out how many euros your $1250 will buy. Your answer should clearly identify the web site, the date and time and the conversion factor used (in euros/dollar) as well as the amount of euros you would get. Imagine that you have done the exchange, so that you now have euros instead of dollars. (If you’d rather your company arranged your business trip to some other country, feel free to change the problem accordingly.)

(b) Two days after you changed your money into euros, your company calls off the trip. You need to change your money back into dollars. Go back to the web to figure out what you would get back in dollars for the euros that you have. Again identify the web site, date and time, and conversion factor, this time in dollars/euro. Once you’ve done the conversion, compare the amount of money you now have with the original amount. Calculate the percentage gain (or loss) due to the change in exchange rate.

(c) If you actually converted dollars to euros and back the bank would charge a fee each time. Suppose that fee is 2% of the amount converted. Go through the problem again, but this time account for the 2% fee. That is, figure out how many euros you would have gotten in the first conversion, with the 2% fee included, then how many dollars back in the second, with another 2% fee. Finally, calculate the total percentage gain (or loss).

(d) If the conversion rate was exactly the same each time (unlikely, but imagine it) would your loss in the conversion to euros and back be 4%?

Travel note. If you’re planning a trip to a foreign country, explore the cheapest way to convert your money. Getting cash at a local bank is probably not the answer. Your debit and credit cards will probably work world-wide – ask about the fees. And tell them that you will be using the card elsewhere so they don’t think it’s been stolen.

Exercise 3.10.41. [S][W] [Section 3.4] [Goal 3.4] Oregon town weighs future with fossil fuel

The Bend, OR Bulletin reported on April 20, 2012, that Boardman, Oregon

... is among at least half a dozen ports in the region weighing whether to ship millions of tons of coal to Asia from the Powder River Basin of Wyoming and Montana. ...  
If all of the projects were built, as much as 150 million tons of coal per year could be exported from the Northwest, nearly 50 percent more than the nation’s entire coal export output last year.

http://www.bendbulletin.com/article/20120422/NEWS0107/204220325/
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(a) Use the data in this paragraph (not a web search) to estimate how many tons of coal the United States exported in 2011.

(b) Use the web to confirm (or not) your estimate.

Exercise 3.10.42. [W][S][Section 3.4] [Goal 3.1] [Goal 3.4] [Goal 3.6] Seeking savings, some ditch brand loyalty

An article with that headline appeared in The Boston Globe on January 29, 2010, saying (among other things) that

Unit sales of private label goods have jumped 8 percent since 2007, while brand names have declined roughly 4 percent, according to Nielsen Co. 35

We’ve displayed the data in Figure 3.8

![Figure 3.8: Private Labels, Branded Goods](image)

(a) Find the absolute and relative changes in unit sales of private label and brand name goods in the years since 2007.

(b) Are the percentages in the article consistent with the numbers in the graphic?

(c) How have total sales changed between 2007 and 2009?

(d) How has the percentage of private label sales changed in this period?

Exercise 3.10.43. [S][Section 3.4] [Goal 3.2] [Goal 3.3] [Goal 3.4] Research funding

The February 10, 2011 The Boston Globe carried an article that said

The University of Massachusetts received a record amount of funding for research in fiscal year 2010, taking in $536 million, according to preliminary figures. That was an increase of $47 million, or 9.5 percent, over the previous year. 36

34 See comment in instructor’s manual.
35 http://www.boston.com/business/articles/2010/01/29/shoppers_are_ditching_name_brands_for_store_brands/
36 http://www.boston.com/business/articles/2011/02/10/umass_research_funding_reaches_record/
3.10. EXERCISES

Use the information in the article to calculate the total research funding for the UMass system in fiscal year 2009 two ways, by subtracting, and using the "1 + " idea. Do you get the same answer each way?

Exercise 3.10.44. [S][Section 3.4] [Goal 3.4] [Goal 3.3] Long gone?

On March 4, 2011 columnist Alex Beam wrote in The Boston Globe:

PBooks? No one has read a “p-book” – that’s industry jargon for “print books” – in years, according to nice Mr. Bezos of Amazon, who would like to sell you e-books on his Kindle gizmo. How counterintuitive, then, that total US books sales increased 3.9 percent to almost $12 billion in 2010, according to the Association of American Publishers.

I just read two p-books: “The Last Crossing,” by Guy Vanderhaeghe, and Mordecai Richler’s hilarious “Barney’s Version,” which I am told is 20 times better than the movie. And I read them for free, thanks to my local library.

Wait – aren’t libraries obsolete? The subject of my next column, perhaps.

(a) What were book sales in 2009?

(b) Use the data given to estimate the average number of dollars spent and books bought by each adult in the US in 2010.

(c) Discuss how reasonable you find your answers to the previous question.

(d) Can you make quantitative sense of “20 times better” in the second paragraph?

Exercise 3.10.45. [S][Section 3.4] [Goal 3.3] [Goal 3.4] Student loan growth.

On March 21, 2011 John Sununu wrote in The Boston Globe:

For-profit schools . . . served nearly 2 million students in 2010, training nurses, technicians, chefs, and just about any other profession imaginable. They account for 10 percent of American higher education,

Since 2003, the total debt burden from education loans to private and public institutions has grown 18 percent per year and now stands at over $500 billion.

(a) Use the information in the first paragraph to calculate the total number of students in American higher education in 2010.

(b) What was the average student debt in 2010? (Assume the $500 billion figure is for 2010.)

(c) Use the information in the second paragraph to calculate total student loan debt burden for each of the years 2009, 2008 . . . 2003.

(d) Estimate the average student debt for each of the years between 2003 and 2010.

[See the back of the book for a hint.]

Exercise 3.10.46. [U] On September 27, 2011, PCMag reported that

37 http://www.boston.com/lifestyle/articles/2011/03/04/from_landlines_to_e_mail_to_movies_to_print_their_demise_has_been_greatly_exaggerated/

38 http://www.boston.com/bostonglobe/editorial_opinion/oped/articles/2011/03/21/creature_double_feature/

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Visits to Google+ jumped 1269 percent during the week of September 17, while total U.S. site visits were up from 1.1 million to almost 15 million during that same time period, according to Hitwise.

Market leader Facebook is still miles ahead with almost 1.8 billion U.S. visits, followed by YouTube with 530 million and Twitter with 33 million. 

(a) Verify that the numbers in the first paragraph are consistent.

(b) Compare Google+ visits with Facebook visits, or Google+ visits with YouTube or Twitter visits.

(c) Paul Allen calls himself the Google+ unofficial statistician. He is also the founder of Ancestry.com. His post on December 27, 2011 contains a lot of Google+ data [https://plus.google.com/+PaulAllen/posts/ZcPA5ztMza](https://plus.google.com/+PaulAllen/posts/ZcPA5ztMza). In that post, he said that he expects the growth to continue to accelerate. Was he correct? Be careful – he’s saying more than that the growth will continue. He is saying that it will continue to accelerate.

**Exercise 3.10.47.** [U] [Section 3.4] [Goal 3.3] [Goal 3.4] 1,300%!

On September 29, 2011 Hiawatha Bray wrote in *The Boston Globe* that

> The social networking team at Google Inc. has a great sense of timing. They picked last week to tear down the gates at their new Google+ service, which was previously accessible only by invitation. It worked. Google+ visits surged by nearly 1,300 percent on the week of September 24, with 15 million users coming to the site.

How many users were there before the gates fell?

**Exercise 3.10.48.** [S] [W] [Section 3.5] [Goal 3.4] [Goal 3.5] What’s a good basketball player worth these days?

In *The Boston Globe* on June 30, 2010 you could read that

> [A] maximum deal for [LeBron] James would start at $16.56 million (at the minimum) and increase by 8 percent each season over the five-year period.

(a) Use the “1+” trick four times to calculate James’ salary in the fifth season.

(b) In the five seasons, James will get four eight percent salary increases. Compare the result of your calculation to a single $4 \times 8\% = 32\%$ increase.

**Exercise 3.10.49.** [S] [Section 3.7] [Goal 3.6] Find an advertised discount in your local paper or on the web and calculate the discounted price, two ways. First try mental arithmetic to find an approximate answer. Then compute carefully. Compare your results. Make sure you include a copy of the ad when you turn in your answer.

**Exercise 3.10.50.** [S] [R] [Section 3.7] [Goal 3.3] [Goal 3.4] [Goal 3.6] 13 million get unexpected tax bill from Obama tax credit

On December 17, 2010 the Wilmington, NC *StarNews* carried an Associated Press report with that headline saying that

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39 [http://www.pcmag.com/article2/0,2817,2393640,00.asp](http://www.pcmag.com/article2/0,2817,2393640,00.asp)
41 See comment in instructor’s manual.
The Internal Revenue Service reported that the average tax refund was $2,892 in the 2010 filing season, up from $2,663 in 2009. However, the number of refunds dropped by 3.5 percent, to 93.3 million.

(a) What was the percentage increase in the average tax refund from 2009 to 2010?

(b) How many refunds were there in 2009 and in 2010?

(c) The number of refunds decreased while the average refund increased, so you need some arithmetic to decide how the total amount refunded changed from 2009 to 2010. Find that change, in both absolute and percentage terms.

(d) Does the headline match the quote in accuracy? In tone?

Exercise 3.10.51. [S][R]Section 3.7 | Goal 3.2 | Goal 3.6 | Bargain bread.

A local supermarket offers

| Special
| Pain au Levain
| 2 for $5
| Regularly $3.99
| Over 30% off.

(a) Is the claim true?

(b) How much more than 30% is the actual savings?

Exercise 3.10.52. [S][A]Section 3.8 | Goal 3.3 | Goal 3.4 | Goal 3.6 | Tax and discount.

Suppose sales tax is 5% and a $117 item is discounted 40%. Compute the final price and the sales tax two ways:

(a) First find the discounted price, then the sales tax, then the final price.

(b) First find the sales tax, then the discounted (final) price.

(c) Does it matter to you which way the computation is made? Does it matter to the state? (Remember: the state collects the sales tax!)

Exercise 3.10.53. [S]Section 3.8 | Goal 3.6 | Goal 3.4 | Goal 3.5 | Free!?

If a shirt on the clearance rack is marked “50% off original price” and you have a preferred customer coupon giving you 50% off the sale price of any item, do you get the shirt free?

Exercise 3.10.54. [S] Section 3.9 | Goal 3.7 | Goal 3.2 | Paying off your student loan.

We found this information about student loans on the web:

http://www.starnewsonline.com/article/20101217/ARTICLES/101219738
http://www.myfedloan.org/make-a-payment/ways-to-pay/direct-debit-faq.shtml

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Direct Debit is the easiest and most convenient way to make your student loan payments. Why sign up for Direct Debit?

It’s the most convenient way to make your student loan payments.

• **Save money.** You qualify for at least a 0.25% **interest rate reduction** when you use Direct Debit.

• **Don’t worry.** Since we automatically withdraw your monthly payments from your bank account, they’re never late (as long as the funds are available).

• **Save time.** Your time is valuable. The transaction is automatic, so you won’t need to write checks or sign in every month to make your payments.

• **Help the environment.** Going paperless saves trees and reduces solid waste.

(a) Explain how direct debit works. (You may need to look this up. You may want to look it up even if you think you know.)

(b) If you have a student loan on which you pay interest at a rate of 6.8% what will your interest rate be if you sign up for direct debit?

(c) Why should the **Save money** bullet in the above list say “0.25 **percentage point interest rate reduction**” instead of “0.25% **interest rate reduction**”?

(d) If the **Save money** bullet really means what it says, what will the interest rate be on your 6.8% loan if you sign up for direct debit?

**Exercise 3.10.55.** [S][W][Section 3.9] [Goal 3.7] [Goal 3.2] **Benefits take hit in Patrick budget**

In *The Boston Globe* on January 13, 2008, reporter Matt Viser wrote

Looking for ways to trim a looming $1.3 billion state budget gap, Governor Deval Patrick will propose shifting more of the cost of health insurance premiums onto tens of thousands of state employees.

Under his plan, about 37,000 employees would see their monthly premiums increase by 10 percent.

... Right now, most employees pay 15 percent, and the state covers 85 percent. ... those making more than $50,000 would pay 25 percent.

... For the 37,000 employees who would face the 10 percent increase, that would mean additional monthly costs of $51 for an individual plan and $120 for a family plan. [45] [46]

(a) Explain why the this increase of 10 **percentage points** is really a 67 **percent** increase.

(b) Why might Governor Patrick prefer to publicize the 10 instead of the 67?

(c) Write a letter to the editor pointing out the error in the article. [47] Your letter should be short, accurate, and pointed or funny in some way if possible.

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46. The Commonwealth of Massachusetts went ahead and made this change in 2009.
47. See comment in instructor’s manual.
Exercise 3.10.56. [S] Hubway

Figure 3.9 accompanied an article in *The Boston Globe* about the Hubway bicycle exchange program. According to the article:

A typical station with bikes costs about $50,000 and requires $10,000 in annual subsidies, though operations are expected to be covered fully by user fees and advertising as Hubway matures. Currently, there are 1,003 bicycles distributed across the Hubway’s 105 stations. Bike users can pay a one-time fee or buy an annual membership for $85.

(a) What was the percentage increase in annual use (trips) of Hubway bicycles from 2011 to 2012?

(b) Revise your answer to (a) to take into account the fact that Hubway opened for business in July 2011, so it was available for only half of 2011.

(c) On average, how many trips did each active annual member take in 2012?

(d) On average, how many bikes are available at each station?

(e) Compare the revenue from annual subscriptions to the cost of maintaining the Hubway stations.

Exercise 3.10.57. [U][C][W] Slow down; save gas.

A U.S. government webpage on fuel economy says
While each vehicle reaches its optimal fuel economy at a different speed (or range of speeds),
gas mileage usually decreases rapidly at speeds above 50 mph.
You can assume that each 5 mph you drive over 50 mph is like paying an additional $0.25 per
gallon for gas.
Observing the speed limit is also safer.

and provides Figure 3.10

![Figure 3.10: Slow down, save gas](image)

Figure 3.10: Slow down, save gas

(a) Use the graph to estimate the percentage decrease in fuel economy that results when you increase
your average speed from 50 mph to 55 mph. (Think about whether you should measure fuel economy in
miles per gallon or in gallons per mile. You might want to look at [Section 2.2].)

(b) Use the graph to estimate the percentage decrease in fuel economy that results when you increase your
average speed from 55 mph to 60 mph.

(c) Use the graph to estimate the percentage decrease in fuel economy that results when you increase your
average speed from 60 mph to 65 mph.

(d) Are these percentages the same? Do your calculations support the common assumption that (once
you are at a high enough speed) every time you increase your speed by 5 mph you decrease your fuel
efficiency by about 5%?

(e) What assumption is being made about the price of gas in these calculations?

Exercise 3.10.58. [S][Section 3.3] [Goal 3.2] [Goal 3.3] Holiday shows bringing box office joy

The Boston Globe published the data in Figure 3.11 on December 27, 2012.

(a) Verify the four percentage calculations.

(b) For both venues, revenue increased faster than attendance between 2009 and 2012. That means the
average ticket price must have increased. Compute the average ticket price in each year for each venue,
and the percentage change in each case.

Exercise 3.10.59. [U] Taxing on line purchases

On October 28, 2013 The Boston Globe reported that

3.10. EXERCISES

<table>
<thead>
<tr>
<th>BSO Holiday Pops</th>
<th>Christmas Revels</th>
</tr>
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<tbody>
<tr>
<td><strong>Attendance</strong></td>
<td><strong>Attendance</strong></td>
</tr>
<tr>
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<tr>
<td>68,771</td>
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</tr>
<tr>
<td>2012</td>
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</tr>
<tr>
<td>78,916</td>
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<td><strong>Revenue</strong></td>
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</tr>
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</tr>
<tr>
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<td>$770k</td>
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<tr>
<td>+15%</td>
<td>+12%</td>
</tr>
<tr>
<td>+18%</td>
<td>+16%</td>
</tr>
</tbody>
</table>

Figure 3.11: Good news at the box office

If you’re considering making a big-ticket purchase on Amazon.com, you might want to do it before Friday.

That’s when Massachusetts shoppers will start paying the 6.25 percent sales tax on items they buy from Amazon.

The state Department of Revenue expects the tax to raise $36.7 million before the fiscal year ends June 30. [49]

(a) What total taxable sales does Department of Revenue predict for the rest of the fiscal year?

(b) Estimate the average (mean) Massachusetts resident’s Amazon purchases during that time period.

(c) Estimate the number of Massachusetts residents who shop on line. Then estimate their Amazon purchases during that time period.

Exercise 3.10.60. [U][S] How hot was it?

On Friday, March 23, 2012 The Boston Globe displayed a front page graphic saying that the 82 degree record temperature on March 22 was about 35% higher than the historical average of 61 degrees.

(a) Convert these Boston record and average temperatures to their equivalents in degrees Celsius.

(b) Benjamin Bolker lives in Hamilton, Ontario, Canada, where they measure temperature in degrees Celsius. What percentage change would his newspaper, the Hamilton Spectator, have reported for those temperatures?

(c) What is wrong with the Globe’s reporting? Write your answer in the form of a letter to the editor.

Exercise 3.10.61. [S] What should you do after you graduate?

According to Vox.com, “the top 25 hedge fund managers earned a collective $21.1 billion this year.” [50] Vox.com put this figure into context, saying: “it’s about 0.13 percent of total national income for 2013 being earned by something like 0.00000008 percent of the American population.”

(a) Check the calculation that the 25 hedge fund managers are the percent of the population claimed.
(b) Use the data from Vox.com to estimate the total national income in 2013.

(c) The same Vox.com article stated that the earnings of the 25 hedge fund managers were “about 2.5 times the income of every kindergarten teacher in the country combined.” Can you verify this income comparison of hedge fund managers and kindergarten teachers?

**Exercise 3.10.62.** [U][S][Section 3.9] [Goal 3.7] Payday loans in the military


Access [to payday loans]significantly increases the likelihood that an airman is ineligible to reenlist by 1.1 percentage points (i.e., by 3.9%). We find a comparable decline in reenlistment. Payday loan access also significantly increases the likelihood that an airman is sanctioned for critically poor readiness by 0.2 percentage points (5.3%).

(a) What is the likelihood (as a percent) that an airman is ineligible to reenlist when no payday loans are available? What is the percentage when they are?

(b) What is the likelihood (as a percent) that an airman is sanctioned for critically poor readiness when no payday loans are available? What is the percentage when they are?

(c) What is a payday loan? Should servicemen and women be paid so poorly that they need them?

**Review exercises**

Sample routine review questions. When *Common Sense Mathematics* is published these and others may be available in an online homework system/

**Exercise 3.10.63.** [A] Some routine percentage calculations.

(a) Calculate 40% of 250.

(b) Calculate 130% of 79.

(c) 85 is what percentage of 140?

(d) 62 is what percentage of 30?

(e) 30% of what number is 211?

(f) 115% of what number is 52?

**Exercise 3.10.64.** [A] For each problem, calculate the 6.25% sales tax and also the final price. Redo the calculation again using the 1+ trick.

(a) A book with sticker price $12.99.

(b) A computer priced at $499.99.

(c) A necklace priced at $32.00.

(d) A DVD selling for $5.50.