Common Sense Mathematics
Instructor's Manual

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Introduction

We understand that if you're one of those people who skips the instructions when assembling the new porch furniture, or the manual when you get a new cell phone or the on line help for a software application, you may not ever read this[1] That said, we'll try to make it worth your while.

Common Sense Mathematics as a course text

Both you and your students should understand from the start that this is *not a math course*. We try to make that clear in the Preface.

Both instructors and students find that hard to believe. And as instructors we often find it hard to remember. Since we're mathematicians, we're tempted to think the mathematics is both more important and more useful than it really is. And it's something we *know how to teach*[2] so we fall back on it when the real quantitative reasoning issues seem too messy to tackle.

The chapters fall naturally into three groups. You can construct an hour exam after each. (You can't really make much use of ten minute in class quizzes.)

- Chapters 1-5 deal with numbers a few at at time. The central concepts are absolute and relative change and percentages, working with units, and estimation.
- In Chapters 6-10 we work with sets of of numbers. Chapter 6 on averages introduces weighted averages as a more useful concept than the simple mean. It also provides a bridge to our introduction of Excel in Chapter 7, where we use it to study the mean, median and mode for real data sets, to ask "what-if" questions and to draw histograms. The following chapters introduce linear and exponential functions in useful real world contexts, focussing

[1] Please pardon the self-referential paradox.
[2] Or at least think we know how to teach.

on implementing and graphing them in Excel rather than the more traditional algebraic treatments. The algebra does enter surreptitiously in the form of cell references in Excel.

- Chapters 11-14 cover probability, starting with dice and coins where counting outcomes solves problems, ending with a insurance and extended warranties where probabilities are essentially statistical. The next two chapters address the frequency of rare events like runs and the ease with which you can construct misleading arguments based on the misuse of conditional probability – of course all done without formal definitions.

- The last chapter is on the mathematics underlying various voting schemes.[3]

The outline above sketches the technical topics addressed as the course progresses. The text presents each of those topics in the context of a story to be understood. Almost all the stories are built around real numbers. Occasionally we felt we needed to make up a problem in order to present a particular technique.

In principle, you'll should replace many of the stories in the text with current ones as you teach. That's difficult; we rarely do it ourselves. More often we teach the example in the text, so the students have something to read, but try to construct homework problems based on current news stories.

Unfortunately, many of the instructors teaching quantitative reasoning are underpaid adjuncts stitching together multiple jobs to eke out a living. It's unfair to ask them to spend the extra time it can take to teach from our text. We hope that the increased satisfaction can be its own reward. And it does become easier with time.

Goals

Teaching from *Common Sense Mathematics* is more work than teaching from a standard text with problems at the end of each section that focus on the techniques rather than the important ideas.

We've tried to provide some help making those ideas explicit. At the start of each chapter you'll find that chapter's goals – the essential ideas we hope the students will begin to master. The exercises are tagged to suggest which goals they tend to address – often across chapters.[4]

[3] This material is in draft form only. It's been class tested in other courses at UMass Boston, but not yet in the quantitative reasoning course.

[4] This draft of *Common Sense Mathematics* contains just drafts of the goals. Exercises haven't yet been tagged.

Constructing a syllabus

Here's how we organize our syllabus at UMass Boston. The semester is fourteen weeks long, and there are fourteen chapters, which suggests that a chapter a week is about the right pace. But we find it impossible to do all of each chapter. We choose a few of the sections or topics that we think will particularly interest the particular group of students that semester, and think about them thoroughly.

The idea is to take advantage of the fact this isn't a regular mathematics course like calculus or linear algebra, or even college algebra, in which some set of topics must be covered to prepare students for the next course. We are (rather ambitiously) trying to prepare students for life. Learning to think a few ideas through is more important than exposure-for-the-record to lots of ideas.

Vocabulary

Teaching from drafts of *Common Sense Mathematics* we found that students often had so much trouble with vocabulary that they couldn't even get to the quantitative reasoning.

Here's a blog entry that addresses that issue, from the sixth class of the semester.

> I spent almost all of the rest of the class working the Exercise on Goldman Sachs bonuses (I promised that on Tuesday). I knew that the difficulty was in reading the words around the numbers more than in the manipulations themselves. I was surprised at how important just plain vocabulary problems were. In particular, some students thought "consistent" meant "the same from year to year" (which Goldman Sachs' data aren't) and not (when applied to numbers) something like "fit together the way they should". Later some people didn't quite grasp that "salaries plus bonuses", "compensation" and "what GS paid employees" were all referring to the same quantity.
>
> The same kind of problem came up in a previous class about the meaning of "wholesale". Since I can't anticipate all the words students might not know (both in the course and after they leave) I hope I've convinced them that they can't make sense of paragraphs with numbers in them unless they check out the meanings of words they're unsure of. That said, I will try not to use fancy ones too often.
>
> One student called the need to think about both the words and the numbers a *perfect storm*. I hope not.
>
> ...
>
> Notes later. When I described today's class to my highly educated wife at dinner time, she said she'd have had the same trouble as some of the students with the mean-

ing I attached to “consistent”. She did agree after we looked it up in our (hard copy) dictionary that I’d used it correctly – but that it was unreasonable of me to assume the class would have been able to. She suggested I bring a dictionary to class, and a thesaurus too. I pointed out that we already have a dictionary in class – on line – and that we should have used it right then and there. At dictionary.com the first meaning is

```
1. agreeing or accordant; compatible; not self-contradictory: His
views and actions are consistent.
```

which would have cleared things up right away – particularly “not self-contradictory”.

The term paper

We assign one each semester; students choose a topic that they care about, with some guidance from us about what kinds of topics are suitable.

We allow students to work together in pairs if they wish.

Details here to follow.

A class about runs

If you can find time in your schedule for this class exercise your students may learn a lot about Poisson processes – runs happen.

Everyone understands that the 50% chance of heads when flipping a coin once doesn’t mean that heads and tails will alternate. But they often think, (subconsciously) that when they see a lots of heads in a long sequence something happens to help the tails “catch up.” If you need to argue with someone who believes this, try to convince him that flipping the same coin a hundred times in a row can’t be different from flipping a hundred coins at the same time.

We’ve designed an in-class experiment you can do to demonstrate this belief.

Describe this experiment in a way suitable for a textbook rather than as lecture notes after the fact. Maybe scan in one typical student response, report the statistics from the class, suggest that the reader do the experiment for herself (before reading the section ...).

Now look at a grid consisting of 64 blocks, eight across and eight down. As you go through the grid (moving left to right) mentally toss a coin in your head. With each toss, mark “H” for heads or “T” for tails in the grid. When you have finished filling it in, go back and count how many Hs and how many Ts occur in your grid.

It is unlikely that someone will have exactly 32 heads and 32 tails, but it is likely that your counts will be near that.

Now we look at the grid and look for runs of four heads in a row. That is, if your first row looks like:

```
H T T H H H H T
```

then this would contain one run of four Hs. If your first row looks like:

```
H T T H H H H H
```

then you count *two* runs of four Hs. Note that there are five Hs in a row. Within this block of five Hs, we pick out two different runs: (H H H H)H and H (H H H H).

Go through and count the runs of four Hs, using this definition. Most students had a few runs of four Hs, typically fewer than five or six runs. Why is this? The mind wants to have a balanced number of Hs and Ts, so doesn’t let you put too many in a row. Your vision of a fair coin is that it should alternate between H and T fairly often.

Now count down each column and look for runs of four Hs. You’ll find many more such runs. What is going on here? The mind only remembers a certain amount, and basically after 8 or so flips it forgets what went before. That’s why the rows are 8 blocks long, so that by the time you move to the next row you will have forgotten what you put in the previous row. Thus, we are getting a more random distribution of Hs and Ts and, as happens in nature, we will get runs of Hs and Ts.

The key point here is that runs do happen. We think of the probability of getting heads on a coin toss as being $\frac{1}{2}$, and so we expect that we will not have a long run of heads, but the rules of probability say that this will happen occasionally. You need to be aware that it is a real possibility.

How we read the newspaper

What kinds of problems work, and don’t.

Why we avoid the sports pages, and the business pages.

What's missing, and why

Politics/controversy?

You will have to decide how much controversy you want to allow or encourage in your classroom. Politics makes for engagement, but class discussions can easily degenerate into assertions of belief. You should not be imposing yours on your students.

Some topics of current interest are too complex. You can't really teach much serious analysis of climate change and global warming.

Missing mathematics:

- Laws of exponents.
- Formal definitions of roundoff.
- Algebra. See the reasons at `http://www.glasbergen.com/?s=algebra`
- Drill problems.

An instructive exploration

In this section I describe the train of thought prompted by the bar chart in Figure 0.1 from *The Boston Globe* on July 15, 2010.[5]

The explanatory text is hard to read in the scanned image. Here's what it says:

> Lottery revenue per person returned to communities in fiscal 2011.
> **C**omments or ideas: mcarroll@globe.com
> **S**napshot statistics appear weekly. For a complete report, go to `http://www.boston.com/news/specials/government_center/#snapshots`.

What first grabbed my attention was the peak for Framingham – an otherwise undistinguished suburb of Boston. I thought perhaps that represented a statistical anomaly, a big winner in that town, which might be the case if "revenues returned to communities" meant (or included) payoffs on winning tickets and not just money returned from the state to the towns. I put that question aside for a while.

[5] I could write this material in the authorial "we" but decided to use the first person instead since it is essentially a record of my thoughts – Ethan Bolker.

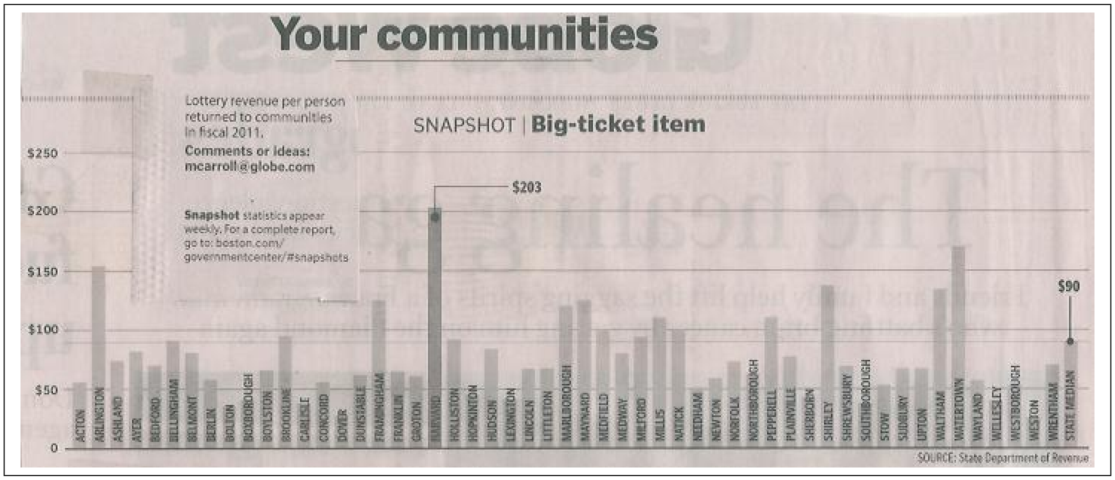

Figure 0.1: Massachusetts Lottery Returns

I wondered if I could ask students to verify the claimed “STATE MEDIAN” value of $90 per person. Looking at the graph, it seemed pretty clear that well over half the bars aren’t that high. That points to an interesting ambiguity – the $90 doesn’t seem to be the median for the data values displayed on the chart (the per capita returns for each community). It’s probably the median for the communities weighted by the size of the communities. To check that, you’d need community populations in addition to the data in the chart.

Then I looked for Boston in the list, since its value would have the highest weight. But Boston isn’t there! I think I know why: the Snapshot feature appears in the paper in the section headed **Y**our communities – an attempt by the Globe to appeal to suburban readers. In fact, more towns are missing than are present – there seem to be bars only for the Western suburbs – none from the North or South shores, or the middle or Western parts of the state. Perhaps that’s because I live in a Western suburb.

With most of the towns missing I realized that I couldn’t check the computation for the state median even if I knew the populations. So I decided to look for the raw data. (In the class we sometimes ask the students to do that; more often we do it for them.) The reporter provided the link `http://www.boston.com/news/specials/government_center/#snapshots`, from which I navigated to `http://www.boston.com/yourtown/massfacts/snapshot_massachusetts_lottery_aid_2011/`

The first thing I looked for was the graphic, so I could replace the hard-to-read scanned image with an easy-to-read downloaded one. It wasn’t there – I often find that graphics in the hard copy of the paper don’t appear on the web site.

But I did find this quotation

When it comes to the state Lottery, there are definite winners among communities. The Lottery each year distributes aid to communities, based on a formula. The communities that received the most aid per resident tended to be urban and less wealthy. For instance, tops for fiscal 2011 were Somerville, North Adams, and Boston, all of whom got more than $245 per person. Smaller, wealthier communities received much less. Chilmark, Aquinnah, and Nantucket received $6 or less per person. **The state median was $90**.

and the raw data for all the cities and towns. I grabbed that from the web page and pasted into a csv file so that I could examine it in Excel. That required just a little editing:

- Delete the extraneous information surrounding the data.
- Replace all the commas with nothing.
- Replace all the tabs with commas and (a space).
- Replace all the dollar signs with nothing.

Here it is: `http://www.cs.umb.edu/~eb/qrbook/Instructor/lotteryReturns.csv` .

Note that the data include the total returned to each community as well as the per capita amount, so I could recover the populations. There's enough information to check the median value of $90. I will get around to that one of these days, and report back here.

The web page also provides a link to the *many eyes* data visualization tool at `http://manyeyes.alphaworks.ibm.com/manyeyes/`. That might be worth exploring some other time.

Concluding thoughts:

- If I want to use this example in the book, the place to put it is in the chapter on averages, where we introduce Excel. I could ask about building a histogram, building a bar chart (not a histogram) that looks like the Globe graphic, checking the median (requires thought, probably serious hints), checking the listed rankings by sorting the data.

- Consider posting my puzzlement at the Globe web site, which encourages reader participation, but gets very little.

- Watch the weekly Snapshots carefully next semester, try to find one to base a class or homework problem on, then ask the students to comment at the web site as a part of their homework.

- Write Matt Carroll (the Globe reporter responsible for the weekly snapshot), describing this adventure. Perhaps ask him to class in the fall as a guest speaker.

Page by page comments on the text

The text that follows offers pedagogical tips, class exercises and some extra problems. It's generated from the TEX source for *Common Sense* so that we can edit it where it's relevant. When page numbers there change the page references here change too.

Chapter 1

Fermi Problems

Section 1.1, page 24

This chapter and the next are closely related. You might want to combine them when planning classes and homework assignments. It was often hard to decide which exercises belonged here, which there.

Real Fermi problems ask for estimates from scratch. We concentrate instead on developing Fermi problem techniques to verify claims in the media. That's both easier for students, and follows directly from our focus on working with numbers in the news – educating our students to be consumers rather than producers of quantitative information.

The text in this Chapter stresses techniques: quick mental arithmetic, counting zeroes, some scientific notation and the metric prefixes.

Note that many Fermi problems that used to require estimation skills now succumb to a web query. For example, it's easy to find a reliable count of the number of kindergarten teachers in the United States.

Section 1.2, page 25

When we first used the word "consistent" in class in this context we discovered that many of the students didn't know what it meant. Students' limited vocabularies turn out to be a source of confusion in many quantitative reasoning problems. It's important to be aware of that, and to encourage dictionary use.

Section 1.2, page 25

Writing out the unit conversions this way foreshadows doing that more formally in a later chapter.

Section 1.2, page 26

The class will probably want to go directly to "divide by 24." It's probably worth a minute or two to write it out formally to prepare for unit conversions later on.

Section 1.2, page 27

A nice thing to do now is to ask the students to take their pulses and report the results. Collect the data so you can make them available in a spreadsheet for the class to play with later when you introduce Excel. You might want to introduce median and mode quickly now – but don't spend arithmetic time now on the mean.

Section 1.3, page 28

When we teach this material in a course we divide the class into teams of three. Each team starts on one of the seven estimates with instructions to move on to the next one when done, circling back to google search if they reach the end of the list. The teams start in different places, so after about half an hour the class has found two or three estimates for each of the seven tasks. The different answers for each task should have the same order of magnitude. If they don't, we try to figure out why not.

Section 1.6, page 41

30 This is a particularly provocative example since almost everyone's first guess – including ours – is that it's an exaggeration.

Section 1.6, page 42

32 We haven't done this problem with a class yet. If you do, let us know what happened.

The author is probably right about the number. Does that mean that 1/6 of the population are criminals ... (Some of the fingerprints on file are for people who've died, and some are for foreigners - but probably very few are of living children.)

The real story is probably that they have lots of fingerprints on file for folks who aren't criminals - e.g. people who have registered with selective service.

Section 1.6, page 45

39 The material in this question makes a good class, combining history, physics and quantitative reasoning about an important subject many current college students know nothing about.

Section 1.6, page 46

42 We showed the film "Powers of 10" in class once but our students didn't find it as interesting as we do. Much as we like it, we haven't used it since.

Section 1.6, page 55

67 The highighted posts in *The New York Times* comments on this article are really interesting. You could build a whole class around them.

Chapter 2

Units and unit conversions

Section 2.4, page 66

We will replace this made up problem with a real one, chosen to make the same pedagogical points. You can consider doing that now, perhaps asking your students to find one for homework.

Section 2.8, page 74

The bad news is that this is another made up problem (with easy numbers). The good news is that it's accompanied by some practical advice for painting.

Section 2.10, page 91

.36 This problem is interesting because it's political, the computations are easy and the answers don't make sense.

Chapter 3

Percentages, Sales Tax and Discounts

Section 3.1, page 95

In principle, the content of this chapter is a review of material on percentages students learned in high school. In fact the review is necessary. Moreover, the presentation is a little more sophisticated, and, we hope, a little more useful, than what they've seen already. So even for those who remember it, there's value added here.

Section 3.1, page 96

It's hard to persuade students to learn the "multiply by 1+change" trick. But it's well worth the effort, which will pay big dividends later when looking at inflation, interest rates and exponential growth. And it helps wake up students who might otherwise see this material as just boring review of things they know or knew.

Section 3.2, page 98

Speaking mathematically, we'd rather have the relative change be the fraction new/old, paralleling the definition of absolute change as new-old. But speaking practically, for the target student audience it's better to have the relative change be the difference (new-old)-1.

Section 3.5, page 100

You should replace the discussion in this and the following section with real rather than invented numbers. When you find a good example, let us know and we'll use it in the text here.

Section 3.8, page 108

21 Much to our surprise, we found that about a third of the class didn't know what "wholesale price" meant when we first assigned this ptoblem on markups. Now there's a hint in the back of the book.

Section 3.8, page 109

ch semester we try to find a current news story that students might respond to with a letter to the editor, or an online comment at the appropriate web site. We assign a draft letter or comment as an exercise, discuss the results in class, and offer extra credit for submitted or published comments.

Section 3.8, page 111

26 This exercise is an advance look at the mathematics of compound interest.

Section 3.8, page 112

29 We made up the questions for this exercise after a quick reading. They might be harder than they seem. You should probably try to answer them before assigning them. If you find them hard you can give hints, rewrite them, or leave them difficult.

Section 3.8, page 114

31 This exercise might make an interesting class discussion. Assign it first, then build a class around student solutions.

Section 3.8, page 114

32 This problem comparing private label to branded goods turned out to be more interesting than we thought. There are several decreases (negative values) to deal with. The answer in the solutions manual shows how the 1+ trick lets you find the relative increase without first finding the absolute increase.

Section 3.8, page 115

33 The Gulf oil spill in the summer of 2010 generated lots of data along with the oil. If it had happened during the semester we might have used it daily.

Chapter 4

Inflation

Section 4.2, page 128

It's difficult to keep the printed text up to date on current events. When you teach this section, study inflation from last year to this year. Then the students have two separate treatments to learn from – yours in class and the one in the book.

Section 4.4, page 130

This section is new – not yet class tested.

Section 4.4, page 131

The inflation calculator gives \$215.40 for 1983 and \$206.48 for 1984. The average is

$$(214.40 * 206.48 * 222.32)^{1/3} = 214.302433$$

but you don't want to teach that!

Section 4.7, page 134

4 This would make an interesting spreadsheet exercise when we get to spreadsheet calculations and graphing. Then you can ask if there is any year in which the minimum wage went up fast enough to account for inflation.

Chapter 5

Average values

Section 5.1, page 142

Consider pointing out that the units of the average are the same as the units of the things you are averaging, since the weights are dimensionless. But that observation might confuse students while they are working to master the new concept.

Section 5.2, page 142

Students find this section on grade point average computation compelling – many often say they had no idea how it was done and are delighted to have found out. It makes the weighted average concept clear in a context that really matters.

Section 5.3, page 144

We find that many students solve this equation by writing some version of

$$\frac{90 \times 2.8 + 30\,G}{120} = 3.0 = 360 = 108 = 3.6.$$

You can of course see what they're *thinking*, and the answer is the right value of G. Their prose (if that's what you can call it) conflates "=" meaning "is the same number as" with "=" meaning "is the same equation as". We constantly ask for "more words" and can't seem to get them. If you know how, please let us know.

Section 5.3, page 145

It's worth taking a little time to practice the guess-and-adjust-your-guess strategy. It may not be as efficient as algebra in situations like this, but it's much more generic and much less arcane. Students can understand and appreciate it and might even remember it.

Section 5.6, page 148

This paradox is a well known phenomenon. See Chapter 6, A Small Paradox, in *Is Mathematics Necessary?*, Underwood Dudley (ed.), Mathematical Association of America, 2008, and further references there.

Chapter 6

Income Distribution

Section 6.0, page 157

’ve written parts of this and the next few chapters as spreadsheet tutorials. We teach in a computer lab where the students can walk through the examples a step at a time.

Section 6.4, page 161

We recommend drawing these bar charts by hand on the board, and asking students to do the same from time to time on paper.

Section 6.4, page 163

Excel can use seperate vertical scales to plot two data series. We think it’s more useful and more interesting to teach this *ad hoc* solution.

Section 6.8, page 169

This hand drawn figure may seem unprofessional, but in fact we think it’s useful. It’s more like what a student could produce than a fancy graphic would be. We think we should have more pictures like this.

Section 6.13, page 178

.5 This exercise may be worth assigning for the Excel practice, and for reinforcing the computations of various averages from summarized data. But the conclusions aren't very striking.

Section 6.13, page 182

.11 We haven't assigned this Benford's law exercise yet. We think it would be very interesting, but perhaps not worth the time it takes away from other more useful topics.

Chapter 7

Electricity Bills

Section 7.2, page 198

We usually treat the slope and intercept as given data, since that's how they appear in the world. Computing the slope as $\Delta y/\Delta x$ belongs in an algebra class, but not here.

Section 7.3, page 199

Since the entries in the Tamworth Electricity Bill table are out of order, Excel has drawn some of the segments in the graph twice. You can see that if you look carefully. You might want to point this out, or not.

Section 7.6, page 204

We're undecided about whether to expand this discussion of the difference between power and energy. It takes a lot of teaching time (and energy) to convey the distinction convincingly in class. Perhaps those resources are better spent on other parts of the curriculum. We have included problems that explore the issue further. We should write even more, particularly Fermi problems about green energy – wind, solar, conservation.

If you do choose to spend more time on the difference between power and energy, and the confusion because the name "watt-hour" contains a unit of time, consider discussing the light-year, which is a measure of distance, not time. So "light-years ago" is never right.

Section 7.7, page 205

In the tax rate brouhaha in the 2008 election we recall reading a story about a dentist who complained that he would need to be careful not to let his income exceed $250,000 – where candidate Obama drew no-new-taxes line – lest his overall tax rate increase. If you find the story let us know and we'll turn it into an exercise.

Section 7.8, page 210

9 This reference to the Hiawatha Bray article is the first of several possible exercises on the same theme – if you combine them and assign them and let us know what happened we'll rewrite them.

Section 7.8, page 217

21 The article on the quarry water cooling system also asserts that

> The system ... cost only about $700,000 more than a traditional cooling system, meaning Biogen Idec should get a return on its investment in eight to 10 years.

Discussing payback time might be interesting – or too difficult.

Chapter 8

The Tower of Pisa

Section 8.0, page 223

s, we teach how to find regression lines (using Excel). But our approach stresses skepticism throughout. Rather than teaching this as a tool they can use, we treat it as a tool often misused.

Section 8.1, page 223

This part of this chapter, like the start of the last one, is written as an Excel tutorial. If possible, students should follow along, checking the steps in the link above as they read or as you lecture. They can use the link in the text to see what we've done, or they can recreate everything, starting with just the data in `http://www.cs.umb.edu/~eb/qrbook/Instructor/pisaData.xls` .

Section 8.2, page 226

This error surprised us when it occurred during a class we hadn't prepared carefully. That turned out to be useful – the students saw their teacher seeing that a number made no sense, then looking for a

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Section 8.5, page 236

15 The students may all be interested in global warming, so they will want to think about it. The consensus among climate scientists is that it's real and anthropogenic, but the real science is complex. You can't draw reliable conclusions from simple regressions like the ones in this exercise. So treat this material respectfully and cautiously.

Chapter 9

Compound Interest

Section 9.4, page 247

Of course you don't need to rely on experiments to know that the doubling time is independent of the initial value. It's very easy to prove with a little bit of algebra. But this is a book about quantitative reasoning, not about algebra. For its intended audience the experiments are more convincing than the more formal mathematics many people find mysterious.

Section 9.4, page 249

If we were teaching algebra and not quantitative reasoning we might use a negative exponent to write $(1/2)^{10}$ as 2^{-10}. But we're not, so we don't, so we avoid the time it would take to remind students about working with negative exponents.

Section 9.5, page 249

The material in this section on bacterial growth is at the edge of what we think students in a quantitative reasoning course need. It deals with real data, not the artificial doubling time problems in most books. Carrying through the discussion in sufficient detail to allow them to solve similar problems would take time better spent on other topics. But if there's time in the syllabus there are ideas here it's worth exposing. They tie together all the themes of the chapter.

Section 9.6, page 257

17 We could construct the Fermi problems based on this radioactive waste data ourselves, and ask the students to solve them. But by this time in the course we hope they can start from the numbers and create their own.

Chapter 10

Paying off a Debt

Section 10.4, page 270

Many qr courses spend a lot of time on this formula. We think it's not time well spent. What is important is the *idea* of frequent compounding.

Chapter 11

Probability

Section 11.0, page 279

robability is hard, often counterintuitive. We deal with it in three chapters. This one is about the basic quantitative notion, focussing first on the easy cases coming from games of chance, but not spending significant time on the combinatorics. In the next chapter we take on repeated independent events, the bell curve, and rare events. In the one after that we take on conditional probability, but without formulas. Throughout the discussion we often find that there are *ideas* about probability that should be thought about but that don't fit nicely into simple numerical examples, either real or imagined.

Our choice of "invented" instead of "discovered" mathematics in the chapter introduction is deliberate. You might want to discuss that philospohical question in class.

Section 11.2, page 281

Consider not even mentioning the formulas for converting from odds to probabilities and back lest the students latch on to them as more important than they are.

Section 11.4, page 284

We're very fond of this section on roulette but the students often seem not to be. And it's not clear that it's useful. Maybe you should skip it. Maybe we should. Perhaps try out the later section on pari-mutuel betting instead.

Section 11.8, page 293

.8 Ben Bolker suggests analyzing this hiring dilemma using a payoff matrix, with utilities associated with each state (awful, ok, great). Then we could compute an expected value for each action (hire known, hire unknown) in terms of the various probability and payoff assumptions. This would be cool in Excel.

Chapter 12

Break the Bank

Section 12.5, page 304

Of course

$$0.99^{100} = 0.366032341 \approx \frac{1}{e} = 0.367879441$$

but you don't want to go there with this class.

Chapter 13

How Good Is That Test?

Section 13.1, page 313

This chapter focusses on two way contingency tables in order to discuss several important common logical pitfalls dealing with everyday probabilities. We think that approach makes more sense, and is easier to remember and apply, than an explicit treatment of dependent events and Bayes' theorem. That's too technical for our goals in this quantitative reasoning text, and so better left for a full course in probability and statistics. In fact, many of the examples in this chapter employ qualitative rather than quantitative reasoning.

If you want to go further into the analysis of dependence (perhaps leading to Bayes' theorem) consider two way tables as the entry point. Independence corresponds to tables whose rows (and hence columns) are proportional. Those are the only ones that can be modeled using areas of parts of a square, as in the last chapter.

Causation corresponds to tables with a 0 in one quadrant.

Chapter 14

Voting

Index

List of Figures

List of Tables