10

Borrowing and Saving

When you borrow money — on your credit card, for tuition, for a mortgage — you pay it back in installments. Otherwise what you owe would grow exponentially. In this chapter we explore the mathematics that describes paying off your debt.

Chapter goals:

Goal 10.1. Examine how debit and credit cards work.

Goal 10.2. Study balance and interest when paying off a loan periodically.

Goal 10.3. Calculate monthly mortgage payments and examine the costs and benefits of home ownership.

Goal 10.4. Understand periodic compounding, APR and other interest terms.

Goal 10.5. Understand the basics of saving money with a long-term goal like retirement.

10.1 Debit and credit cards

In the old days people shopped with paper money. Now the answer to "paper or plastic?" is more likely to be "plastic" — or "neither," if you pay with an app.

The infrastructure that supports the convenience of a cash-free economy doesn't come free. In this section we will look at how it is paid for.

Debit cards.

You open a bank account and deposit money. The bank gives you a debit card — magic plastic that you present at a coffee shop in exchange for a $3 latte. The bank sends the shop $3 from your account, after subtracting a processing fee.

> [A] merchant who accepts a swiped Visa debit payment from a customer would pay either 0.80% plus $0.15 or 0.05% plus $0.21 in Interchange fees for that transaction. [R1]

The fee for your $3 latte would be about 18 or 21 cents, depending on the option, so about 6%. The percentage would be less on a larger purchase.

Merchants take debit card fees into account when setting prices, so you pay them, indirectly. They are a reasonable price to pay for the convenience of the card, as long as you have the money in your account. If you don't, the bank can pay the merchant anyway, collect from you later, and charge you an *overdraft fee* for the extra convenience.

The online bank *Chime* reported that

> in 2014, the Consumer Financial Protection Bureau (CFPB) found that the majority of overdraft fees were charged on transactions of $24 or less. With a median fee of $34 at the time, the same type of charge on a loan for a similar three day period would result in an annual percentage rate (APR) of 17,000%. [R2]

Let's check the arithmetic. If the bank honors your $24 purchase even when you have no money in your account they are lending you that money until you pay back $\$24 + \$34 = \$58$ three days later. The $34 fee corresponds to an interest rate of

$$\frac{\$34}{\$24} \approx 1.42 = 142\%.$$

Since there are 120 three day periods in a year, the annual percentage rate is about

$$120 \times 142\% \approx 17,000\%.$$

That agrees with what Chime reported.

You can avoid borrowing money at that interest rate.

> Banks get to decide either to cover or reject a transaction that would make your balance negative, but you can control one thing. Opting out of an overdraft coverage program means that your bank cannot cover one-time debit card or ATM transactions or charge overdraft fees on them. [R3]

BANK OF THE PELOPONNESE
CREDIT CARD STATEMENT

ACCOUNT NUMBER	NAME	STATEMENT DATE	PAYMENT DUE DATE
314159265359	Glyphne Muse	02/01/20	03/01/20

CREDIT LINE	CREDIT AVAILABLE	NEW BALANCE	MINIMUM PAYMENT DUE
$1,200.00	$1,074.76	$125.24	$20.00

REFERENCE	SOLD	POSTED	ACTIVITY SINCE LAST STATEMENT	AMOUNT
2P71828182		1/25	PAYMENT THANK YOU	-166.80
1P41421356	1/12	1/16	MERCURY TRANSPORT	14.83
0P69314718	1/13	1/16	HAMMER & CHISEL HARDWARE	30.55
1P73205080	1/18	1/18	MADAME DELPHI	27.50
6P02214E23	1/20	1/22	OLYMPIA DINER	12.26
6P6260E-34	1/28	1/30	CALLIOPE CONSULTING	40.10

Previous Balance	(+)	166.80	**Current Amount Due**	125.24
Purchases	(+)	125.24	**Amount Past Due**	
Cash Advance	(+)		**Amount Over Credit Line**	
Payments	(-)	166.80	**Minimum Payment Due**	20.00
Credits	(-)			
FINANCE CHARGES	(+)			
Late Chartes	(-)			
NEW BALANCE	(-)	125.24		

FINANCE CHARGE SUMMARY	PURCHASES	ADVANCES
Periodic Rate	1.65%	0.54%
Annual Percentage Rate	19.80%	6.48%

Figure 10.1. A credit card statement [R4]

Forego the latte if you can't pay for it right now:

Opt out of overdraft coverage.

Credit cards.

When you pay with a debit card you spend money you already have. When you pay with a credit card you promise to pay later. Your bill explains the interest you're charged for borrowing that money. Figure 10.1 shows a sample credit card statement.

If you have a credit card you get a statement like this once a month. Glyphne Muse[1] (the owner of this card) charged $125.24 in merchandise and services during January. She's decided not to use this card any longer, and will settle her debt by paying $20 each month. When will she be debt free, and how much interest will she have paid?

She makes a minimum payment of $20 in February for her January purchases, so her balance is $\$125.24 - \$20 = \$105.24$. She's paid no interest so far. But now that changes. The credit card company charges interest on the balance she carries in February. The FINANCE CHARGE SUMMARY shows a periodic (that is,

[1] Eleanor Bolker discovered Glyphne, the muse of graffiti.

Month	Balance	Interest
Jan	125.24	0.00
Feb	105.24	1.74
Mar	86.98	1.44
Apr	68.41	1.13
May	49.54	0.82
Jun	30.36	0.50
Jul	10.86	0.00
total		5.80

Table 10.2. Seven months to pay it off

Starting balance	105.24
Interest rate	0.0165
Fixed payment	20

Period	Balance	Interest
0	=START	=C15*RATE
=B15+1	=C15*(1+RATE)-PAYMENT	=C16*RATE
=B16+1	=MAX(C16*(1+RATE)-PAYMENT,0)	=C17*RATE
=B17+1	=MAX(C17*(1+RATE)-PAYMENT,0)	=C18*RATE
=B18+1	=MAX(C18*(1+RATE)-PAYMENT,0)	=C19*RATE
=B19+1	=MAX(C19*(1+RATE)-PAYMENT,0)	=C20*RATE
=B20+1	=MAX(C20*(1+RATE)-PAYMENT,0)	=C21*RATE
=B21+1	=MAX(C21*(1+RATE)-PAYMENT,0)	=C22*RATE
=B22+1	=MAX(C22*(1+RATE)-PAYMENT,0)	=C23*RATE
=B23+1	=MAX(C23*(1+RATE)-PAYMENT,0)	=C24*RATE
=B24+1	=MAX(C24*(1+RATE)-PAYMENT,0)	=C25*RATE
=B25+1	=MAX(C25*(1+RATE)-PAYMENT,0)	=C26*RATE
=B26+1	=MAX(C26*(1+RATE)-PAYMENT,0)	=C27*RATE
=B27+1	=MAX(C27*(1+RATE)-PAYMENT,0)	=C28*RATE
=B28+1	=MAX(C28*(1+RATE)-PAYMENT,0)	=C29*RATE
=B29+1	=MAX(C29*(1+RATE)-PAYMENT,0)	=C30*RATE
=B30+1	=MAX(C30*(1+RATE)-PAYMENT,0)	=C31*RATE

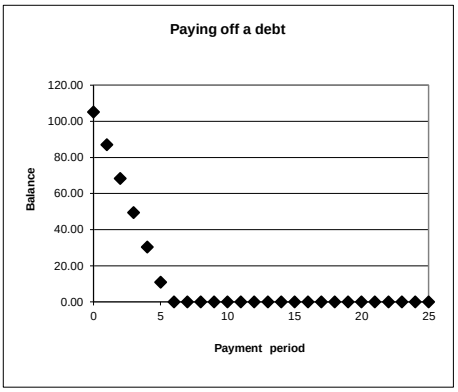

Figure 10.3. Paying off credit card debt

monthly) rate of 1.65% so she will pay $\$105.24 \times 0.0165 = \$1.74$ in interest. The 1+ trick tell us that at the beginning of March she owes

$$\$105.24 \times 1.0165 = \$106.98.$$

After her \$20 payment on March 1 her balance is

$$\$105.24 \times 1.0165 - \$20 = \$86.98.$$

The credit card company used \$1.74 of the payment for the February interest. The rest they subtracted from her balance.

Table 10.2 tells the rest of the story. Figure 10.3 shows the Excel formulas we used in `PayOffDebt.xlsx` to calculate the values in that table, along with a graph showing how the balance decreases each month at a slightly faster rate, until it reaches 0.

Glyphne's last payment is for the \$10.86 balance.

In seven months she's paid \$5.80 in interest. That doesn't seem too terrible. It's $\$5.80/\$125.24 = 0.046311 \approx 4.6\%$. But don't be fooled. This is a monthly statement, so that 4.6% isn't the annual interest rate. Glyphne didn't borrow that money for a year. Some of it she had for seven months, some for just one.

The law requires the credit card company to tell you the *APR* (*annual percentage rate*) somewhere on the monthly statement. You can find it on this one in the **FINANCE CHARGE SUMMARY** section: it's 19.80%. You can check their arithmetic: $1.65\% \times 12 = 19.80\%$. We will have more to say about the APR in Section 10.4.

That large interest rate is why the credit card company wants you to pay just the small minimum each month. The full balance appears at the top of the statement labelled NEW BALANCE — but the only payment shown is the MINIMUM PAYMENT DUE. You have to know that it's best for you to pay the full balance at once.

There are other ways credit card companies make you pay for the convenience of borrowing their money. One kicks in if you miss a payment by even a day or so. Then they charge a substantial late payment fee and may also increase the already large interest rate. The law requires credit card companies to print a warning on your statement. Here's what one says:

> **Late Payment Warning:** If we do not receive your minimum payment by the date listed above you may have to pay up to a \$39.00 late fee and your APRs will be subject to increase to a maximum Penalty APR of 29.99%.

Moreover, the late payment will show up on your credit report, so when you go to a bank later to take out a mortgage on the condo you want to buy, they may charge you a higher interest rate too.

Does all this mean that using a credit card is a bad idea? No, as long as you're careful. Then you can take advantage of some of the good things credit can do for you:

- Glyphne's statement shows that she paid her last balance of \$168.80 on time in full, avoiding all finance charges. So, in fact, she borrowed that money for a month from the credit card company at no cost. If she kept it in a savings account until it was time to pay his bill she'd even have made a few pennies in the meanwhile.
- Some credit cards give you back a reward at the end of the year — perhaps 1% of your total purchase dollars or some frequent flyer airline miles.
- Merchants pay credit card service fees like those for debit cards, but usually larger — perhaps 2%. When making a large purchase you may be able to

negotiate a discount for paying by cash or check. Even when you pay the full price you may want to save the local merchant the fee.

- If you miss a payment your credit rating may suffer. But just avoiding credit errors won't get you a good credit rating. For that you have to prove you can manage debt — by having a credit card and paying the balance in full when due. Then when it's time to borrow money for a car or a condo your good credit rating may get you a lower interest rate.
- If you have a balance on an existing card that you can't afford to pay off immediately, consider opening a second card and transferring the balance. The new card company may offer you 0% interest for a while to encourage you to switch. If you do that and then don't use the new card you can pay off the old balance over time without any further interest charges. Be sure to read the small print before you do this — the transferred debt may be interest free, but often there's a charge (perhaps three or four percent) to make the transfer.
- Federal legislation passed in response to the 2009 financial crisis forced credit card companies to change their policies so that "payments above the Minimum Payment Due will be applied first to higher interest rate balances." That notification appeared on one of the authors' statements, along with the kind thought that "This may help you to pay off your highest interest rate balances more quickly and reduce your interest charges." They did not reveal how much money they spent lobbying in Washington against the regulation.
- Finally, you may find a credit card issued by one of your favorite charities. Then the charity collects a small fraction of the fees the merchants pay.

Do remember:

Pay your full balance on time every month.

You can even arrange to have that happen automatically from your bank account, so you don't have to remember and you save the cost of a stamp. Just make sure there's enough money in the bank.

10.2 Can you afford a mortgage?

There's a $250,000 condominium in Denver you want to buy. You've managed to scrape together $50,000 for the down payment (savings, your parents, . . .) but will have to take out a mortgage for the $200,000 balance. Can you afford it? There are many websites that provide a place to start. We visited `smartasset.com/mortgage/colorado-mortgage-calculator` filled out the Mortgage Calculator and discovered that on June 3, 2019, in Denver, Colorado you could get a 30 year

fixed rate mortgage at 4.38% annual interest with a monthly payment of $999 or a 15 year fixed rate mortgage at 3.88% with a monthly payment of $1467.

In this section we'll look at what those numbers mean, see how they are calculated and discuss a few important issues (some quantitative, some not) that you should think about when making a decision like this one.

Paying off a mortgage is like paying off a credit card balance when you make no new purchases. There's an annual rate. Your balance at the end of a month includes interest computed at one twelfth of the annual rate. Each month you pay all the current interest and some of the principal. Since the principal is decreasing, there's less interest each month so more of the payment goes toward the principal. One difference is that the credit card company sets the minimum payment; then it takes as long as it takes to pay off the balance, while the mortgage payment is figured out in advance so that everything is paid off at a particular time — usually 15 or 30 years.

The mortgage company uses this formula to calculate the monthly payment:

$$P \times \frac{r/12}{1-\left(1+\frac{r}{12}\right)^{-12y}} \tag{10.1}$$

where P is the principal (the amount of your mortgage), r is the annual interest rate, and y is the length of the mortgage, in years.

It is probably the most complicated formula in *Common Sense Mathematics*. We won't explain where it comes from, and you need not memorize it. But you can understand some parts of it. It has the form

$$P \times (\text{complex expression involving } r \text{ and } y)$$

which tells you that your monthly payment is proportional to P. The complex part is the expression in parentheses — the *effective monthly rate*. That's the number of dollars in your payment for each dollar you borrow. There the $r/12$ finds the monthly rate from the annual rate. The product $12y$ is the number of months in y years.

You can use that formula to check that the Wells Fargo calculator finds the right monthly payment of $984 for a 30 year $200,000 loan at 4.250% interest. We did the arithmetic in Excel, with the formula

```
=(STARTBALANCE*INTERESTRATE/12)/(1-(1+INTERESTRATE/12)^(-12*YEARS))
```

in cell `C11` on the `mortgage` worksheet in `PayOffDebt.xlsx`. There you can see the principal balance at the end of each year and the total interest paid. On that 30 year mortgage it's $154,196.72.

When you borrow you always pay back more than the amount you borrowed — in this case, $150,000 in interest in addition to the $250,000 principal. Should that frighten you? Maybe or maybe not. Is it worth it? Perhaps, for several reasons.

- It would take a long time to save up the full purchase price (to avoid borrowing). Saving would be difficult because you would be paying rent the whole time. So you can think of the mortgage payments as money spent instead of rent.
- The condo may well be worth more after 15 or 30 years than the total you paid for it — even including the interest on the mortgage.
- Inflation is pretty nearly inevitable over the years. These computations are all made in dollars computed in the year you make the purchase, but the actual value of that money when you pay it to the bank in later years will be less, in then current dollars. Think of it this way: your salary is likely to increase at least as fast as inflation, so the fixed monthly mortgage payments will be a smaller and smaller percentage of your take home pay.
- That said, you do want to minimize the amount of interest you pay, by paying attention to the significant difference between a 15 and a 30 year mortgage. The short one has a lower interest rate (2.5% instead of 3.5%) and a much lower total interest cost: about $40,000 instead of $123,000. So you should choose it if you can manage the extra $430 per month in payments.
- You will also get a lower rate if you have established a good credit rating in the years before you apply for your mortgage. So start now to use a credit card wisely.

Words of warning. This discussion shows how, in principle, you pay off a loan by paying some interest and some principal periodically. That's just one of the financial things you'll need to understand when you think about buying a house or condo. Just asking the bank or shopping online for an interest rate isn't sufficient. As with most other topics in this book our hope is to provide a quantitative foundation for further questions. Some of those will address these issues:

- There are other up front costs: legal fees, title searches, inspections, points.
- The cost of owning is more than just the cost of the mortgage. You must be prepared for expenses that your landlord would cover if you were renting — things like real estate tax, insurance, repairs.
- Variable rate mortgages generally start out with lower rates than fixed rate mortgages — but payments can balloon when the initial rate expires.

There are many books and web pages that may help — here's just one we found with a simple search: `www.ourfamilyplace.com/homebuyer/checklist.html` .

Starting balance	0
Interest rate	6.0000%
Fixed payment	-1200

Period	Balance	Interest
0	0.00	0.00
1	1200.00	72.00
2	2472.00	148.32
3	3820.32	229.22
4	5249.54	314.97
5	6764.51	405.87
6	8370.38	502.22
7	10072.61	604.36
8	11876.96	712.62
9	13789.58	827.37
10	15816.95	949.02
11	17965.97	1077.96
12	20243.93	1214.64
13	22658.57	1359.51
14	25218.08	1513.08
15	27931.16	1675.87
16	30807.03	1848.42

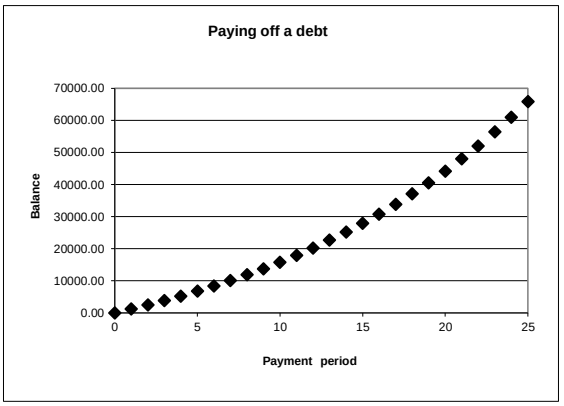

Figure 10.4. Saving for retirement

10.3 Saving for college or retirement

In Chapter **??** we studied how money accumulates when you invest a big chunk and let the interest compound. But you rarely save a big chunk of money all at once. A more realistic way to save, for college (for your children) or for retirement, is to put away a fixed amount on a regular basis.

The kind of calculations we made above to study paying off a debt will help now to study how money saved regularly accumulates. Suppose you can invest $1200 a year and make your payment at the end of the year.

You think you can get a 6% return on your investment, since you're willing to take some short term risk for the sake of long term return. At the end of the second year you will have

$$\$1200 \times 1.06 + \$1200 = \$2472$$

and after the third

$$\$2472 \times 1.06 + \$1200 = \$3820.32.$$

These calculations look just like the ones we made for paying down credit card debt, except that in this case we add the periodic payment to the balance rather than subtracting it. That means we can use the debt payment spreadsheet to see how our money accumulates by entering a negative "payment" to be added to the growing balance. Figure 10.4 shows the result (we haven't changed the labels).

In 25 years you will have accumulated nearly $66,000. Of that amount, you contributed just $\$1,200 \times 25 = \$30,000$. The rest, more than $35,000, is interest. Making precise sense of the total accumulation in cell `C40` and the total interest in cell `D41` is a little tricky. You have to think carefully about whether these values are computed at the beginning or the end of the year, before or after interest is credited. You don't need to do this detailed analysis to understand the principle.

10.4 Effective interest rate

We discovered in Chapter **??** that compounding is a good thing for your investments. When you have a balance on your credit card it's a good thing for the credit card company. The sample bill in Section 10.1 lists the Annual Percentage Rate (APR) as 19.08% and the Periodic Rate as $19.08\%/12 = 1.65\%$.

At the end of the year interest charged monthly will have been compounded twelve times. Since

$$1.0165^{12} = 1.217,$$

the *effective annual rate* (EAR) is 21.7% — almost 2 full percentage points higher than the already substantial advertised APR of 19.8%. In Europe credit cards must report the EAR where we see the APR.

The effective rate works for you rather than against you when you invest rather than borrow. In the last section we looked at saving for retirement by saving \$1200 a year at 6% interest, paid once a year. You shouldn't be surprised that monthly compounding will earn you interest on your interest if you save \$100 each month instead. Then the computation

$$\left(1+\frac{0.06}{12}\right)^{12} = 1.0616778119 \approx 1.0617. \tag{10.2}$$

tells you your effective interest rate is about 6.17%. It's not a coincidence that the left side of this equation matches part of the complicated formula in Equation 10.1.

The effective annual rate for the already outrageous debit card fee APR of 17,000% is unimaginable:

$$142^{120} \approx 1.88 \times 10^{258},$$

which is 188 followed by 256 zeroes.

Whether you think of the true interest rate as the APR or the EAR the moral is clear. Don't spend small sums of money you don't have. Tell your bank to refuse payment when your account is empty. Go without that particular cup of coffee.

10.5 Instantaneous compounding

We have just spent some time working with expressions like the one in Equation 10.2, which quantifies the advantage of montnly compounding: 6% is effectively 6.17%.

If monthly compounding is good then daily compounding must be even better. To see what six percent annual interest compounded daily leads to, compute

$$\left(1+\frac{0.06}{365}\right)^{365} = 1.06183131.$$

That corresponds to an effective annual rate of 6.183%. Hourly compounding gives

$$\left(1+\frac{0.06}{8765}\right)^{8765} = 1.06183633,$$

which is just a tiny bit better. Compounding every minute results in 1.06183654, which differs only in the seventh decimal place. These computations suggest that as you compound more and more often you do better and better, but by less and less. There seems to be a limit. In fact there is. You can find it with the magic number e and the Excel function `EXP` we discussed in Section **??**. If you compound 6% annual interest *every instant* the computation

$$e^{0.06} = \mathrm{EXP}(0.06) = 1.061836547$$

tells you the effective interest rate to nine decimal places.

To compare monthly and instantaneous compounding in terms that are easier to understand, suppose you invested a thousand dollars. Then the Google calculator tells you

```
1 000 * ((e^0.06) - ((1 + 0.06/12)^12)) = 0.158734681
```

so the difference after one year is about 16 cents. That's chump change for a thousand dollar investment.

Finally, suppose you could find someone to pay you 100% interest annually. Then without compounding, one dollar would double, and become two. If you compounded instantaneously your dollar would turn into $e =$ `EXP(1)` = 2.72 dollars in a year.

10.6 Exercises

Exercise 10.6.1. [W][Section 10.1][Goal 10.1] Your credit report.

> The Fair Credit Reporting Act (FCRA) requires each of the nationwide credit reporting companies — Equifax, Experian, and TransUnion — to provide you with a free copy of your credit report, at your request, once every 12 months. The FCRA promotes the accuracy and privacy of information in the files of the nation's credit reporting companies. The Federal Trade Commission (FTC), the nation's consumer protection agency, enforces the FCRA with respect to credit reporting companies. [R5]

To get your reports, visit `www.annualcreditreport.com/index.action`.

That's not a clickable link. You have to type it into your browser. Here's why, from `www.annualcreditreport.com/aboutThisSite.action`.

> AnnualCreditReport.com is the official site to get your free annual credit reports. This right is guaranteed by Federal law. To verify that this is the official site, visit `www.consumerfinance.gov/askcfpb/311/how-do-i-get-a-copy-of-my-credit-report.html`.
>
> Don't be fooled by look-alike sites. You can be sure that you are on the right site if you type `www.annualcreditreport.com` in your browser address line. Don't come to this site by clicking on a link in another site or in an email.

Now you have your credit *report*. That's not the same as credit *score*.

> Based on the information in your credit report, lenders calculate your credit score so they can assess the risk you pose to them before they decide whether they will give you credit. The higher your score, the less risk you pose to creditors.
>
> The information in your credit report is used to calculate your FICO (the acronym stands for Fair, Isaac and Company) score. Your score can range anywhere from 300-850. Aiming for a score in the 700s will put you in good standing. A high score, for example, makes it easier for you to obtain a loan, rent an apartment or lower your insurance rate. [R6]

Look for a place on the web that will give you an estimate of your credit score. Wherever you go, be sure to read the fine print, and don't pay for anything. Some credit and debit card companies provide an updated score to their customers.

Without revealing any of your personal information, write about how easy or difficult it was to estimate your credit score. What did you learn as you did this research?

Exercise 10.6.2. [S][Section 10.1][Goal 10.1] How long to pay it off?

Starting in 2010 credit card companies were required to provide the information in Table 10.5 each month. The numbers there are for a bill with a balance of \$2,020.37, a minimum payment amount of \$40.00 and an annual percentage rate of 12.24%.

If you use the `PayOffDebt.xlsx` spreadsheet to work on this exercise you will need the `mortgage` worksheet, since the `plain` worksheet only covers 25 payment periods.

(a) Verify the three year time to pay off the balance at a rate of \$67 per month.

(b) Show that a constant monthly payment of \$40 is much more than is needed to pay off the balance in 18 years. How can the 18 year claim be correct?

If you make no additional charges using this card and each month you pay . . .	You will pay off the balance shown on this statement in about . . .	And you will end up paying an estimated total of . . .
Only the minimum payment	18 years	$3,843
$67	3 years	$2,426 (Savings=$1,417)

Table 10.5. Paying off a credit card balance

(c) The 2010 Consumer Credit Law allows banks to raise the minimum payment on an account to a constant amount sufficient to pay off the balance in five years. What would that minimum payment be for this bill?

(a) Verify the three year time to pay off the balance at a rate of $67 per month.

The spreadsheet computes a monthly payment of $67.34 when I put in 18 years for the length of the "mortgage". That's close enough.

(b) Show that a constant monthly payment of $40 is much more than is needed to pay off the balance in 18 years. How can the 18 year claim be correct?

The spreadsheet computes a monthly payment of $23.20 when I put in 18 years for the length of the "mortgage". That's just over half the $40 minimum payment for the current bill.

The company says that paying the minimum each month will settle the debt in 18 years. If the minimum stayed at $40 I'd be done much sooner than that. The company could be right because I think the minimum payment will be smaller when the bill is smaller.

(c) The 2010 Consumer Credit Law allows banks to raise the minimum payment on an account to a constant amount sufficient to pay off the balance in five years. What would that minimum payment be for this bill?

The spreadsheet tells me that's $45.19. Interesting that it's in the same ballpark as the current minimum payment.

Exercise 10.6.3. [U][R][Section 10.2][Goal 10.3] Build your own mortgage.

Redo the computations in Section 10.2 for a house or condo of your choice in your town. Start with a reasonable cost and down payment. Find rates from at least two separate on line sites; check them with the formula and the `PayOffDebt.xlsx` spreadsheet.

Exercise 10.6.4. [U][Section 10.2][Goal 10.3][Goal 10.2] Using the debt payoff spreadsheet.

The debt payoff spreadsheet can reproduce some of the computations from the exponential growth spreadsheet we introduced in Chapter **??**. Test that by setting the monthly payment to 0 and the annual interest rate to 12 times the growth rate you want to study.

In particular, what happens if the annual growth rate is 1200%, the starting balance is 1, and the monthly payment is 0?

Exercise 10.6.5. [R][S][Section 10.2][Goal 10.2] [Goal 10.3] Jumbo loans.

On November 20, 2010 a story in *The Boston Globe* headlined "Rates for big loans tumble" said that

> Over the past year, the average interest rate for so-called jumbo loans — $523,750 and up in the Boston area — has fallen from 6 percent to about 5 percent for a 30-year, fixed-rate mortgage. That translates into a monthly savings of about $375 on a $600,000 loan. [R7]

(a) What monthly payment will retire the loan when the interest rate is 6%?

(b) What monthly payment will retire the loan when the interest rate is 5%?

(c) Is the newspaper's claim of a $375 monthly saving correct?

(a) What monthly payment will retire the loan when the interest rate is 6%?
By playing around with the spreadsheet I found that a monthly payment of $3,597 leaves a balance of $305 after 30 years. That's close enough to zero.

(b) What monthly payment will retire the loan when the interest rate is 5%?
$3,220 per month just about does the job.

(c) Is the newspaper's claim of a $375 monthly saving correct?
$\$3,597 - \$3,220 = \$377$, which is close enough to what the paper says.

Exercise 10.6.6. [R][S][Section 10.2][Goal 10.2] [Goal 10.3][Goal 10.4] Mortgages in the news.

A March 4, 2011 article in *The New York Times* headlined "Without Loan Giants, 30-Year Mortgage May Fade Away" claimed that the monthly payment on a 30 year mortgage at six percent interest would be $600 but just $716 for a 20 year mortgage. [R8]

On the same day in an article in *The Boston Globe* headlined "The end of 30-year fixed- rate mortgage?" you could read that

> The difference between a 15- and 30-year mortgage amounts to well over $600 per month on a $300,000 loan, a substantial amount that may prevent wide swaths of the middle class from buying homes. [R9]

Verify the calculations in each of these quotations.

The spreadsheet tells me that a 30 year mortgage for $100,000 at 6% annual interest calls for a monthly payment of $599.55 which leads to $115,838.19 total interest paid. A 20 year mortgage requires a montly payment of $716.43 for $71,943.45 in total interest. The first assertion is correct (with rounding to whole dollars).

For $300,000 loan the difference between the monthly payments for a 30 and a 15 year mortgage at 6% is

$$\boxed{\$2{,}531.57 - \$1{,}798.65 = \$732.92} \ ,$$

which is indeed "well over $600 per month." I'm surprised the article didn't say "well over $700 per month."

Exercise 10.6.7. [U][W][Section 10.3][Goal 10.5] [Goal 10.4] Retirement planning.

Find an online retirement income calculator. Use it with data you imagine for yourself. Write down what you do as you proceed. (Screenshots would be nice.) Record what it tells you at the end.

Their calculator is much more sophisticated than the simple one in Excel we introduced in this chapter. See if you can use ours to get answers that match what it told you.

Exercise 10.6.8. [S][Section 10.4][Goal 10.4] Payday loans.

The Boston Globe on New Year's Day 2009 reported that a New Hampshire law will cap the interest rate on payday loans at 36 percent per year.

> Payday lenders typically charge $20 per $100 for two-week loans backed by the borrower's car title or next paycheck. That amounts to 1.43 percent interest per day, an annual rate of 521 percent. [R10]

The cap will limit the daily rate to about 0.1 percent, so just $1.38 — a dime a day — on that two week $100 loan.

(a) What is a "payday loan"?

(b) Verify the computation that 1.43% interest per day is 521% interest annually.

(c) If the 1.43% daily interest is compounded daily then the true annual rate of interest is in fact much more than 521%. How much is it?

[See the back of the book for a hint.] Start with the calculation $(1+0.0143)^{365}$. The answer is hard to believe.

(d) Verify that paying interest of \$1.38 on a two week loan of \$100 is just about a "dime a day" and corresponds to a daily interest rate of about a tenth of a percent. What annual rate does that represent?

(e) Visit a payday loan website and report on what you discover there about interest rates.

(a) A payday loan is a short term loan, usually at a very high interest rate, to someone who needs cash between now and the time he gets his next paycheck, when he will pay off the loan (together with the exorbitant interest).

(b) $1.43 \times 365 = 521.95$, so the computation is nearly correct. The right answer rounded to the nearest percent is 522%, not 521%. If you borrowed \$1 for a year you would have to pay almost \$522 in interest.

(c) If the 1.43% daily interest is compounded daily then to find the true annual interest rate I calculated $1.0143^{365} = 178.133377$. That corresponds to an annual rate of 17,713%. That's a staggering number. If you borrowed just \$1 for a year you'd have to pay back that dollar and nearly \$18,700 in interest.

The number is so large that I didn't believe my answer and had to redo the problem several times before I trusted the computation.

(d) The capped daily rate of 0.1% corresponds to an annual interest rate of 36.5% (with no compounding). To find the APR corresponding to daily compounding, compute $1.001^{365} = 1.44025131$. That corresponds to a 44% annual rate.

(e)
$$\frac{\$1.38}{2 \text{ weeks}} = \frac{\$1.38}{14 \text{ days}} = 0.0985714286 \frac{\$}{\text{day}}$$
which is indeed just about a dime a day. It corresponds to a daily interest rate of 0.99%, which is indeed just about a tenth of a percent.

(f) No solution provided to this part of the exercise. The payday loan sites come and go very quickly.

Exercise 10.6.9. [S][C][Section 10.4][Goal 10.5] [Goal **??**] Supporting a hospital bed.

The headline "Charity sues R.I. hospital over donation in 1912" accompanied an article in the City and Region section of *The Boston Globe* on February 23, 2008. The article described a gift intended to provide a free bed in perpetuity for needy patients.

There you could read that

> Mark E. Swirbalus, a Boston lawyer representing Children's Friend, said that "as far as we know, the hospital never set aside a bed and never set aside the money." The $4,000, if conservatively invested by the hospital in 1912, would be worth about $1.5 million today, he said. [R11]

(a) Is Swirbalus's claim about a "conservative investment" correct?

[See the back of the book for a hint.] What annual interest rate would you need to turn $4,000 into $1.5 million in 94 years? Compare that rate to the increase due just to inflation.

(b) What the hospital should have done was invest the money and use just the interest each year to fund the bed. That would work — if only there were no inflation that made the cost of the bed increase.

Suppose the hospital got a 6.5% percent yearly return on investment and annual inflation was 3.5%. Explain why it would be able to spend about $120 on the bed in 1912, and could keep spending at that rate as the years went on.

(c) If the cost of providing a hospital bed in 1912 was $120, what would it be in 2008 if all you had to do was adjust for inflation?

(a) Is Swirbalus's claim about a "conservative investment" correct?

I need to find out if $4,000 "conservatively invested" in 1912 would produce $1.5 million in 2008.

I'm going to find out what interest rate r I'd need to get that much growth in the 94 years from 1912 to 2008. I will do that by guess and check in the formula

$$1,500,000 = 4,000(1+r)^{94}.$$

$r = 0.06$ is too small. $r = 0.07$ is too large. $r = 0.065$ is gives me $1,489,097$, which is close enough. So a compounded return of 6.5% per year would do the job. Is that conservative?

Well, inflation alone would turn $4,000 into about $87,000. (I used the inflation calculator for 1913 to 2009 since it goes back only to 1913, not 1912.) That turns out to correspond to an annual return of about 3.5%.

I don't know enough about investing to decide whether getting a return that's twice the rate of inflation is "conservative".

(b) What the hospital should have done was invest the money and use just the interest each year to fund the bed. That would work — if only there were no inflation that made the cost of the bed increase.

Suppose the hosptital got a 6.5% percent yearly return on investment and annual inflation was 3.5%. Explain why it would be able to spend about $120 on the bed in 1912, and could keep spending at that rate as the years went on.

That $120 is 3% of the $4000. After it's been spent there is only $3880 to invest. At 6.5% interest that will yield $1.065 \times \$3880 = \$4132$ in 1913. Three percent of $4132 is $124. But that's just about what $120 in 1912 inflates to in 1913, so the hospital can spend what they need to and invest the rest for the next year.

(c) If the cost of providing a hospital bed in 1912 was $120, what would it be in 2008 if all you had to do was adjust for inflation?

I did the real work for this in the first part of the exercise. I know $4,000 inflates to $87,000, so three percent of $4,000 will inflate to $0.03 \times \$87,000 \approx \$2,600$.

That's probably nowhere near enough to provide one hospital bed in 2008.

Exercise 10.6.10. [S][Section 10.2][Goal 10.3] Half the time, more than twice the benefit.

Show that taking out a 15 year mortgage instead of a 30 year mortgage (for the same loan amount at the same annual rate) doesn't double your monthly payment and more than halves the total interest you pay on your loan.

(The advantages are usually even greater since you can usually negotiate a lower interest rate for a shorter mortgage.)

I used the spreadsheet with a $100,000 mortgage at 3.5%

For a 30 year mortgage the spreadsheet told me that a monthly payment of $449.04 would cost $61,656.09 in total intererest while for 15 years the figures are $714.88 for the monthly payment and $28,678.86 for total interest.

The monthly payment is larger, of course, but not even close to twice as large. The total interest paid is indeed less than half as much.

The changes are even more dramatic when I try 6% instead of 3.5%.

Exercise 10.6.11. [S][Section 10.3][Goal 10.5] Saving $50,000.

(a) Use the spreadsheet `PayOffDebt.xlsx` to figure out how much you'd have to save per year at 3% interest (compounded annually) to have a balance of $50,000 in your account at the end of 25 years.

(b) Then use the mortgage tab in that spreadsheet to answer the same question if you save a fixed amount each month rather than each year.

(a) Use the spreadsheet to figure out how much you'd have to save per year at 3% interest (compounded annually) to have a balance of $50,000 in your account at the end of 25 years.

By experimenting I discovered that saving $1375 per year I would accumulate $50131.49 in 25 years. That's close enough.

(b) Then use the mortgage tab in the spreadsheet to answer the same question if you save a fixed amount each month rather than each year.

I'm not sure how to do this. I discovered that a 25 year mortgage for $35,000 at 3% interest would require a monthly payment of $166. When it was paid off I would have paid $14,792 in interest. Adding that to the $35,000 comes to close to the $50,000 I am aiming for. I hoped and thought that this paying down would give me the right answer. But when I multiply the monthly payment of $116 by 12 I get an annual cost of $1992. That makes no sense given my answer to the previous question.

Exercise 10.6.12. [S] What merchants pay for credit card services.

In May, 2014 *The Nilson Report* said that total spending for credit, debit and prepaid purchases in 2013 was $4.530 trillion, broken down as follows:

credit $2.399 trillion
debit $1.949 trillion
prepaid $0.182 trillion. [R12]

The Boston Globe reported that "merchants in the United States spent $71.7 billion on fees [for these transactions] last year."

(a) What is the average merchant fee, as a percentage?

(b) Make sense of the $4.530 trillion total: think about it in units like dollars per person per day, dollars per transaction, ...

(a) What is the average merchant fee, as a percentage?

$$\frac{\text{total fees}}{\text{total spending}} = \frac{\$71.7 \text{ billion}}{\$4.530 \text{ trillion}} = 0.01582781456 \approx 1.6\%.$$

That answer makes sense to me.

(b) Make sense of the $4.530 trillion total: think about it in units like dollars per person per day, dollars per transaction, ...

I think it's easier to work with dollars per person per day than to estimate the number of transactions.

$$4.530 \frac{\text{trillion \$}}{\text{year}} \times \frac{1 \text{ year}}{365 \text{ days}} \times \frac{1}{300 \text{ million people}} \approx \$41 \text{ per person per day.}$$

That number is probably the right order of magnitude. There are lots of small purchases with debit cards and a few large ones with credit cards that increase the mean. If the average transaction was for $20 that would be two transactions per person per day.

Exercise 10.6.13. [U][Section 10.2][Goal 10.2][Goal 10.3][Goal 10.4] Excel templates from the internet.

At `www.excely.com/template/loan-calculator.shtml` you can download a Loan Calculator Excel Template.

(a) Do the calculations there match those in `PayOffDebt.xlsx`?

(b) Find out where that template uses the Excel's built-in `PMT` function. Compare how it works to the formula in the spreadsheet `PayOffDebt.xlsx`.

Exercises added for the second edition.

Exercise 10.6.14. [U][S][Section 10.1][Goal 10.1] Credit Card Fees.

On October 16, 2017 Bloomberg News reported on the Supreme Court's decision to take a case on American Express credit card fees. There you could read that "[Merchants pay] $50 billion in fees to credit-card companies each year." Those fees come from "the 'astronomical number' of credit-card transactions each year — 22 billion totaling more than $2 trillion in 2011." [R13]

In June of 2018 the Court sided with American Express, ruling that the company could demand that merchants not ask customers to use a different credtit card. [R14]

(a) Does the figure 22 billion transactions per year make sense?

(b) What is the average dollar value of a credit card transaction? Does your answer seem reasonable?

(c) The average you computed in the previous question is the mean. Would you expect the mean and mode to be smaller or larger? Why?

(d) What is the average percentage fee charged merchants for credit card transactions?

(a) Does the figure 22 billion transactions per year make sense?

If I estimate that 2/3 of the 330 million people in the U.S. make those transactions then that's 100 transactions per person per year. That number makes sense to me.

(b) What is the average dollar value of a credit card transaction? Does your answer seem reasonable?

$$\frac{\$2 \text{ trillion}}{22 \text{ billion transactions}} \approx 91 \frac{\$}{\text{transaction}},$$

That seems reasonable to me.

(c) The average you computed in the previous question is the mean. Would you expect the mean and mode to be smaller or larger? Why?

I suspect that most transactions are for a lot less than \$91, so the mode and the median will be less than \$91.

The \$91 mean is skewed by a small percentage of large transactions.

(d) What is the average percentage fee charged merchants for credit card transactions?

$$\frac{\$50 \text{ billion in fees}}{\$2 \text{ trillion in purchases}} = 0.025 = 2.5\%.$$

Exercise 10.6.15. [U][S][Section 10.1][Section 10.4] What does a missed payment really cost?

In Section 10.1 we quoted a late penalty fee with an APR of 29.99%. What is the actual EAR?

The monthly interest rate is $29.99\%/12$. When that is applied for 12 months you calculate the increase by multiplying by

$$\left(1 + \frac{0.2999}{12}\right)^{12} \approx 1.3448$$

which is almost 35.5%.

Exercise 10.6.16. [U][W] Predatory lending.

Take one of the Predatory Lending Awareness Quizzes at `extension.missouri.edu/cfe/wcap/quizzes.htm` . Write about what you discovered. Will that change your behavior?

Exercise 10.6.17. [N][U] The debit card trap.

On August 20, 2009 *The New York Times* editorialized that

> A study by the Center for Responsible Lending, a nonpartisan research and policy group, describes what it calls the "overdraft domino effect." One college student whose bank records were analyzed by the center made seven small purchases including coffee and school supplies that totaled $16.55 and was hit with overdraft fees that totaled $245.
>
> Some bankers claim the system benefits debit card users, allowing them to keep spending when they are out of money. But interest rate calculations tell a different story. Credit card companies, for example, were rightly criticized when some drove up interest rates to 30 percent or more. According to a 2008 study by the F.D.I.C., overdraft fees for debit cards can carry an annualized interest rate that exceeds 3,500 percent. [R15]

We haven't made up any questions yet to go with this interesting quote.

Exercise 10.6.18. [S][Section 10.4][Goal 10.1] [Goal 10.2][Goal 10.4] Regulating the credit card industry.

The Boston Globe reported on May 13, 2009 on the Senate's deliberations on credit card rules.

One change, since signed into law, requires lendsrs to apply payments first to the part of the balance with the highest interest rates.

A second change, proposed as an amemndment by Senator Bernie Sanders (Independent, Vermont) would limit interest rates to 15%. [R16]

Suppose a credit card user has a balance of $100 at 24% for purchases and $1000 at 0% for a debt she transferred from another credit card. She makes no new purchases, and pays off her loan at the rate of $100 per month.

(a) When will her loan be paid off and how much interest will she have paid under the 2009 rules — payments are applied to the purchases balance first.

(b) Under the new rules, which are now law?

(c) If Sanders' amendment had passed? (It didn't.)

(a) When will her loan be paid off and how much interest will she have paid under the 2009 rules — payments are applied to the purchases balance first.

Under the 2009 rules it will take her 10 months to pay off the $1,000 transfer. During that time her $100 balance will have been accruing interest at the rate of 2% per month. Then she will owe $\$100(1.02)^{10} = \$121.90$. (I could have found the same answer with the spreadsheet.) Then she will pay that off in two

months, so she'll have one month's interest on the unpaid balance of \$21.90: another $0.02 \times \$21.90 = \$0.44$. Her total interest payments will be $\$21.90 + \$0.44 = \$22.34$.

(b) Under the new rules she will pay off the high interest part of her bill in the first month, with no interest charge, and then the rest in 10 months, again with no interest charge.

(c) If Sanders's amendment had passed her interest rate would be capped at 15% annually, so if she pays off the zero interest balance first her unpaid balance will become $100 \times (1 + 0.15/12)^{10} = 113.23$. The last interest charged will be just $(0.15/12) \times 13.23 = 0.17$ for a total interest payment of \$113.40.

She'll pay no interest if she pays off the purchases first.

Exercise 10.6.19. [N][Section 10.1][Goal 10.1] Reward cards.

In *The Boston Globe* on December 18, 2009, Candice Choi wrote about credit card reward programs:

`www.boston.com/business/personalfinance/articles/2009/12/18/rewards_cards_may_be_a_bit_less_rewarding_after_you_consider_the_higher_fees/.`

Exercise 10.6.20. [U] Smoke and mirrors.

A visitor at `money.stackexchange.com` asks

> I'm reading about Household International's fraudulent mortgage interest rate. According to Michael Lewis, Household disguised a 15 year, fixed-rate loan as a thirty-year loan. The sales would offer to replace a client's existing \$67,300 mortgage (8.5% interest rate) with a bigger but seemingly cheaper one: \$86,300 at an "effective rate" of 7.6%. The sales pitch goes something like this: "If I can put together a loan that pays out like a 7.579%-a-year loan, but has a total term of 18.63 years — would you be interested?"
>
> Can someone explain how exactly does the trick work? How did the sales misrepresent the 7.6% interest rate, which is seemingly lower than the client's existing 8.5% interest rate? [R17]

The original story is at `www.forbes.com/forbes/2002/0902/062.html` .

The stackexchange site has an answer that can be checked with the mortgage calculator spreadsheet.

Exercise 10.6.21. [U][N] Paying for credit.

In the June 26, 2018 edition of *The New York Times* you could read that

> In addition to annual fees, interest charges and other, more mysterious fees that consumers pay, credit card companies also levy fees on merchants: usually a flat fee per sale and a commission of 2 percent to 3.5 percent or more. If you spend $200 at a store and use a credit card, you could be sending as much as $7 of that payment to the credit card company.
>
> Merchants, no fools, pass those fees on to consumers by making their products more expensive. This yields a credit card "tax" that everyone pays (even those, usually the poor, who don't have credit cards).
>
> There is no reason to expect credit card companies to offer their services free. But the credit card tax paid by American retailers and consumers is the highest in the world. Credit card "swipe" fees account for an estimated $42 billion every year in the United States. The Europeans pay less, because they see this as an obvious market failure and limit the commission to 0.3 percent, meaning that you would pay 60 cents instead of $7 in fees for that $200 purchase. [R18]

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