Chapter 5

Average Values

We start by remembering that to compute an average you simply add the values and divide by the count. We quickly move on to weighted averages, which are more common and more useful. They’re a little harder to understand, but worth the effort. They help explain some interesting apparent paradoxes.

Chapter goals:

Goal 5.1. Compute means using weighted averages.

Goal 5.2. Investigate what it takes to change a weighted average.

Goal 5.3. Understand paradoxes resulting from weighted averages.

5.1 Average test score

Suppose a student has taken ten quizzes and earned scores of

\[ 90, 90, 80, 90, 60, 90, 90, 70, 90, 80. \]

To keep the computations simple and focus on the ideas, we’ve made up this short unrealistic example. Later we will return to our prejudice in favor of real data.

To find her average score you add the ten scores and divide by ten:

\[
\frac{90 + 90 + 80 + 90 + 60 + 90 + 90 + 70 + 90 + 80}{10} = \frac{830}{10} = 83.
\]

There are only four different values in this list: 60, 70, 80 and 90. But her average score isn’t just \((60 + 70 + 80 + 90)/4 = 75\). A correct calculation must take into account the fact that she had lots of grades of 90 but just one 60. Here is a way to do that explicitly:

\[
\frac{1 \times 60 + 1 \times 70 + 2 \times 80 + 6 \times 90}{10} = \frac{830}{10} = 83.
\]
To work with the fraction of times each score occurs rather than the number of times just divide the 10 in the denominator into each of the terms in the numerator:

\[
\frac{1}{10} \times 60 + \frac{1}{10} \times 70 + \frac{2}{10} \times 80 + \frac{6}{10} \times 90 = 83.
\]

Viewed that way, we see exactly how each of the four different quiz grades contributes to the average with its proper weight.

We can rewrite that weighted average showing the weights as decimal fractions

\[
0.1 \times 60 + 0.1 \times 70 + 0.2 \times 80 + 0.6 \times 90 = 83
\]

or percentages:

\[
10\% \times 60 + 10\% \times 70 + 20\% \times 80 + 60\% \times 90 = 83.
\]

In each case all the tests are accounted for so the weights expressed as fractions or as decimals sum to 1. As percents they sum to 100\%.\(^{\text{3}}\)

The same strategy finds the average value of a card when (as in blackjack) the value of the face cards (Jack, Queen and King) is 10. Imagine that you choose a card at random, write down its value, return it to the deck, shuffle, and do it again – many times. What will the average value be? It’s sure to be greater than the simple average of the numbers 1 to 10 (which is 5.5) since there are more cards with value 10 than any other. This is a weighted average, where the weight of each value is its probability.\(^{\text{2}}\) There are 4 chances in 52 that you see (say) a four, and 16 chances in 52 that you see a card with a value of 10. The average value is

\[
\frac{4}{52} \times 1 + \frac{4}{52} \times 2 + \cdots + \frac{4}{52} \times 9 + \frac{16}{52} \times 10 = \frac{1 + 2 + \cdots + 9 + 4 \times 10}{13} \approx 6.54.
\]

The first step in the computation puts all the fractions over a common denominator and cancels a 4. We could have started there by thinking about the cards one suit at a time.

5.2 Grade point average

The UMass Boston registrar posts the rules used to compute student grade point averages at the web site [http://www.umb.edu/registrar/grades_transcripts/grading_system](http://www.umb.edu/registrar/grades_transcripts/grading_system). Here is what that web page said in the spring of 2009.\(^{\text{3}}\)

How the Grade Point Average (GPA) is computed

Each letter grade has a grade point equivalent. List your grades in a column, then each grade point equivalent next to the letter grade. Multiply each grade point equivalent by the number of credits for each class. Total all products and divide by the total number of credits. The answer will be your grade point average for that semester.

The site then displays Table 5.1 showing a sample computation for a student who took 11 courses for a total of 33 credits and earned one grade of each kind.

---

1. See comment in instructor’s manual.
2. Our official study of probability starts in Chapter 11 but there’s no reason not to think about this straightforward problem now.
3. See comment in instructor’s manual.
5.3 Improving Averages

The “average” in “Grade Point Average” is a weighted average of the grades, with credits as the weights.

Suppose a student has taken four courses worth 4, 3, 3 and 2 credits and earned grades of B+, A, B+ and C-, respectively. Since she has earned a total of 12 credits, her GPA is

\[
\frac{4 \times 3.3 + 3 \times 4 + 3 \times 3.3 + 2 \times 1.7}{12} = \frac{37}{12} = 3.13.
\]

That’s a solid B average in spite of the C-.

To see the weights more clearly, rewrite the computation showing the fraction of credits for each course:

\[
\frac{4}{12} \times 3.3 + \frac{3}{12} \times 4 + \frac{3}{12} \times 3.3 + \frac{2}{12} \times 1.7 = 3.13.
\]

The Registrar’s web site tells you to multiply the grade equivalent by the number of credits. We do the multiplication in the other order – fraction of credits times grade equivalent – to make the weights more visible.

5.3 Improving averages

Suppose a student at the end of her junior year has a GPA of 2.8 for the 90 credits of courses she has taken so far. What GPA must she earn for the 30 credits she will take as a senior so that she can graduate with a GPA of 3.0?

The ordinary average of 2.8 and 3.2 is 3.0, so she might think at first that a 3.2 as a senior will do the trick. We know that can’t be right, since there are more credits in her first three years than in her last. She will need more than a 3.2 to bring her GPA up to 3.0. We need to figure out how much more.

If her senior year GPA is \(G\) then her combined GPA is the weighted average

\[
\frac{90 \times 2.8 + 30 \times G}{120}.
\]
5.3. IMPROVING AVERAGES

What value of $G$ will make the arithmetic in this expression come out at least 3.0?

We’ll answer this question three ways. Each method has its advantages and disadvantages.

If you remember even a little bit of algebra you can solve the equation

$$\frac{90 \times 2.8 + 30 \cdot G}{120} = 3.0$$

for the unknown $G$. Multiply both sides by 120 to get

$$90 \times 2.8 + 30 \cdot G = 360$$

so

$$30 \cdot G = 360 - 90 \times 2.8 = 360 - 252 = 108$$

so

$$G = \frac{108}{30} = 3.6.$$  

Once you see the answer you can see why it is right. The 3.0 GPA she wants will be just one fourth of the way from the 2.8 GPA she has so far to what she needs as a senior, since she has already earned three fourths of her credits – the 2.8 carries 75% of the weight. That leads to another way to solve the problem. If you noticed the one fourth at the start you could do it in your head: $G$ must be three times as far from 3.0 as 2.8 is, so it must be 3.6.

Finally, you can answer the question even if you’ve forgotten your algebra and don’t see the answer right away. Just guess, check your guess, and guess again as long as you have to.

Try a first guess of $G = 3.0$ in expression (5.1):

$$\frac{90 \times 2.8 + 30 \times 3.0}{120} = 2.85,$$

so 3.0 is too small. How about 4.0 (straight A work as a senior)?

$$\frac{90 \times 2.8 + 30 \times 4.0}{120} = 3.1,$$

which is more than she needs. The answer is somewhere between 3.0 and 4.0. Try 3.5:

$$\frac{90 \times 2.8 + 30 \times 3.5}{120} = 2.975.$$  

That’s almost enough. Guess 3.6 next:

$$\frac{90 \times 2.8 + 30 \times 3.6}{120} = 3.0.$$  

Bingo! Got it!

Guess-and-check isn’t as efficient as algebra, but it’s easy to remember, and it works in places where algebra won’t help. That makes it a better life skill.  

---

4See comment in instructor’s manual.
5See comment in instructor’s manual.
5.4 The consumer price index

The CPI calculator says that the inflation rate from 2006 to 2007 was 2.85%. Does that mean that a candy bar that cost $1.00 in 2007 sold for $1.0285 (or $1.03) in 2007? Unlikely! That would be true if the inflation rate applied literally to every purchase. But it doesn’t. The inflation rate is an average of the changes in costs of various items. The Bureau of Labor Statistics surveys the population to find out how much we spend on various goods and services to discover what weights to use when averaging the changes in prices:

The CPI market basket is developed from detailed expenditure information provided by families and individuals on what they actually bought. For the current [2010] CPI, this information was collected from the Consumer Expenditure Surveys for 2005 and 2006. In each of those years, about 7,000 families from around the country provided information each quarter on their spending habits in the interview survey. To collect information on frequently purchased items, such as food and personal care products, another 7,000 families in each of these years kept diaries listing everything they bought during a 2-week period.

Over the 2 year period, then, expenditure information came from approximately 28,000 weekly diaries and 60,000 quarterly interviews used to determine the importance, or weight, of the more than 200 item categories in the CPI index structure.

Table 5.2 shows the weights and the percentage changes for the several categories for urban consumers in the Northeast for the period 2006-2007.

<table>
<thead>
<tr>
<th>Category</th>
<th>weight (%)</th>
<th>2006-2007 change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food and beverages</td>
<td>15.009</td>
<td>4.0</td>
</tr>
<tr>
<td>Housing</td>
<td>44.377</td>
<td>3.0</td>
</tr>
<tr>
<td>Apparel</td>
<td>3.697</td>
<td>-1.5</td>
</tr>
<tr>
<td>Transportation</td>
<td>16.030</td>
<td>1.2</td>
</tr>
<tr>
<td>Medical care</td>
<td>5.780</td>
<td>4.8</td>
</tr>
<tr>
<td>Recreation</td>
<td>5.387</td>
<td>-0.4</td>
</tr>
<tr>
<td>Education and communication</td>
<td>6.455</td>
<td>2.7</td>
</tr>
<tr>
<td>Other goods and services</td>
<td>3.265</td>
<td>2.6</td>
</tr>
<tr>
<td>All items</td>
<td>100</td>
<td>2.58</td>
</tr>
</tbody>
</table>

The percentages in the second column add to 100, as they should. It’s interesting to note from the third column that average prices in Apparel and Recreation actually decreased. The 2.58 in the last row, last column is the weighted average of the changes in each category, calculated this way:

\[
0.15009 \times 4.0 + 0.44377 \times 3.0 - 0.03697 \times 1.5 + \cdots + 0.03265 \times 2.6 = 2.583642 \approx 2.58.
\]

6 Inflation in the cost of candy is more likely to occur as manufacturers make the bars smaller but sell them for the same price.

7 In fact, each of these changes is a weighted average of subcategories within each category. For example, food and beverages are broken down into those consumed at home and those consumed away from home.
5.5 NEW CAR PRICES FALL . . .

That figure for the 2006-2007 inflation rate for urban consumers in the Northeast doesn’t exactly match the nationwide average of 2.85% from the inflation calculator. That’s because the nationwide average is itself a weighted average of the regional averages, and prices in the Northeast seem to have increased less than those elsewhere.

5.5 New car prices fall . . .

On September 5, 2008 the business section in The Columbus Dispatch carried an Associated Press story headlined New-vehicle prices plunge, report says. The article said that

(t)he average price of a new vehicle in the second quarter fell 2.3 percent from a year earlier to $25,632. . . .

Since average costs (the Consumer Price Index) increased in that year, we were puzzled. We were pretty sure car prices had gone up too. When we read further we found that

. . . Truck-based vehicles such as pickup trucks, minivans, and SUVs accounted for less than half of all sales in the second quarter for the first time since 2001 . . .

Aha! That means vehicle prices could all have risen even while the average fell! We’ve made up some numbers to show how the arithmetic might work. Suppose truck-based vehicles accounted for 45% of the sales in 2008—that’s “less than half,” and that the average prices for truck-based vehicles and cars were $32.2K and $20.2K respectively, as shown in Table 5.3.

<table>
<thead>
<tr>
<th>model</th>
<th>average price ($K)</th>
<th>percent of market</th>
</tr>
</thead>
<tbody>
<tr>
<td>car</td>
<td>20.2</td>
<td>55</td>
</tr>
<tr>
<td>truck</td>
<td>32.2</td>
<td>45</td>
</tr>
</tbody>
</table>

Table 5.3: Vehicle sales: second quarter 2008

Then the average price for a vehicle would be

\[0.55 \times 20.2K + 0.45 \times 32.2K = 25,600 = 25.6K,\]

which is close to the reported average of $25,632.

Now imagine what the numbers might have been a year earlier, when, perhaps, truck-based vehicles outsold cars 55% to 45%, as in Table 5.4.

<table>
<thead>
<tr>
<th>model</th>
<th>average price ($K)</th>
<th>percent of market</th>
</tr>
</thead>
<tbody>
<tr>
<td>car</td>
<td>19.6</td>
<td>45</td>
</tr>
<tr>
<td>truck</td>
<td>31.6</td>
<td>55</td>
</tr>
</tbody>
</table>

Table 5.4: Vehicle sales: second quarter 2007

http://www.dispatch.com/content/stories/business/2008/09/05/new_vehicle_prices_0905.ART_ART_09-05-08_C10_AQB7T2V.html

© 2014 Ethan Bolker, Maura Mast 116
The average price for a vehicle would have been

\[ 0.45 \times 19.6K + 0.55 \times 31.6K = 26,200 \approx 26.2K. \]

With these assumptions, the average price fell by $600, a relative decrease of $600/26,200 = 0.0229 \approx 2.3\%$, as the article reports. But average prices for cars and trucks separately both increased.

This is really a warning about averaging averages. The average vehicle cost is an average of the average car cost and the average truck cost. The weights in that second average matter.

### 5.6 An averaging paradox

The average professor’s class size is smaller than the average student’s class size.

How can that be? Sometimes the best way to understand a paradox is to imagine a simple extreme example rather than trying to untangle complex real data.

Suppose a small department (with just one professor) offers just two classes. One is a large lecture, with 100 students. The other is a seminar on a topic so narrow that no students sign up. Then the average class size (from the professor’s point of view) is \((100 + 0)/2 = 50\) students, while each student’s average class size is 100.

### 5.7 Exercises

**Exercise 5.7.1.** [S][Section 5.1][Goal 5.1] Can she earn a B?

Suppose a student’s final grade in a Biology course is determined using the following weights:

- quizzes are worth 5%
- exam 1 is worth 20%
- exam 2 is worth 20%
- lab reports are worth 15%
- research paper is worth 15%
- final exam is worth 25%.

Just before the final, she has earned the following grades (all out of 100):

| Lab report grades: 75, 90, 85, 69, 70, 75, 80, 75 |
| Quiz grades: 85, 8, 0, 60, 70, 80, 80, 75 |
| Exam 1: 80 |
| Exam 2: 70 |
| Paper: 85 |

(a) What is her lab report average?

---

10See comment in instructor’s manual.
(b) What is her quiz average?

(c) What is her course average just before the final?

(d) For a B she needs a course average of at least 82%. What is the lowest grade she can get on the final and achieve that goal?

Exercise 5.7.2.  [S][Section 5.1]  [Goal 5.1] Fundraising.

On October 20, 2011 the Elizabeth Warren campaign provided *The Boston Globe* with the data we used to draw Figure 5.5. The first bar in the chart shows the total dollar contributions to her campaign, broken down according to where the money came from (Massachusetts vs. out of state). The second bar shows the total number of donors, broken down the same way.

![Elizabeth Warren's Fundraising](image)

**Figure 5.5: Where did the money come from?**

(a) What was the average donation (per donor)?

(b) What was the average donation from Massachusetts?

(c) What was the average donation from outside Massachusetts?

(d) Check that appropriate weighted average of your answers parts (b) and (c) gives the answer you found in (a).

Exercise 5.7.3.  [Section 5.2]  [Goal 5.1] A grade point average that matters.

Check that your own GPA has been computed correctly using the rules in effect at your school.

If you are in your first semester and don’t yet have a GPA, imagine the grades you expect at the end of the semester and figure out what your GPA would be.

If you are in your last semester then this exercise will take you a long time. Instead you may check your GPA for just one semester, or for the courses in your major.

---

5.7. EXERCISES

If all the courses you took carry the same number of credits, check that you get the correct GPA if you compute the average the old-fashioned way, without using any weights.

**Exercise 5.7.4.** [W][S] Section 5.3 [Goal 5.2] Ways to raise your GPA.

A UMass Boston student has earned 55 credits toward his degree but has a GPA of just 1.80. The registrar has informed him that he is on probation and will be suspended unless his overall GPA is at least 2.0 after one more semester.

(a) If he takes 12 credits, what is the minimum GPA he must earn in that semester to avoid being suspended?

(b) Answer the same question if he takes 9 credits.

(c) What if he took just 6 credits?

(d) (Optional) What would you do if you found yourself in a similar situation – take fewer courses in hopes of doing really well in them, or take more courses so that you could afford to do not quite so well in each?

[See the back of the book for a hint.]

**Exercise 5.7.5.** [S][Section 5.3] [Goal 5.1] [Goal 5.2] Gaming the system

In the *Chess Notes* column in *The Boston Globe* on Tuesday, March 10, 2008 Harold Dondis and Patrick Wolff wrote that

... the sensation of the [Amateur East] tourney was the team with the highest score, GGGg (no relation to the song Gigi but standing for the three Grandmasters and one future Grandmaster.) The players were our Eugene Perelshteyn, Roman Dzindzichasvili, Zviad Izoria, all Grandmasters, and 5-year-old Stephen Fanning who rounded out the team. Was this a valid lineup? Well, yes, it was. The rules of the Amateur provide that the average rating of a team could not exceed 2200. GGGg’s three Grandmasters were well above that, but Stephen Fanning ... had a current rating of 178, which brought the average rating to 2017.

The Grandmasters of GGGg ... delivered wins, while their fourth board, who would have won the prize as the cutest chess player (if the sponsors had had the foresight to establish such a prize) struggled to make legal moves, sometimes failing to do so. Naturally, there followed an extensive debate as to whether the victorious ensemble had gamed the system.

(a) If the three Grandmasters had the same rating, what would it have been?

(b) Did GGGg game the system?

(c) How might the tourney organizers change the rules to prevent this kind of team from winning?

**Exercise 5.7.6.** [S][Section 5.3] [Goal 5.1] [Goal 5.2] Good day, sunshine.

On June 23, 2009 *The Boston Globe* reported that

June 2009 in Boston might turn out to be the dimmest on record. So far in the month, the sun was shining only 32% of the time. The record low was in 1903, when the sun shone only 25% of the time. 

5.7. EXERCISES

(a) What percentage of sunshine for the remaining days of the month would make 2009 at least a tie for the dimmest June?

(b) Did that happen?

Exercise 5.7.7. [S][Section 5.3 | Goal 5.1 | Goal 5.2] The Hightower Lowdown.

In *The Hightower Lowdown* (Volume 12, Number 5, May 2010) you can read

- 5 MILLION PEOPLE (about 10% of the workforce) are out of work.
- UNEMPLOYMENT IS HEAVILY SKEWED BY CLASS. Among the wealthiest 10% of American families (incomes above $150,000), only 3% are unemployed – a jobless rate that rises as you go down the income scale. Among the bottom 10%, more than 30% are out of work.

What average unemployment rate for the middle 80% of families fits with the given values for the top and bottom 10% to work out to the overall (weighted) average unemployment rate of 10%?

Exercise 5.7.8. [S][A][Section 5.3 | Goal 5.1 | Goal 5.3] Who wins?

Alice and Bob are both students at ESU. In September they start a friendly competition. In June they compare transcripts. Alice had a higher GPA for both the Fall and Spring semesters. Bob had a higher GPA for the full year.

(a) Explain how this can happen, by imagining their transcripts – number of credits and GPA for each, for the two semesters and for the full year, as in this table:

<table>
<thead>
<tr>
<th></th>
<th>Fall credits</th>
<th>Fall GPA</th>
<th>Spring credits</th>
<th>Spring GPA</th>
<th>Year GPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alice</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bob</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Who wins?

[See the back of the book for a hint.]

Exercise 5.7.9. [S][Section 5.4 | Goal 5.1 | Goal 5.2] Your rate may vary.

(a) Suppose a student in 2007 estimated that her expenses were distributed among the CPI categories as listed in Table 5.6. (She had no housing expenses since she lived at home.) Calculate the inflation rate she would have experienced relative to the year before.

(b) Compare the answer in (a) to the national average of 2.58% and explain why the rate the student experienced was higher or lower.

(c) Estimate the weights in *your* life now for the various categories used to compute the CPI in Table 5.2. Then figure out how much your cost of living would have increased from 2006-2007 if you’d been living then with the same lifestyle.

---

13 Thanks to an anonymous reviewer for this problem.
Table 5.6: How one student spent her income

<table>
<thead>
<tr>
<th>Category</th>
<th>weight (%)</th>
<th>2006-2007 change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food and beverages</td>
<td>15</td>
<td>4.0</td>
</tr>
<tr>
<td>Housing</td>
<td>0</td>
<td>3.0</td>
</tr>
<tr>
<td>Apparel</td>
<td>15</td>
<td>-1.5</td>
</tr>
<tr>
<td>Transportation</td>
<td>25</td>
<td>1.2</td>
</tr>
<tr>
<td>Medical care</td>
<td>5</td>
<td>4.8</td>
</tr>
<tr>
<td>Recreation</td>
<td>10</td>
<td>-0.4</td>
</tr>
<tr>
<td>Education and communication</td>
<td>15</td>
<td>2.7</td>
</tr>
<tr>
<td>Other goods and services</td>
<td>15</td>
<td>2.6</td>
</tr>
<tr>
<td>All items</td>
<td>100</td>
<td>?</td>
</tr>
</tbody>
</table>

Exercise 5.7.10. [S][C][Section 5.4] [Goal 5.1] [Goal 5.2] Regional differences in the CPI.

We saw in Section 5.4 above that the average 2006-2007 inflation rate for the Northeast urban consumer was 2.58% while the national average was 2.85%.

(a) Estimate the fraction of the population of the United States that counts as urban in the Northeast.

(b) Use your estimate to estimate the average inflation rate for the rest of the country.

Exercise 5.7.11. [S][Section 5.4] [Goal 5.1] [Goal 5.2] Eat out or in?

The overall change in the Consumer Price Index is a weighted average of changes in various categories. On November 19, 2011 The Boston Globe on reported the data displayed in Figure 5.7.

(a) The change in the CPI for food is a weighted average of the change in the cost of food at home and the change in the cost of food away from home. Find the weights.

(b) Is it reasonable to say that about 40% of food expenses are for food away from home?

\[14\] http://www.bostonglobe.com/business/2011/11/19/rising-food-prices-mean-more-costly-thanksgiving/1lGoSN1dznVRfn1H0hTXdI/1graphic.html
The overall change in the CPI is a weighted average of the change for food and the change for everything else. Do you have enough information to find the weights?

Exercise 5.7.12. [U][Section 5.5] [Goal 5.2] We made up the numbers in our car price example in Section 5.5. Do some research to find the real ones.

Exercise 5.7.13. [S][C][Section 5.5] [Goal 5.1] Projected cost of F-35 program up to $382B

On June 2, 2010 USA Today reported a story from Bloomberg News:

The projected cost of Lockheed Martin’s F-35 Joint Strike Fighter, the most expensive U.S. weapons program, is $382 billion, 65% higher than the $232 billion estimated when the program started in 2002, according to a projection from independent Pentagon analysts sent to Congress.

The Pentagon’s cost-analysis office reports that the price per plane – including research, development and construction costs – is $112.4 million, the official told Bloomberg. That’s about 81% more than the original estimate of $62 million.

The production cost alone of each plane is estimated at $92.4 million, almost 85% higher than the $50 million projected when the program began, the Defense Department analysis said.

(a) Check the arithmetic to see that the three percentage increases reported are correct.

(b) Can you explain how the overall cost can have increased by just 65 percent when the overall cost per plane increased by 81 percent and the construction cost alone has increased by almost 85 percent?

(c) Rewrite the article using costs corrected for the inflation between 2002 and 2010.

One of the readers commented that

The Defense Department’s cost estimates, cited by Bloomberg News, have never withstood scrutiny, and the latest ones are equally suspect. A report on the F-35 from the U.S. Government Accountability Office (March, 2008, available at [http://www.gao.gov/new.items/d08388.pdf](http://www.gao.gov/new.items/d08388.pdf)) found the cost of buying and operating an estimated 2,458 aircraft had reached $950 billion—about $122 million per aircraft for acquisition and $264 million for operation over an expected life-cycle. The GAO needs to update its figures; they are unlikely to have gone down.

(d) How would you reconcile the figures in the two quotations?

Exercise 5.7.14. [U][Section 5.6] [Goal 5.1] Estimate the average class size at your school, in each of the two ways discussed in the text. Which figure will the school’s publicity office post on the web site to attract new students?

Exercise 5.7.15. [U][Section 5.6] [Goal 5.3] [Goal 5.1] Your car is more crowded than you think.

Table 5.8 reports results from a 1969 Personal Transportation Survey on “home-to-work” trips in metropolitan areas.

(a) The survey stated that the average car occupancy was 1.4 people. Check that calculation.

---


16 The data and the problem are from David Hemenway, *Why your classes are larger than “average”*, Mathematics Magazine, 55, #3 (1982)
5.7. EXERCISES

<table>
<thead>
<tr>
<th>number of riders</th>
<th>percentage of cars</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>73.5</td>
</tr>
<tr>
<td>2</td>
<td>18.2</td>
</tr>
<tr>
<td>3</td>
<td>4.7</td>
</tr>
<tr>
<td>4</td>
<td>1.9</td>
</tr>
<tr>
<td>5</td>
<td>1.1</td>
</tr>
<tr>
<td>6</td>
<td>0.5</td>
</tr>
<tr>
<td>7</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 5.8: Rush Hour Car Occupancy

(b) Show that the average number of riders in the car of a typical commuter is 1.9 people.

(c) Suppose you could persuade drivers of single occupant cars to switch to five person car pools in order to increase the average number of riders per car from 1.9 to 2.

What would the percentage of single occupant cars be then? How many people would be in the car of a typical commuter?

[See the back of the book for a hint.]

Exercise 5.7.16. [S] [Section 5.6] [Goal 5.1] [Goal 5.3] Why Your Friends Have More Friends Than You Do

Imagine a small social network – perhaps Facebook when it was just starting out – with 100 people. One of them is friends with the other 99, but none of those 99 is a friend of any of the others.

(a) What is the average number of friends in this network?

(b) Explain why the statement “your friends have more friends than you do” is true for almost everybody in this network.

This network is just a toy one for making easy computations. But the paradox is true for real social networks, whenever there are some people with many friends and some with few. It’s surely true for Facebook.

For more on this interesting paradox, see the article Why Your Friends Have More Friends Than You Do by Scott L. Feld, American Journal of Sociology, Vol. 96, No. 6 (May, 1991), pp. 1464-1477, written long before Facebook.

Review exercises

Sample routine review questions. When Common Sense Mathematics is published these and others may be available in an online homework system/

Exercise 5.7.17. [A] Find the average of each set of numbers.

(a) 40, 20, 30, 50, 40, 40, 55, 45, 60, 30

(b) 0, 0, 0, 5, 10, 10, 10, 10

(c) 5, 5, 5, 5, 5, 5, 5, 5, 5, 5
5.7. EXERCISES

(d) 0, 0, 0, 100, 100, 100
(e) 0.2, 0.4, 0.33, 0.45, 0.2, 0.1, 0.1, 0.1, 0.2, 0.4
(f) 5, 3, 6, 1, 4, 4, 4, 7, 6, 6, 4, 5, 1
(g) 86, 72, 86, 90, 91, 86, 75, 88, 42, 89, 90

Exercise 5.7.18. [A] Find each weighted average.

(a) Three test scores were 75, five test scores were 88 and two test scores were 90.
(b) Over the past month, I took three 5-mile runs, four 7-mile runs and ten 3-mile runs.
(c) Last week, the store sold 25 DVDs at $14.99 each; 13 DVDs at $12.99 each and 19 DVDs at $7.99 each.

Exercise 5.7.19. [A] Find the grade point average for each student, using the chart given in the book.

(a) Bob earned an A in a 3-credit course, a B- in a 3-credit course, a B+ in a 4-credit course and an A- in a 3-credit course.
(b) Mary earned a C+ in a 3-credit course, an A- in a 4-credit course, a B+ in a 2-credit course, a B+ in a 3-credit course and a B in a 3-credit course.
(c) Alice earned a D+ in a 2-credit course, an A in a 3-credit course, an A- in a 4-credit course and a B+ in a 4-credit course.

Exercise 5.7.20. [A] Find the semester and cumulative gpa for each student, using the chart in the book.

(a) Mike has a 2.9 cumulative GPA and 45 credits. This semester he took three 4-credit courses and earned grades of B-, B+ and B+. He also took a 3-credit course and earned a C grade.
(b) Hilda has a 2.5 cumulative GPA and 16 credits. This semester she took a 3-credit course and earned a C+, a 4-credit course and earned a B-, and a 5-credit course and earned an A-. 