Common Sense Mathematics
Instructor’s Manual

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About *Common Sense Mathematics*

The art of teaching requires the instructor to guide his student to work independently to discover principles for himself, and in time to acquire the power of principles to the manifold situations which may confront him.

James Brander Matthews (1852 – 1929)

**Introduction**

We understand that if you’re one of those people who skips the instructions when assembling the new porch furniture, or the manual when you get a new cell phone or the on line help for a software application, you may never read this.\(^1\) That said, we’ll try to make it worth your while.

Both you and your students should understand from the start that this is *not a traditional math text* for a traditional math course. We try to make that clear in the Preface, and in the first few classes at the beginning of each semester. Students rarely believe us. Many complain half way through the semester that this math course isn’t like any other they’ve taken. Where are the formulas? As instructors we often find it hard to believe too. Since we’re mathematicians, we’re tempted to think (semi)formal mathematics is both more useful and more important than it really is. That comforts us, since it’s something we *know how to teach*\(^2\) so we fall back on it when the real quantitative reasoning issues seem too messy and frustrating to tackle.

There is real mathematical content here – lots of overlap with what you find in most quantitative reasoning and some liberal arts texts, perhaps with an occasional favorite topic missing, often something more advanced that you might not expect to find. But the mathematics is embedded in discussions of issues most of which most students find genuinely interesting.\(^3\)

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1. Please pardon the self-referential paradox.
2. Or at least think we know how to teach.
3. But you can’t please all of the people all of the time . . .
There’s much to gain from the fact that this isn’t a regular mathematics course like calculus or linear algebra, or even college algebra, where some set of topics must be covered to prepare students for the next course. At UMass Boston there is no next course – students are here just to satisfy the quantitative reasoning requirement. So we are (rather ambitiously) trying to prepare students for life. Learning to think a few ideas through is more important than exposure-for-the-record to lots of ideas.

The text presents each topic in the context of quotations from the media to be understood. Only occasionally we thought it better to make up a problem in order to present a particular idea. In principle, you should replace many of the stories in the text with current ones as you teach. That’s difficult; we do it ourselves less often than we’d like. There are, after all, advantages to teaching from the text: the material there is class tested. You avoid the risk of having to muddle through an example you thought would work – until you tried it the first time. Moreover, the students have something to read to go along with the class discussion. Save your ingenuity for new homework exercises based on current local news stories.

Unfortunately, many of the instructors teaching quantitative reasoning are underpaid adjuncts stitching together multiple jobs to eke out a living. Perhaps it’s unfair to ask them to spend the extra time it can take to teach from our text. We hope that the increased satisfaction can be its own reward. Fortunately, it does become easier with time.

**Constructing a syllabus**

The semester at UMass Boston is fourteen weeks long, which suggests that a chapter a week is about the right pace. At that pace we find it impossible to cover all of each chapter. Instead we choose a few sections or topics that we think will particularly interest the particular group of students that semester, and think about them thoroughly. Sometimes we build a class around one of the exercises.

The chapters fall naturally into three groups.

- Chapters 1-4 deal with numbers a few at at time. The central concepts are estimation, working with units, absolute and relative change and percentages. The first chapter, on Fermi problems, sets the tone for the entire text.

- In Chapters 5-10 we work with sets of of numbers. Chapter 5 on averages introduces weighted averages as a more useful concept than the simple mean. It also provides a bridge to Chapter 6, where we use Excel to study the mean, median and mode for real data sets, to ask “what-if” questions and to draw histograms. The following chapters introduce linear and exponential functions in useful real world contexts, focusing on implementing and graphing them in Excel rather than on more traditional algebraic
treatments. The algebra does enter surreptitiously in the form of cell references in Excel. We think a spreadsheet program rather than a graphing calculator for more advanced calculations is a much better long term time investment for the students.

- Chapters 11-13 cover probability. The first starts with dice and coins where counting outcomes solves problems and ends with insurance and extended warranties where probabilities are essentially statistical. The next two chapters address the frequency of rare events like runs and the ease with which you can construct misleading arguments based on the misuse of conditional probability – of course all done without formal definitions.

**Where’s the math?**

Answer: embedded in the real applications. You will find (among other things, and not necessarily in order of presentation or importance)

- Estimation and mental arithmetic
- Scientific notation
- Unit conversions and the metric system
- Weighted averages
- Descriptive statistics (mean, median, mode)
- The normal distribution
- Linear equations and linear models
- Exponential equations and exponential models
- Regression
- Elementary probability
- Independent events
- Bayes’ theorem
- Logical thinking
- The geometry of areas and volumes

Some mathematics you might expect is missing – primarily because it fails to pass the “should your students remember this ten years from now?” test:

- Formal logic and set theory (and most other formal mathematics)
- Laws of exponents (“laws” of just about any kind)
- Quadratic equations
- The algebra of polynomials
- Traditional “word problems”
Homework exercises

That’s William Carlos Williams’ *The Red Wheelbarrow.* We quote it here for the “depends”. We believe every interesting question has the same answer: “it depends” – if you can find the answer by calculating or looking it up, the question isn’t interesting. So answers to exercises call for complete sentences – even complete paragraphs. Just circling the (presumably correct) numerical answer isn’t sufficient. Each answer should effectively restate the question. We suggest that students write enough prose so that they can use their homeworks to study for the exams without having to return to the text to see what the questions were. We return corrected homeworks promptly, and post solutions (available in the solutions manual!) that model what we expect from them. We encourage students to turn in word-processed documents. Since writing mathematics in those documents is tedious, we suggest that they leave blank space for the calculations and equations and fill those in *legibly* by hand.

There are many more exercises in the text than you can use in any one semester. There are more ideas for exercises in this manual. There are more still in the on line resources for *Common Sense Mathematics* – about which more below.

We find that weekly homework tends to work best. We’ve taken to spending some of the time flipping the classroom: having students start homework exercises in groups in class, asking for help when they need it. We always encourage students to start each assignment when it’s posted, so that they can ask questions before the due date.

Most homework exercises relate to material already taught. We also find it helpful to assign some on topics we haven’t yet reached in class. Students are then much better able to participate in discussions.

Goals

Teaching from *Common Sense Mathematics* is more work than teaching from a standard text with problems at the end of each section that test the mastery of basic techniques rather
than the understanding of important ideas.

We’ve tried to provide some help making those ideas explicit. At the start of each chapter you’ll find that chapter’s goals – the essential ideas we hope the students will begin to master, and the particular mathematics needed for that task.

To help you construct assignments, we’ve tagged the exercises to suggest which goals they tend to address and which sections they relate to – often across chapters. At the end of the text you’ll find the same information organized by goal; search there for exercises that address particular goals.

Instructors using *Common Sense Mathematics* have found that their students often need practice with routine arithmetic and algebra in order to come to grips with the ideas that are the focus of the text. To address that need, we have included some review exercises at the ends of the chapters as appropriate. This kind of review is often best addressed with problems with online software support like Webassign [http://www.webassign.com/](http://www.webassign.com/). We recommend that you use them sparingly, and only as necessary – that is, after the need is clear. We sometimes ask students to *read* the review problems and decide for themselves whether they need to work some and turn them in for feedback.

If you assign review problems first, before the applications in the text, your students may think this is a math course like any other – formal skills to master with no thought as to their usefulness or meaning.

Online homework systems are good for routine review, but no help at all with exercises like those in this text. To provide that help we have created SCOPE, at [http://scope.math.umb.edu](http://scope.math.umb.edu), a virtual community where teachers can share experiences and construct assignments with up to date world wide content. Figures 1.1 and 1.2 provide a glimpse of how it works. If you’re using *Common Sense Mathematics* in your classes please join the club and contribute. You can even write your mathematics using rudimentary LATEX.

### Exams

We usually schedule two hour long examinations and a final exam. (It’s hard to make much use of ten minute in class quizzes to test the kinds of analysis we are trying to teach.)

Our exams take place in a computer lab. We allow “open everything” – textbook, class notes, internet, but, of course no texting or email asking for help. When students react with pleasant surprise to this announcement we point out that this is exactly what they will have available when they tackle real problems in life – which is what we want to teach and test.

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5Note: the draft review exercises in have not yet been as carefully constructed and tested as the others in the text. Expect more of them, and better, in the published version.
We remind them that allowing access to all these resources means we can ask harder problems than they may be used to in an exam. In fact it is difficult to construct exams that truly test the use of common sense mathematics.

Here is the preamble to our first exam:

General guidelines

- When you’ve solved a problem (perhaps on scrap paper), write the answer out neatly on the paper with the problem (you can use the other side too). Don’t just circle a number. Show all units, and write complete sentences. If you’ve used any technology, say so.

- The purpose of this course is to help you learn how to use quantitative reasoning principles to solve real problems that matter to you. An exam can’t test that well because you must answer the questions quickly. Here’s a compromise. For homework for Thursday, rethink your answers. If you can write better ones, submit them. (Don’t redo problems you got right the first time.) I will correct both the exam and the resubmissions. Getting a problem right the second time isn’t worth as much as getting it right the first time, but it can make a difference in your grade. The exam is posted on the course web page.

Work independently. You can email me with questions, but don’t consult with friends or classmates or tutors.
Google (and the internet), calculators, class notes and the text are all OK. Make sure you acknowledge any help of this kind. But take care. Time spent searching the web or shuffling through notes is time you’re not using to answer the questions. Of course you can’t use the computer to exchange email with your classmates during the exam. No text messages either, please.

- Remember to show only the number of significant digits (precision) in your answer justified by the numbers you start with and the estimates you make. Remember to use the equal sign only between numbers that are equal, not as a substitute for words that explain what the numbers mean and what you are doing.

The first question on the exam is

1. (5 points) Read the general guidelines - particularly the first two about the form your answers should take, and the chance to improve your answers between now and Thursday. Write “I understand the instructions” here for a free 5 points.

Sad note: All students write a correct answer to this first question but many have not read or understood the instructions!
Vocabulary

From G. K. Chesterton’s *The Scandal of Father Brown*:

Father Brown laid down his cigar and said carefully: “It isn’t that they can’t see the solution. It is that they can’t see the problem.”

Teaching from early drafts of *Common Sense Mathematics* we found that students often had so much trouble with vocabulary that they couldn’t even get to the quantitative reasoning. Here’s a blog entry that addresses that issue, from the sixth class of the semester.

I spent almost all of the rest of the class working the Exercise on Goldman Sachs bonuses (I promised that on Tuesday). I knew that the difficulty was in reading the words around the numbers more than in the manipulations themselves. I was surprised at how important just plain vocabulary problems were. In particular, some students thought “consistent” meant “the same from year to year” (which Goldman Sachs’ data aren’t) and not (when applied to numbers) something like “fit together the way they should”. Later some people didn’t quite grasp that “salaries plus bonuses”, “compensation” and “what GS paid employees” were all referring to the same quantity.

The same kind of problem came up in a previous class about the meaning of “wholesale”. Since I can’t anticipate all the words students might not know (both in the course and after they leave) I hope I’ve convinced them that they can’t make sense of paragraphs with numbers in them unless they check out the meanings of words they’re unsure of. That said, I will try not to use fancy ones too often.

One student called the need to think about both the words and the numbers a perfect storm. I hope not.

... Notes later. When I described today’s class to my highly educated wife she said she’d have had the same trouble as some of the students with the meaning I attached to “consistent”. She did agree after we looked it up in our (hard copy) dictionary that I’d used it correctly – but that it was unreasonable of me to assume my students would have been able to. She suggested I bring a dictionary to class, and a thesaurus too. I pointed out that we already have a dictionary in class – on line – and that we should have used it right then and there. At dictionary.com the first meaning is

1. agreeing or accordant; compatible; not self-contradictory:
   His views and actions are consistent.

[http://gutenberg.net.au/ebooks02/0201031.txt](http://gutenberg.net.au/ebooks02/0201031.txt)
The “not self-contradictory” would have cleared things up right away.

The term paper

We assign one each semester; students choose a topic that they care about, with some guidance from us about what kinds of topics are suitable. We allow students to work together in pairs if they wish. Some instructors allow, encourage or require students to study some appropriate topic in a group and present to the class.

We pace the students through this significant project by asking them to submit several topic ideas about halfway through the semester. Then we give individual feedback on each student’s choice. A few weeks later we require an outline or sketch that poses the questions to be answered (with guesses as to the answers) and identifies data sources. Near the end of the semester students turn in a draft. The final version is due at the final exam.

Here are some excerpts from term paper instructions we offer our students.

One of the important parts of this course is the term paper. Yes, you didn’t expect a term paper in a math course. But this one is about quantitative reasoning about things that matter in the real world. Your paper will give you a chance to practice that.

You will choose a topic, find some data and quantitative information about it, perhaps form a hypothesis, explore “what-if” questions, make estimates, analyze data, and draw conclusions. In other words, you will use many of the techniques and ideas of this course to make a quantitative analysis of a topic that interests you.

If you are going to use lots of data from the web to do your analyses (sports statistics, poverty rates) you should not be typing it into Excel one number at a time. Many websites let you download tables in csv format. “csv” stands for “comma separated values” – and Excel can load those files. Even if csv is not available there are tricks that let you cut data and then paste it into Excel. If you show me your data source I can help with that.

You may work with a classmate and submit a joint paper.

What should I write about?

The best way to do well in this assignment is to write about something that really matters to you. Here are some ideas suggested by classmates from previous semesters. (This is not a list for you to choose from, it’s a guide as to the kinds of topics that might - or might not - work.)

You may want to arrange some peer review.
• Business plans.
What would it take to open a beauty salon? A bike store? A photography business? Can my garage band make enough money to support me? Can my rugby club or softball league break even sponsoring a tournament?
Each of these questions led to a good paper. The authors had to collect information (often from personal or job experience or a friend in the business), build a spreadsheet, ask what-if questions and analyze the outcomes. They were successful because they had access to the data and enough knowledge of the activity to make sense of it.
Your business plan probably has two parts. The first is the estimate for the startup costs. The second is the estimate of the cash flow in and out once the business is up and running. I strongly suggest you focus on the second part. For startup costs, just imagine you will have to borrow the money, and put the monthly loan payment down as an expense in your monthly cash flow spreadsheet. You can vary that amount to see how much you could afford to borrow.
Whatever your business (dog training, personal fitness, growing marijuana, ...) you should search on line for business plans in your kind of business. They will give you some idea of the kinds of things you need to consider. Of course real plans will call for a lot more detail than you can provide in your paper.
• Can I afford to buy a house?
This is a common question and a common topic. Sometimes it works, but most of the time it doesn’t. Much more than the cost of a mortgage is involved. The best papers start by imagining lifestyles and family structure and particular communities to live in and trying to quantify those in some sense before plugging in numbers. There are templates on line that help with the ongoing costs of home ownership.
• Sports.
There are lots of numbers on the sports pages. Students (mostly guys) really care about them. That’s a good place to begin. But it’s only a beginning. I’ve never seen a successful paper that tries to answer questions like “do the teams with the highest salaries win the most?” or “are superstars worth the big bucks?” I have seen a few good sports papers. If you want to try one you have to start with smaller questions. And you must be careful to find real data to think about. You can’t use the paper just to sound off about your own firm opinions.
• Personal budgets.
This sometimes works and sometimes doesn’t. To do it well you have to collect data on your actual income and expenses over a reasonable period of time, estimate things you can’t pin down exactly, take into account large
expenses that don’t happen every week or month, build a spreadsheet and then ask and answer reasonable “what-if” questions. There are many web sites that offer Excel spreadsheets you can personalize and fill in to create your budget. Look at several and find one that matches your needs. Don’t try to build your own from scratch.

• Transportation
Can I afford to buy a car? Is it better to drive to school or take the T? Often asked, usually not answered well. I’ve suggested to students tackling it that they try to quantify the parts of the decision that aren’t monetary: time, convenience, … but no one has taken the suggestion. You should also consider several alternatives – occasional taxis, zipcar – not just a simple comparison of commuting costs.

• Current events topics.
You can do a good paper on a current controversy only if you phrase the questions narrowly enough. One common error is to write a paper that’s just a platform for expressing your own opinions, perhaps quoting experts with whom you agree. I’ve seen that done on the legalization of marijuana, on the incidence of rape or domestic violence, and on the cost of incarceration (from a criminal justice major). You can’t do justice to global warming in a paper for this course. You probably can’t do justice to energy independence. You might be able to manage income inequality. On any of these topics you’d do well to argue both sides of an issue, using data to support contradictory opinions before you come to a conclusion.

• Sharing a paper with another professor
If there’s a topic that would work well in another course you are taking you can consider writing about it if you clear that in advance with me and the other instructor.
A class about GPAs

This posting from our teaching blog shows how we build a class around a couple of homework problems. The students find these particular problems relevant – they care about their GPAs but most don’t really understand how GPA is calculated or what they can do to improve theirs. We use group work and encourage discussion; as this blog posting shows, this sometimes leads to new or different approaches we hadn’t expected.

Class 10 – Thursday, October 3, 2013

From Maura:

I filled in for Ethan today, who couldn’t make it. He gave me two problems to have the class work on, so that’s what we did. Both problems were about weighted averages. I passed them out and asked them to work in groups to solve them. (They are in the book: Exercise 5.7.4 (page 119) and Exercise 5.7.8 (page 120).)

The first problem built on what the class had done with GPA calculations before the exam. The essential problem is that a student has 55 credits and a GPA of 1.8 (this is a cumulative GPA – should have emphasized that). The student will take 12 credits and needs to raise the GPA above 2.0 to avoid academic suspension. What semester GPA does the student need to earn?

While they had done an example last week on calculating semester GPA, the students for the most part weren’t able to make the leap to using cumulative and semester GPA information together. Some got stuck on the number of courses that could make up 12 credits and what individual grades the student should earn. I suggested they keep it simple and just think of a 12-credit course. The other issue was that quite a few groups proposed a semester GPA of 2.2 and argued that since (1.8+2.2)/2 = 2.0, that GPA should work. The tutor and I talked them through the importance of weighting the 1.8 GPA by the 55 credits and how they would do that. Once the groups heard that, they got the right idea. Most of the groups used algebra to solve the problem, but a few did the guess-and-check approach. Both are valid approaches. The algebra approach has the advantage of giving the answer as the lowest possible GPA the student needs to earn. As for guess-and-check, many students took a 4.0 GPA.
semester GPA and established that this would raise the overall GPA above 2.0. Well, yes, but that’s not too realistic for a student who is on academic probation. I encouraged them to refine their guess to get a bit closer to the minimum GPA – most settled on 3.0 as close enough. I asked the groups to put their answers on the board and then we talked them through. While all three groups used the algebra approach, one group used percentages to represent the weights. This was for part (c), where the student takes 6 credits. The group argued that 55 out of 61 credits represents just over 90% of the credits, while 6 out of 61 represents just under 10%. Then they finished the calculation. I liked this approach as it illustrated very clearly how much weight is placed on the 1.8 GPA and should help students see how the 1.8 GPA is is pulling the overall GPA down.

This exercise took a lot longer than I expected – almost 45 minutes. Part of the time was spent talking about what they think they would do in that situation. Would they try 12 credits or focus on only 6? The answers varied, with good reasoning for both sides. I told them that I’m the one who sends out the probation and suspension letters and my experience is that it’s better for students to focus on a small number of classes and do well. I then told them that if they repeat a course, the grade for the repeat is what’s included in their GPA and the first grade is taken out of their GPA. They seemed surprised to hear this. The point is that if you are selective and careful about courses you repeat, you can raise your GPA fairly quickly.

Some of them had already moved on to the second question so I gave them some more time on that. This is a paradoxical one. We have two students, Alice and Bob. Alice’s semester GPA is better than Bob’s in both the fall and spring semesters, but overall Bob has the higher cumulative GPA. The students were asked to invent courses and grades that illustrated this paradox. It’s hard to imagine how this could be, until you start to take it apart. The initial approach of many of the students was to keep everything the same in the two semesters – for example, Alice takes 15 credits each semester and earns a 3.7 GPA each semester, while Bob takes 12 credits each semester and earns a 3.5 each semester. With this approach, Alice’s cumulative GPA will always be higher than Bob’s. To give Bob the edge, he needs to have a lot of credits (that is, a large weight) with a GPA that is higher than one of Alice’s GPAs. And Alice needs to have a lot of credits associated to that GPA. The trick is that Bob’s fall semester GPA could be higher than Alice’s spring semester GPA. When we talked it through in class, people protested that this wasn’t allowed. But in fact it’s legal and the only way to give Bob a higher cumulative GPA. One student put an example on the board for us and we could see how the weights made it work. As an extreme example, I encouraged them to think of Bob taking 15 credits with a high GPA in one semester and only 1 credit with a low GPA in the other semester. Balance Alice’s credits and GPA accordingly, and Bob will end up the winner.

It was fun to revisit this group, several weeks after the beginning of the semester. It’s a good group and I was impressed again at how well they were engaged with the material.
A class about runs

If you can find time in your schedule when you’re teaching Chapter 12 this class exercise may help your students learn a lot about Poisson processes – runs happen.

Everyone understands that the 50% chance of heads when flipping a coin once doesn’t mean that heads and tails will alternate. But they often think, (subconsciously) that when they see a lots of heads in a long sequence something happens to help the tails “catch up.”

We’ve designed an in-class experiment that demonstrates this belief and counter it.

Here’s an outline:

• Don’t describe the purpose of the experiment before you do it!

• Give each student a sheet of paper with an 8 × 8 grid of empty squares.

• Ask each student to *imagine* flipping an imaginary coin over and over again (64 times), filling in his or her grid with “H” or “T” depending on how the imaginary coin falls.

Make sure the students understand that they’re to fill in the squares in “reading order” – left to right in a row, then moving to the start of the next row. Some students will want to fill in random squares – don’t let that happen. If you see students pondering while filling in their grids you can encourage them to flip their mental coins faster.

• When all the grids are done, ask each to count the number of runs of four: four heads in a row, or four tails in a row. Sequences don’t respect the ends of rows – ask students to imagine that all 64 squares were arranged in a single line.

This fragment

\[
\begin{align*}
H & T T T T H H H \\
H & H T & \ldots
\end{align*}
\]

contains three runs – one of tails starting at the second position, two of heads starting at the sixth and seventh positions.

It may take a while to make this counting process clear.
• Query the class to build this data summary at the blackboard.

<table>
<thead>
<tr>
<th>number of runs</th>
<th>number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

• Calculate the average number of runs – a good opportunity to review s.

• Calculate the expected number of runs per students. It’s not hard to convince the class that the probability for $n$ random flips to be all heads or all tails is just $1/2^n$ (you can count all the cases for $n = 1, 2, 3, 4$. There are 61 places on each grid where runs of four might start, so the expected number of runs is $61/8 \approx 7.6$.

You will (almost surely) find your experimental value significantly smaller.

• Now count the runs by column rather than by row, and retabulate. You will find many more, although in our experience not usually enough to reach the expected average.

• Discuss.

The first time most students find very few runs of four. Why? You expect about the same number of heads and tails. Your intuition says don’t put too many of each in a row. Your vision of a fair coin is that it should alternate between H and T fairly often.

When you fill in by rows you switch more often than a real coin would. When you look by columns you’re comparing what’s there to what you wrote 8 or so flips before. You didn’t remember that far back when you were writing, so you get a more random distribution of Hs and Ts and, as happens in nature, we will get runs of Hs and Ts.

The key point here is that runs do happen. We think of the probability of getting heads on a coin toss as being $\frac{1}{2}$, and so we expect that we will not have a long run of heads, but the rules of probability say that this will happen occasionally. You need to be aware that it is a real possibility.

The Ideas section in The Boston Globe on Sunday, June 17, 2012 the carried a story on creating a computer by harnessing human social behavior. One quote caught our eye:

\[9\text{We know there are few blackboards these days. It’s probably a greenboard or a whiteboard.}
\]

\[10\text{https://secure.pqarchiver.com/boston-sub/access/2689315841.html?FMT=FT&FMTS=ABS:FT&type=current&date=Jun+17}
\]
We asked a couple hundred people to complete a string of 1’s and 0’s, and asked them to make it “as random as possible.” As it happens, people are fairly bad at generating random numbers—there is a broad human tendency to suppose that strings must alternate more than they do. And what we found in our Mechanical Turk survey was exactly this: Predictably, people would generate a nonrandom number. For example, faced with 0, 0, there was about a 70 percent chance the next number would be 1.
Page by page comments on the text

The pages that follow offer comments on the text, pedagogical tips and suggestions for the class and sketches or outlines for possible extra exercises. When those exercises have solutions, they appear here.

It’s generated from the \TeX{} source for *Common Sense Mathematics* so that we can edit it where it’s relevant. When page numbers there change the page references here change too.

For class-by-class stories about teaching from *Common Sense Mathematics*, visit our teaching blog at [http://quantitativereasoning.net/ethan-bolkers-quantitative-reasoning-teaching-blog/](http://quantitativereasoning.net/ethan-bolkers-quantitative-reasoning-teaching-blog/)
Chapter 1

Calculating on the Back of an Envelope

Section 1.1, page 1

This chapter and the next are closely related. You might want to combine them when planning classes and homework assignments. It was often hard to decide which exercises belonged here, which there.

Real Fermi problems ask for estimates from scratch. We concentrate instead on developing Fermi problem techniques to verify claims in the media. That’s both easier for students, and follows directly from our focus on working with numbers in the news – educating our students to be consumers rather than producers of quantitative information.

The text in this Chapter stresses techniques: quick mental arithmetic, counting zeroes, some scientific notation and the metric prefixes.

Note that many Fermi problems that used to require estimation skills now succumb to a web query. For example, it’s easy to find a reliable count of the number of kindergarten teachers in the United States.

There are many more exercises in this chapter than you can assign in any single semester. Some of them can form the basis for a discussion that will fill a whole class period. You can consider using those instead of the examples in the text.

Section 1.2, page 2
This section makes an excellent class early in the semester. Tallying the number of students who think millions, billions and trillions is a good place to start.

In class we call this “curly arithmetic”, since \( \approx \) replaces \( = \). The students then cheerfully use the phrase in their homework.

Section 1.3, page 4

A nice thing to do now is to ask the students to take their pulses and report the results. Collect the data so you can make them available in a spreadsheet for the class to play with later when you introduce Excel. You might want to introduce median and mode quickly – but don’t spend arithmetic time now on the mean.

Section 1.3, page 4

When we first used the word “consistent” in class in this context we discovered that many of the students didn’t know what it meant. Students’ limited vocabularies turn out to be a source of confusion in many quantitative reasoning problems. It’s important to be aware of that, and to encourage dictionary use.

Section 1.4, page 5

Work hard in class to make this point.

Section 1.4, page 6

The class will probably want to go directly to “divide by 24.” It may be worth a minute or two to write it out formally to prepare for unit conversions later on.

Section 1.6, page 8

When we teach this material in a course (often as the first class) we divide the students into teams of three. Each team starts on one of the seven estimates with instructions to move on to the next one when done, circling back to google search if they reach the end of the list. The teams start in different places, so after about half an hour the class has found two or
three estimates for each of the seven tasks. The different answers for each task should have the same order of magnitude. If they don’t, we try to figure out why not.

Comments on the Exercises

Section 1.8, page 13

13 We haven’t done this problem with a class yet. If you do, let us know what happened.

The author is probably right about the number. Does that mean that 1/6 of the population are criminals ... (Some of the fingerprints on file are for people who’ve died, and some are for foreigners - but probably very few are of living children.)

The real story is probably that they have lots of fingerprints on file for folks who aren’t criminals - e.g. people who have registered with selective service.

Section 1.8, page 20

35 This is a particularly provocative example since almost everyone’s first guess – including ours – is that it’s an exaggeration.

Extra (ideas for) Exercises

1.8

Exercise 1.8.57. [U][N] How many backspaces?

From

http://blog.stephenwolfram.com/2012/03/the-personal-analytics-of-my-life/

For many years, I’ve captured every keystroke I’ve typed, now more than 100 million of them...

There are all kinds of detailed facts to extract: like that the average fraction of keys I type that are backspaces has consistently been about 7% (I had no idea it was so high!).
Mike Royko wrote a column with that headline for the Chicago Daily News on June 17, 1965. Here are the first paragraphs.

The astronauts had hardly set foot in Chicago Monday when Jack Reilly, the mayor’s official estimator of crowds, announced that 1,000,000 Chicagoans had turned out to greet the space heroes.

A few hours later, Reilly got all excited and announced that he had been wrong. Two million people had turned out.

Later in the afternoon, Mayor Daley got even more excited than Reilly, and took over the crowd-estimating function, although he kept Reilly on the city payroll. Daley said 2,500,000 had turned out to greet the space heroes.

They would have gone on like a pair of drunken pinochle players, shouting “million, billion, trillion, zillion” at each other, had not the astronauts left town.

There is no question that it was an impressive turnout. It may have been, as Reilly (or was it Daley?) said: The biggest welcoming reception in Chicago’s history.

But was it 2,500,000? Or 2,000,000? Or even 1,000,000, which now sounds like almost nothing.

I have taken a flight at finding the answer by using some fundamental and highly questionable arithmetic, which is more than Reilly or the mayor did.

The astronauts were viewed in three basic areas: The airport, along the Kennedy Expressway, and on the parade route downtown. Working backwards, which is the way I like to work, let us start with the downtown parade route.

The city department of streets tells me that the average downtown sidewalk is about 15 feet wide. The average city block is about 450 feet long.

For the entire 31 blocks, this would provide about 420,000 square feet of sidewalk space. Let us assume that all of the spectators were dangerously skinny – half-starved, in fact – and let us cram one into every square foot. It is impossible, but let’s try it. That gives us 420,000 people.

Then there are those who watched from the office buildings. It is impossible to be accurate, but let’s be generous and estimate all of the buildings to be 20 stories high, with about 50 windows fronting the street on a block. And let’s put three people in every window, which is also overdoing it.

This gives us 180,000 window-spectators along the parade route.

Working back to the expressways, there are 31 overpasses, about 450 feet long, with sidewalks about 12 feet wide. By using the same principle of one beanpole per square foot, we have another 341,000 people, most of whom wouldn’t see anything but someone else’s neck.
Then there are those who sat on the slopes of the expressway, where there are slopes, or scrambled up to hang onto the guard rails where the expressway is elevated.

We’ll exaggerate wildly and say that there were 3,000 every mile. This gives us another 60,000 or so.

There were only a thousand or two at the airport, according to witnesses.

The whole thing adds up to about 1,000,000 people.

And since most people need more than one square foot of space, and there weren’t three people in every window, and the overpasses weren’t that impossibly jammed, it appears that there were considerably fewer than 1,000,000 out for the astronauts.

Had there been 2,000,000 or 2,500,000 as Reilly and the mayor suggested, the death toll among the weak and the small would have been too terrible to even think about, even in this column.

It appears, then, that a more accurate guess of the turnout would be in the neighborhood of 500,000 or 700,000.

(a) Comment on Royko’s assumptions. Check his arithmetic.

(b) Is Reilly’s first claim of 1,000,000 greeters a reasonable order of magnitude estimate?

(c) What was the population of the Chicago metropolitan area when this column was written? How can you use that information to provide an alternate critique of the Mayor’s claims?

(d) Estimate the size of the largest crowd that could view a parade in your home town.

Exercise 1.8.59. [R][U][A][N]

Some exercises to practice estimating the result of a sequence of multiplications, free of context – just for practice. Then check with a calculator to see that the answer is in the right ballpark.

Exercise 1.8.60. [U][R] [Section 1.3] Goldman Settles With S.E.C. for $550 Million

That’s what The New York Times reported on July 15, 2010. How much is that per person in the United States?

Exercise 1.8.61. [U][C][G][Section 1.3] [Goal 1.1] [Goal 1.2] [Goal 1.5] [Goal 1.8] Nuclear bombs

“Kiloton” and “Megaton” are terms you commonly hear when nuclear bombs are being discussed. In that context the “ton” refers not to 2,000 pounds, but to the explosive yield of a ton of TNT.
The material in this question makes a good class, combining history, physics and quantitative reasoning about an important subject many current college students know nothing about.

(a) What was the explosive yield of the only (two) atomic bombs ever used in war?

(b) How does the explosive yield of the hydrogen bombs in the current arsenals of the United States and Russia (and other countries) compare to that of those first atomic bombs?

(c) Estimate the destructive power of the world’s current stockpile of nuclear weapons, in terms easier to grasp than kilotons or megatons or gigatons or ....

Exercise 1.8.62. [G][N][Section 1.6] Recycling

The February 28, 2009 issue of The Economist has enough information on waste and recycling to generate as many Fermi problems as you can imagine. And there are ideas there that could lead to interesting possible term papers, if you still need ideas.


In this special report

* Talking rubbish
* You are what you throw away
* Down in the dumps
* Modern landfills
* The science of waste
* The value of recycling
* Waste and money
* Tackling waste
* Sources and acknowledgments

http://www.economist.com/opinion/displaystory.cfm?story_id=13135349

Exercise 1.8.63. [U] [Section 1.5] Cheerio.

Here’s an exercise and its solution from a student. ²

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¹See comment in instructor’s manual.
²(Edward McConaghy, Fall 2012)
How many boxes of Cheerios are sold each year?
If 1 person out of 30 (10,483,000 out of U.S. Population: 314,490,000) had a 2oz bowl (8 servings per box) of Cheerios 3 times a week during a year, that would equal 220,143,000 pounds or boxes (Cheerios happen to come in one pound boxes).

Critique the solution.

**Exercise 1.8.64.** [U][N][Section 1.5] *I.R.S.’s Taxpayer Advocate Calls for a Tax Code Overhaul*

From *The New York Times*, January 9, 2013:

In her legally required annual report to Congress, the national taxpayer advocate, Nina E. Olson, estimated that individuals and businesses spend about 6.1 billion hours a year complying with tax-filing requirements. That adds up to the equivalent of more than three million full-time workers, or more than the number of jobs on the entire federal government’s payroll.  

(a) Check the math: does 6.1 billion hours a year “add up to the equivalent of more than three million full-time workers”?

(b) How many federal employees are there? Estimate or use the web to find out. Compare your answer to three million.

(c) The IRS estimates how long it takes an individual to prepare and file annual income taxes. They may also give an estimate for how much time a business needs to comply with tax-filing requirements. Find these estimates and see if they help you verify Olson’s claim of 6.1 billion hours.
Chapter 2

Units and Unit Conversions

Section 2.1, page 28

This might be a place to discuss the distinction between number and amount – discrete vs. continuous measurement, and the accompanying grammatical questions. It makes sense to say “I bought five pounds of potatoes” or “I bought five potatoes” – note the different units. But you can’t say just “I bought five.” But be careful. Students may find this both pedantic and distracting.

Section 2.8, page 40

The bad news is that this is another made up problem (with easy numbers). The good news is that it’s accompanied by some practical advice for painting.

Comments on the Exercises

Section 2.9, page 56

38 This problem is interesting because it’s political, the computations are easy and the answers don’t make sense.
Extra (ideas for) Exercises

2.9

Exercise 2.9.60. [G][Goal 2.4] [Section 2.8] What does it cost to go green?

One way to save energy is to install solar panels to cover all or part of your roof. Solar panels are expensive, but there are some clever ways to reduce or eliminate the cost. In this project, you will act as a consultant to a homeowner who is thinking of installing solar panels. Your job is to figure out how much of the roof will be covered and what the options are for paying for it. Make sure you look at the option of the homeowner paying for the panels up front, using rebates and credits, and calculate how long it will take to earn that back through reduced electricity costs. Use information from your local area for this, and state your assumptions clearly.

(a) Estimate the area of the roof where the panels will be installed. Solar panels work best when placed on south-facing roofs, so work that into your calculation.

(b) Do some research about options for installing solar panels. One approach is to sign a long-term contract with a solar services provider to purchase the power that the panels generate. Or a homeowner could apply for a grant to cover some of the cost. The federal government also allows homeowners to take a tax credit of up to 30% of the cost of installing panels. There are other options and it’s your job to look into them.

(c) Write a proposal for the customer that outlines the different options. For each option, calculate the up-front costs and the long-term costs (or benefits).

(d) What would you recommend? Write a clear recommendation, justifying your answer with the data you have found and calculated.

Exercise 2.9.61. [Section 2.8] [Goal 2.4] [Goal 2.1] Fire and Ice

In 2013, the Northwest camps of Wayne County Community College District installed a thermal energy cooling system on its campus. This system consists of large water-filled tanks. The water in the tanks is frozen at night, when electricity is less expensive, and used in the cooling system during the day. The college installed its tanks underground in a space between two buildings, measuring approximately 60 by 90 feet.

(a) Standard cooling tanks (from the Calmac corporation, for example, at http://www.calmac.com/products/icebankc_specs.asp are 89 inches wide, 91 inches long and 69.5 inches tall. Use this information to determine the maximum number of tanks that the college could install in that space. Make a sketch of the layout of the installation.
(b) Another type of tank from the same company is cylindrical, measuring 48 inches tall and with a diameter of 73.75 inches. How many tanks of this style could be installed into the space? Make a sketch of the layout of the installation.

**Exercise 2.9.62.** [U] Section 2.1 [Goal 2.1] Back in the saddle again

Bradley Wiggins won the 2012 Tour de France, spending 87 hours 34 minutes 47 seconds in the saddle to travel 3,497 km.

(a) What was his average speed, in km/hour?

(b) What was his average speed, in mi/hour?

(c) Did he travel as far as the distance from Maine to Florida? If not, how far would he have gone on that route?

(d) Did he travel as far as the distance from California to New York? If not, how far would he have gone on that route?

**Exercise 2.9.63.** [U] To Match Walton Heirs’ Fortune, You’d Need to Work at Walmart for 7 Million Years

The businessman Sam Walton founded the WalMart company in 1962. Since then the company has expanded globally and earns hundreds of billions of dollars in sales each year. Sam Walton died in 1992 and at that time was among the richest people in the United States.

Just how rich are the Waltons? According to the latest edition of the Forbes 400, released yesterday, the six wealthiest heirs to the Walmart empire are together worth a staggering $115 billion.

... The average Walmart worker earns just $8.81 an hour. At that wage, the union-backed Making Change at Walmart campaign calculates that a Walmart worker would need:

- 7 million years to earn as much wealth as the Walton family has (presuming the worker doesn’t spend anything)
- 170,000 years to earn as much money as the Walton family receives annually in Walmart dividends
- 1 year to earn as much money as the Walton family earns in Walmart dividends every three minutes

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(a) Verify Making Change at Walmart’s calculations.

(b) What is the average annual salary of a Walmart worker?

**Exercise 2.9.64. [U]** The 2012 Nobel Prize in Physics

Serge Haroche and David J. Wineland won that prize for experimental work in quantum physics. One of their experiments built an optical clock.

The precision of an optical clock is better than one part in $10^{17}$ which means that if one had started to measure time at the beginning of the universe in the Big Bang about 14 billion years ago, the optical clock would only have been off by about five seconds today.  

Check the arithmetic in this paragraph: does one part in $10^{17}$ work out to five seconds in 14 billion years?

**Exercise 2.9.65. [S]** Keeping wikipedia solvent.

This appeal appeared December 2012 on the Wikipedia website:

Dear Wikipedia readers: We are the small non-profit that runs the #5 website in the world. We have only 150 staff but serve 450 million users, and have costs like any other top site. To protect our independence, we’ll never run ads. We take no government funds. We run on donations. We just need 0.3% of readers to donate an average of about $30. We’re not there yet. Please help us forget fundraising and get back to Wikipedia.

If everyone reading this gave the price of a cup of coffee, our fundraiser would be done within an hour. If Wikipedia is useful to you, take one minute to keep it online another year by donating whatever you can today.

(a) Use the data in the first paragraph to estimate Wikipedia’s fundraising goal. Is your answer believable?

(b) Estimate the percentage of the fundraising goal that goes toward staff salaries.

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3 http://www.motherjones.com/mojo/2012/09/sam-waltons-fortune-walmart-employees-7-million-years
4 © 2014 Ethan Bolker, Maura Mast 36
(c) Use the data in the second paragraph to estimate the number of users who visit Wikipedia in an hour. (The average cost of a cup coffee on campus is $2.00.) For a little bit of extra credit, spend no more than five minutes on the internet to see if you can confirm your estimate.

(a) Use the data in the first paragraph to estimate Wikipedia’s fundraising goal. Is your answer believable?

$$0.3\% \times 450 \text{ million users} \times \frac{$30}{\text{user}} = $40,500,000 \approx $40 \text{ million}.$$ 

That sounds reasonable to me.

(b) Estimate the percentage of the fundraising goal that goes toward staff salaries.

I suspect that the 150 employees running this important web site are pretty highly trained in technical matters, so I will estimate their average annual salary as $100K. Then it will take $15,000,000 to cover salaries. That’s $15,000,000/$400,000,000 = 0.375 \approx 38\% \text{ or about one third of their fundraising goal.}$

If you estimate the average annual salary as just $50K then the fraction is closer to 20\%, or one fifth.

Any answer in this range makes sense to me.

The rest of the money is for “costs like any other top site” – things like computers and office space.

(c) Use the data in the second paragraph to estimate the number of users who visit Wikipedia in an hour. (The average cost of a cup coffee on campus is $2.00.) For a little bit of extra credit, spend no more than five minutes on the internet to see if you can confirm your estimate.

It would take 20 million $2 contributions to raise $40,000,000, so that’s the number of hourly visits if their estimate is right.


In June 2007 a one-time measurement was made of number of visitors to all Wikimedia projects combined ... resulted in an average view rate of more than 2500 Wikimedia pages per second, 24 hours per day.

That converts to a rate of 9 million pages per hour, which is in the same ballpark as my estimate.

**Exercise 2.9.66.** [U][Goal 2.4] [Goal 1.2] [Section 2.5] Claim your money, please!

One of the authors received this in email:
I am thompson Cole the newly appointed United Nations Inspection Agent in JFK Airport New York. During our Investigation, we discovered an abandoned shipment on your name through a Diplomat from London, which was transferred to our facility here in JF Kennedy Airport and when scanned it revealed an undisclosed sum of money in a Metal Trunk Box weighing approximately 55kg each.

I believed each of the boxes will contain more that $4M or above in each and the consignment is still left in storage house till today through a registered shipping Company, Courier Dispatch Service Limited a division of Tran guard LTD. The Consignment are two metal box with weight of about 55kg each (Internal dimension: W61 x H156 x D73 (cm). Effective capacity: 680 L.) Approximately.

Mr. thompson Cole goes on to ask for identification, and offers to “bring it by myself to avoid any more trouble. But we will share it 70% for you and 30% for me. But you have to assure me of my 30%.”

(a) Does a box with the dimensions specified contain approximately 680 liters?

(b) What denominations (bills or coins) would you expect to find inside to make a total of $4M?

(c) Does a weight of 55kg seem reasonable for that box full of money?

(d) What things about the email make you suspicious?

Exercise 2.9.67. [U] Cleaning up Everest.

Nepal’s government announced on March 3 that it would require every climber returning from the summit of Mount Everest to bring back at least 18 pounds of garbage, the first concerted effort to eliminate the estimated 50 tons of trash that has been left on the mountain over the past six decades. The waste includes empty oxygen bottles, torn tents and discarded food containers.

(a) How many people would have to bring back 18 pounds of garbage to eliminate that 50 tons?

(b) Estimate How many years would it take to bring back the garbage if everyone who made it to the top did what he or she should.

(c) The quotation says only people who reach the summit would be required to bring back garbage. Answer the previous question if everyone on each expedition had to do that.


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[See the back of the book for a hint.] Use the web to find out enough about the history of Mount Everest expeditions to estimate the average number of climbers and the average number of people who actually reach the summit each year.
Chapter 3

Percentages, Sales Tax and Discounts

Section 3.1, page 63

In principle, the content of this chapter is a review of material on percentages students learned in high school. In fact the review is necessary. Moreover, the presentation is a little more sophisticated, and, we hope, both more useful and more interesting, than what they’ve seen already. So even for those who remember it, there’s value added here.

Section 3.4, page 67

Speaking mathematically, we’d rather have the relative change be the fraction new/old, paralleling the definition of absolute change as new-old. But speaking practically, for the target student audience it’s better to think of the relative change as the difference (new-old)-1. But note that when computing exponential growth later in the book we’ll want to go back to new/old.

Section 3.4, page 67

It’s hard to persuade students to learn the “multiply by 1+change” trick. But it’s well worth the effort, which will pay big dividends later when looking at inflation, interest rates and exponential growth. And it helps wake up students who might otherwise see this material as just boring review of things they once knew and think they still know.
Comments on the Exercises

Section 3.10, page 72

.5 The Gulf oil spill in the summer of 2010 generated lots of data along with the oil. If it had happened during the semester we might have used it daily.

Section 3.10, page 80

.31 Much to our surprise, we found that about a third of the class didn’t know what “wholesale price” meant when we first assigned this problem on markups. Now there’s a hint in the back of the book.

Section 3.10, page 85

.42 This problem comparing private label to branded goods turned out to be more interesting than we thought. There are several decreases (negative values) to deal with. The answer in the solutions manual shows how the 1+ trick lets you find the relative increase without first finding the absolute increase.

Section 3.10, page 87

.48 This exercise is an advance look at the mathematics of compound interest.

Section 3.10, page 89

ch semester we try to find a current news story that students might respond to with a letter to the editor, or an online comment at the appropriate web site. We assign a draft letter or comment as an exercise, discuss the results in class, and offer extra credit for submitted or published comments.

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Extra (ideas for) Exercises

3.10

Exercise 3.10.65. [S][Goal 3.4] Living Without a Cellphone

On September 28, 2012 The Wall Street Journal reported that

In the second quarter, the number of cellphone subscribers on contract plans rose just 0.5% from the year before, to 217 million, according to UBS AG. The number of prepaid customers grew about 11% to 74 million.

(a) What percentage of the people using cellphones have prepaid service now?

(b) How many subscribers had contract plans in the second quarter of 2011 ("the year before")?

(c) How many prepaid customers were there in the second quarter of 2011?

(d) How did the percentage of people using a prepaid service rather than a contract plan change in the last year?

(a) What percentage of the people using cellphones have prepaid service now?

The percentage is

\[
\frac{74}{74 + 217} = 0.254295533 \approx 25\%.
\]

It makes no sense to keep more than two significant digits.

(b) How many subscribers had contract plans in the second quarter of 2011 ("the year before")?

This is a place where the 1+ trick comes in handy. Since the current number is 0.5% more than last year’s number,

\[1.005 \times 217 = \text{last year’s number}\]

so

\[
\text{last year’s number} = \frac{217}{1.005} = 215.920398.
\]

Almost all those digits are nonsense. The answer is about 216 million.

\[1\] Suggested exercise (Lea Ferone, October 2012)
(c) How many prepaid customers were there in the second quarter of 2011?
I used the same method and calculated $74/1.11 = 66.6666667 \approx 67$ million.

(d) What percentage of the people using cellphones had prepaid service last year?
The percentage of people using a prepaid service last year was
\[
\frac{67}{67 + 216} = 0.236749117 \approx 24\%
\]

(e) How did the percentage of people using a prepaid service rather than a contract plan change in the last year?
The percentage increased by about one percentage point from 24% to 25%. Not much at all.

Exercise 3.10.66. [W][R][S] Doublespeak

In his *Letter from Rangoon* in the August 25, 2008 New Yorker, George Packer wrote:

The minister of planning ... gave a long speech that attempted to rebut Petrie’s remarks, using the U.N.’s own statistics. “Some of it was really funny,” Petrie recalls. “He said, for example, 'The U.N. states that a third of the children under five are malnourished. That’s absolutely not true. It’s 31.2 per cent. The U.N. states that three-quarters of an average family’s income is used on food. That’s actually not true. It’s 68.7 per cent.' He was using our statistics to say there was no poverty – that everything was fine.”

(a) Where is Rangoon?

(b) The quote describes how Myanmar’s minister of planning tried to rebut the conclusions of a U.N. report. Do you think he succeeded?

(c) Why did Petrie think the attempt was “really funny”?

(d) Estimate the number of malnourished children in Burma in 2008.

(e) (optional) What is doublespeak? What’s the origin of the term? Is the minister of planning’s response doublespeak?

(a) Where is Rangoon?

Rangoon, now known as Yangon, is the largest city in Myanmar (formerly known as Burma). It used to be the capital.

http://www.newyorker.com/reporting/2008/08/25/080825fa_fact_packer

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(b) The quote describes how Myanmar’s minister of planning tried to rebut the conclusions of a U.N. report. Do you think he succeeded?

No, I don’t think so. The report is describing how much poverty there is in Myanmar. All the minister can do to contradict the report is to criticize the perfectly reasonable approximations it uses to describe the situation in words. 31.2 percent rounds to one third, 68.7 percent is nearly three quarters.

(c) Why did Petrie think the attempt was “really funny”?

The minister was nitpicking the numbers but refusing to acknowledge what they meant.

(d) Estimate the number of malnourished children in Burma in 2008.

According to [http://www.indexmundi.com/burma/population.html](http://www.indexmundi.com/burma/population.html) the population of Burma in 2008 was 47,758,180, or approximately 48 million. The site says the numbers come from estimates made by the US Census Bureau. I see no reason to doubt this figure. Since Myanmar is a very poor country, and poor countries tend to have a disproportionate fraction of small children, I’ll estimate that 20% of the population, or about 10 million, are children. That would mean about three and a half million undernourished children.

(e) (optional) What is doublespeak? What’s the origin of the term? Is the minister of planning’s response doublespeak?

According to [http://dictionary.reference.com/browse/doublespeak](http://dictionary.reference.com/browse/doublespeak) doublespeak is “evasive, ambiguous language that is intended to deceive or confuse.” I think the minister’s comment qualifies.

They say the source of the word might be an analogy to “doublethink”. That cool word, meaning “the act of simultaneously accepting two mutually contradictory beliefs as correct” (according to Wikipedia, which in this case is trustworthy) was invented by George Orwell in his novel “1984”.

Exercise 3.10.67. [C][U] Colleges and the numbers game

On February 4, 2008 Ralph Whitehead Jr. wrote in *The Boston Globe* that

The share of the nation’s 18-year-olds who are from households where no adult holds a four-year degree is 60 percent. If Princeton looked like America, its first-generation number would be 60 [percent], not 11. Its number is about one-fifth of a representative number. Blacks make up 12 percent of America. If Princeton’s enrollment represented one-fifth of black America, the black share of its students would be under 2 percent. If it did the same for women, the female share of its students would be under 10 percent. [*http://pqasb.pqarchiver.com/boston-sub/access/1423321841.html?FMT=FT*](http://pqasb.pqarchiver.com/boston-sub/access/1423321841.html?FMT=FT)
We find these numbers quite confusing. Can you figure out what Whitehead is trying to say? This exercise might make an interesting class discussion. Assign it first, then build a class around student solutions.

**Exercise 3.10.68.** [U][Goal 1.2] [Goal 1.4] [Goal 3.3] *Writing Your Dissertation in Fifteen Minutes a Day*

Author Joan Bolker’s book with that title was published in 1998. The hundred thousandth copy was bought in 2011.

Approximately what percentage of the graduate students writing dissertations in those years bought the book?

**Exercise 3.10.69.** [U][N] ADHD

The web page [http://www.cdc.gov/ncbddd/adhd/data.html/](http://www.cdc.gov/ncbddd/adhd/data.html/) from the Centers for Disease Control (CDC) reports this data on Attention-Deficit / Hyperactivity Disorder (ADHD):

The American Psychiatric Association states in the Diagnostic and Statistical Manual of Mental Disorders (DSM-IV-TR) that 3%-7% of school-aged children have ADHD. However, studies have estimated higher rates in community samples.

Parents report that approximately 9.5% or 5.4 million children 4-17 years of age have ever been diagnosed with ADHD, as of 2007.

The percentage of children with a parent-reported ADHD diagnosis increased by 22% between 2003 and 2007.

Rates of ADHD diagnosis increased an average of 3% per year from 1997 to 2006 and an average of 5.5% per year from 2003 to 2007.

Boys (13.2%) were more likely than girls (5.6%) to have ever been diagnosed with ADHD.

(a) Make sense of these statistics.

(b) Read and comment on Bronwen Hruska’s opinion piece *Raising the Ritalin Generation* at [http://www.nytimes.com/2012/08/19/opinion/sunday/raising-the-ritalin-generation.html](http://www.nytimes.com/2012/08/19/opinion/sunday/raising-the-ritalin-generation.html)

**Exercise 3.10.70.** [N] Food security


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4See comment in instructor’s manual.
Exercise 3.10.71. [N][G] Poor Smokers in New York State Spend 25\% of Income on Cigarettes, Study Finds


The discussion at Andrew Gelman’s blog http://andrewgelman.com/2012/09/poor-smokers-in-new-york-state-spend-25-of-income-on-cigarettes-study-finds/#comments is a gem, dissecting the assumptions behind the headline and the study.
Chapter 4

Inflation

Section 4.1, page 96

Computing the effective increase after taking inflation into account is subtle. You can’t just subtract the percentages. We return to this question in the section below addressing what a raise is worth. It’s probably best to finesse the question here by making only qualitative statements – e.g. “faster than inflation.”

Section 4.2, page 96

It’s difficult to keep the printed text up to date on current events. When you teach this section, study inflation from last year to this year. Then the students have two separate treatments to learn from – yours in class and the one in the book.

Section 4.4, page 99

The inflation calculator gives $215.40 for 1983 and $206.48 for 1984. The average is

\[(214.40 \times 206.48 \times 222.32)^{1/3} = 214.302433\]

but you don’t want to teach that!
Comments on the Exercises

Section 4.7, page 108

25 This would make an interesting spreadsheet exercise when we get to spreadsheet calculations and graphing. Then you can ask if there is any year in which the minimum wage went up fast enough to account for inflation.

Extra (ideas for) Exercises

4.7

Exercise 4.7.31. [N][U][Section 4.5] [Goal 4.1] [Goal 4.2] [Goal 4.4] COLA

The website [http://www.fedsmith.com](http://www.fedsmith.com) featured an article in October 2011 about the projected 2012 cost of living increase for federal retirees. It noted that

> For the first time in three years, there will be an increase in the cost of living adjustment for federal retirees and those receiving Social Security checks. The amount of the increase: 3.6% for CSRS retirees and Social Security recipients. . . . The last COLA increase was 5.8% in 2009 (based on 2008 inflation figures). The two years without any increase in the cost of living had not happened since the automatic increase formula for Social Security was established in 1975. Prior to these two years without any increase, the lowest annual adjustment was 1.3% in 1998. The reason for the lack of an increase the past two years was because, as reported by the federal government for this purpose, there was no inflation.  

Need questions here. This may be too complicated because the COLA does not seem to be equal to the inflation rate. The idea of zero inflation and no cost of living raise is an interesting one.

Exercise 4.7.32. [N][Goal 3.3] [Goal 2.1] [Goal 4.2] 48 Tons of Silver Recovered From World War II Shipwreck

On July 18, 2012 ABC News reported that

> An American company has made what is being called the heaviest and deepest recovery of precious metals from a shipwreck.

The recovery is being made under a contract awarded by the U.K. government, which will keep 20 percent of the cargo’s value, estimated to be in the tens of millions of dollars. The Gairsoppa became U.K. property after the government paid the owners of the ship an insurance sum of 325,000 in 1941. Records indicate the silver was valued at 600,000 in 1941.

The Tampa, Fla.-based Odyssey Marine Exploration, Inc. announced Wednesday that it had recovered 48 tons of silver [and] that about 240 tons of silver may be recovered by the end of the operation.

Possible questions:

(a) some routine percentage calculations
(b) value of silver then and now? did it match inflation rate?
(c) currency conversion

Exercise 4.7.33. [U][N][W][Goal 1.2] [Goal 1.4] Apple Becomes the Most Valuable Public Company Ever, With an Asterisk

From The New York Times blogs, August 20, 2012:

With a surge in its share price, Apple broke the record for the biggest market capitalization, $620.58 billion, set by Microsoft on Dec. 30, 1999, … That record was for an intraday market value – a company’s total outstanding shares multiplied by its stock price at some point during the trading day. On Monday, Apple had an intraday market value of $623.14 billion when its shares traded at $664.74.

To exceed Microsoft’s record for market value at the close of a trading day, Apple shares needed to close at $657.50 or higher, … which they did. The stock closed at $665.15.

Another analyst [noted] that Microsoft’s 1999 market value is still far higher than Apple’s when adjusted for inflation. The Microsoft of late 1999 would be worth $850 billion in today’s dollars … The Microsoft of August 2012 is worth $257 billion.

To beat Microsoft’s inflation-adjusted market value, Apple needs to close at $910 … Perhaps not this week.

http://abcnews.go.com/International/record-setting-silver-recovery-made-world-war-ii/story?id=16806534#.UAglLMZHcbTo

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Chapter 5

Average Values

Section 5.1, page 112

Decide whether you want to point out that the units of the average are the same as the units of the things you are averaging, since the weights are dimensionless. Sometimes we do, sometimes we don’t. The observation may confuse students while they are working to master the new concept.

Section 5.2, page 112

Students find this section on grade point average computation compelling – many often say they had no idea how it was done and are delighted to have found out. It makes the weighted average concept clear in a context that really matters.

Section 5.3, page 114

We find that many students solve this equation by writing some version of

\[
\frac{90 \times 2.8 + 30 \times G}{120} = 3.0 = 360 = 108 = 3.6.
\]

You can of course see what they’re thinking, and the answer is the right value of \( G \). Their prose (if that’s what you can call it) conflates “=” meaning “is the same number as” with “=” meaning “is the same equation as”. We constantly ask for “more words” and can’t seem to get them. If you know how, please let us know.
One of the readers of an early version of *Common Sense Mathematics* suggested that we start the solution with guess-and-check and finish with algebra, to reinforce the value of a viable understood strategy compared to one that may be murky and must be remembered. Consider that when you lecture on the material.

**Section 5.3, page 114**

It’s worth taking a little time to practice the guess-and-adjust-your-guess strategy. It may not be as efficient as algebra in situations like this, but it’s much more generic and much less arcane. Students can understand and appreciate it and might even remember it.

**Section 5.6, page 117**

This paradox is a well known phenomenon. See Chapter 6, A Small Paradox, in *Is Mathematics Necessary?*, Underwood Dudley (ed.), Mathematical Association of America, 2008, and further references there.

**Comments on the Exercises**

**Extra (ideas for) Exercises**

5.7

**Exercise 5.7.21. [U][N][Section 5.1] [Goal 5.1] Improving Reading Skills Should be a state priority**

On June 29, 2010 the *Cape Cod Times* reported that “Almost half of Massachusetts third-graders are not proficient readers.”

Last year, 65 percent of low-income third grade students scored below proficient on the MCAS reading test. And overall, the percentage of third-graders receiving below-proficient scores has hovered around 40 percent over the last decade.  

There’s a weighted average hidden here. If we knew either the percentage of low-income third graders in the population or the percentage of non-low-income students who scored below proficient we could find the other percentage.
Exercise 5.7.22. [U][C][Section 5.5] [Goal 5.1] Average Boston homeowner can expect $220 tax increase

The property tax bill for the average home in Boston is increasing by $220 in the current fiscal year, the city announced yesterday.

... Ronald W. Rakow, the city’s commissioner of assessing, said in a telephone interview that the bill for the average single-family residence will come to $3,155 in fiscal 2011, which began in July, up from $2,935 in fiscal 2010. He said the rate is “still very competitive, compared to the statewide average” of $4,390.

“Due to the current economic climate, reductions in the [city’s] other revenue sources, most notably state aid, left the [city] with little choice but to increase the property tax levy by the maximum allowed under Proposition 2 1/2,” the state law capping property tax rate increases at 2.5 percent without an override, Mayor Thomas M. Menino’s office said in a statement yesterday.

Menino’s office said the tax rate for residential properties in fiscal 2011 is $12.79 per $1,000 of assessed value, up from $11.88.

The rate for commercial properties is $31.04 per $1,000 of assessed value, up from $29.38 in the last fiscal year.

... The tax rate for homes in the city was $11.12 per $1,000 of assessed value in fiscal 2006, $10.99 in fiscal 2007, $10.97 in fiscal 2008, and $10.63 in fiscal 2009, records show.  

Figure 5.1 accompanied the article.

Lots more questions to ask

(a) Verify that the rate increases for residential property for the years reported were consistent with Proposition 2 1/2?

(b) Is the rate increase for commercial property consistent with Proposition 2 1/2?

(c) What does “average homeowner” mean? Median, mean or mode?

(d) To do: write a question about total assessment and tax income - probably a weighted average.

1 http://www.boston.com/news/local/massachusetts/articles/2010/12/18/average_boston_homeowner_can_expect_220_tax_increase/
Figure 5.1: Boston Real Estate Assessments
Chapter 6

Income Distribution – Excel, Charts and Statistics

Section 6.1, page 126

We find students come to our course with a wide range of technology experience. They can all manage word processing, web searching and email. Some have used Excel. Those for whom it is new find this chapter hard going. Often pairing experienced and inexperienced students in the lab classroom works well.

Section 6.5, page 131

We strongly recommend drawing these bar charts first by hand on the board, and asking students to do the same from time to time on paper.

Section 6.5, page 133

Excel can use separate vertical scales to plot two data series. We think it’s more useful and more interesting to teach this ad hoc solution.

Section 6.8, page 136
This hand drawn figure may seem unprofessional, but in fact we think it’s useful. It’s more like what a student could produce than a fancy graphic would be. We think we should have more pictures like this.

Section 6.9, page 137

We find this a particularly valuable section – it forces students to come to grips with the real meanings of each kind of average. Just memorizing definitions suffices for small data sets, but with grouped data the mean is a weighted average and real understanding is required.

We recommend thinking about mode, median and mean in that order.

Section 6.9, page 137

You will need to take time here to explain both the miracle – Excel guessed that updating references was what was wanted – and its value in saving time and typing. It takes getting used to.

Section 6.12, page 142

This is one of those places where it’s hard to find the right compromise between appropriate simplifications and actual untruths about what the concepts mean and the numbers say. It might well take a whole class period to expand on the discussion here. You have to decide whether that’s how you want to spend class time.

Comments on the Exercises

Section 6.14, page 145

er’s an important and subtle distinction here between weights as a percentage of revenue and weights as a percentage of trips. Worth a discussion in class if you have time and a good enough class.

Section 6.14, page 147

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58
We haven’t assigned this Benford’s law exercise yet. We think it would be very interesting, but perhaps not worth the time it takes away from other more useful topics.

Section 6.14, page 149

This exercise may be worth assigning for the Excel practice, and for reinforcing the computations of various averages from summarized data. But the conclusions aren’t very striking.

Section 6.14, page 156

Your students will probably not think this topic is useful or interesting but it might be fun if introduced in class.

Extra (ideas for) Exercises

6.14


The chart in Figure 6.1 shows the salary distribution of tenured faculty at the University of Texas Houston. Vertical scale represents the number of faculty, the horizontal axis shows salary ranges in increments of $20,000.
(a) Recreate this data in Excel as both a table and a histogram.

(b) What is the modal salary for UTH faculty?

The mode is the highest bar – about $60-$80K.

(c) What is the median salary for UTH faculty?

The halfway point to the 203 faculty members (I added up the heights of the bars in Excel) occurs partway through the second bar, so the median is about $70K.

(d) What is the mean salary for UTH faculty?

The mean salary is about $81K – computed in Excel as a weighted average. The spreadsheet shows mean 81.03448276


Where The One Percent Live: The 15 Richest Counties In America

Living in Arlington isn’t cheap, so you’d better be making at least the median household income to live in this county just outside Washington, D.C. ¹

How does this statement contradict itself? ²

Exercise 6.14.41. [U]|Section 6.8 [Goal 6.3] The article on new car and truck prices that we studied in Section 5.5 first asserts that

... the average price of a new vehicle in the second quarter [of 2008] fell 2.3 percent from a year earlier to $25,632 ...
The result is the average new vehicle now costs less than 40 percent of an average household’s median annual income, the analysts said, whereas from 1991 to 2007, it would cost more than half of the median income.  

Verify as much of this last assertion as you can.

**Exercise 6.14.42. [U][A][Section 6.5] [Goal 6.4] [Goal 6.5] [Goal 6.6] Enrollments**

The final enrollment report for the past year at an unnamed small college provided the following information about students: 450 students freshmen; 421 students sophomores; 400 students juniors and 511 seniors.

(a) Create an Excel spreadsheet containing this data. Label the columns appropriately.

(b) Ask Excel to calculate the total number of students enrolled during the past year. Label this result.

(c) Create a properly labeled bar chart of the student data.

(d) A corrected enrollment report noted that there were 419 juniors. Make that adjustment in your spreadsheet and check that the other information (total number of students, bar chart) is correctly updated.

(e) Using this new information, ask Excel to calculate the percentage of students who are freshmen, sophomores, juniors and seniors. Copy and paste so that you type as few formulas as possible.

(f) Create a new bar chart displaying the percentages.

(g) Convert your bar chart to a pie chart.

(h) Arrange the data with percentages and the final bar and pie charts so that they print on one page.

**Exercise 6.14.43. [U][A][Section 6.10] [Goal 6.9] SAT exam percentiles**

A student received this notification on his college entrance exam:

**English Language Arts:** 77th percentile

**Mathematics:** 88th percentile

Explain these this report in everyday language.

[See the back of the book for a hint.] Your answer might begin “More than three quarters of the students taking this test . . .”

**Exercise 6.14.44.** [U][Section 6.8] [Goal 6.1] Comparing the states

You can do this exercise using Excel, or with properly documented research. (Your instructor may specify one method or the other.)

Find the mean, median and mode for the populations of the 50 states.

Display the answers to the previous question on a properly labeled histogram. Discuss your findings – is the distribution skewed?

Redo parts (a) and (b) for the areas of the states.

Redo parts (a) and (b) for the population densities (people per square mile).


**Exercise 6.14.46.** [U][Section 6.9] [Goal 6.6] [Goal 5.1] [Goal 6.7] [Goal 6.8] Reputation on stack exchange

Stackexchange ([http://stackexchange.com](http://stackexchange.com)) is a network of online question and answer websites. Users who post questions and provide answers earn reputation based on community feedback. Table 6.2 shows the number of users with reputations in certain ranges on January 6, 2013 for all stackexchange sites and for the particular site [http://tex.stackexchange.com/](http://tex.stackexchange.com/) where the authors have asked and answered questions about the TeX software used to prepare the manuscript for *Common Sense Mathematics*.

Estimate the mode, median and mean for each distribution. This is subtle in several ways. The bucket sizes vary. Data at the top and bottom end of the range are very scarce. Ask about sensitivity to the assumptions made there about the actual means for the top and bottom categories.

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Table 6.2: Stackexchange reputation
Chapter 7

Electricity Bills and Income Taxes – Linear Functions

Section 7.1, page 161

In a course devoted more to real life applications than to modeling, you can move directly to the section on taxes. There’s no explicit need there for the slope-intercept description of a linear model.

Section 7.3, page 164

We usually treat the slope and intercept as given data, since that’s how they appear in the world. Computing the slope as \( \Delta y/\Delta x \) belongs in an algebra class that’s leading to calculus, but not here.

Section 7.4, page 166

Since the entries in the Tamworth Electricity Bill table are out of order, Excel has drawn some of the segments in the graph twice. You can see that if you look carefully. You might want to point this out, or not.

Section 7.6, page 170
We’re undecided about how much time to spend on power and energy. It takes a lot of teaching time (and energy) to convey the distinction convincingly in class. Perhaps those resources are better spent on other parts of the curriculum. But the topic is important and ultimately interesting, because students care about climate change and alternative power sources. We have included problems that explore the issue further.

If you do choose to spend more time on the difference between power and energy, and the confusion because the name “watt-hour” contains a unit of time, consider discussing the light-year, which is a measure of distance, not time. So “light-years ago” is never right. The knot – one nautical mile per hour – is also a rate, like power, that doesn’t mention time.

Section 7.7, page 174

In the tax rate brouhaha in the 2008 election we recall reading a story about a dentist who complained that he would need to be careful not to let his income exceed $250,000 – where candidate Obama drew no-new-taxes line – lest his overall tax rate increase. If you find the story let us know and we’ll turn it into an exercise.

Comments on the Exercises

Section 7.8, page 177

Exponential decay might be a better model. Consider returning to this exercise when you reach that topic later in the text.

Section 7.8, page 185

22 The article on the quarry water cooling system also asserts that

The system ... cost only about $700,000 more than a traditional cooling system, meaning Biogen Idec should get a return on its investment in eight to 10 years.

Discussing payback time might be interesting – or too difficult.
Extra (ideas for) Exercises

7.8

Exercise 7.8.39. [U][N][Section 7.6] [Goal 7.1] [Goal 7.7] Atop TV Sets, a Power Drain That Runs Nonstop

The New York Times reported on June 26, 2011 that

Those little boxes that usher cable signals and digital recording capacity into televisions have become the single largest electricity drain in many American homes, with some typical home entertainment configurations eating more power than a new refrigerator and even some central air-conditioning systems.

A new study has found that some home entertainment systems eat more energy than refrigerators or central air-conditioning systems.

There are 160 million so-called set-top boxes in the United States, one for every two people, and that number is rising. Many homes now have one or more basic cable boxes as well as add-on DVRs, or digital video recorders, which use 40 percent more power than the set-top box.

One high-definition DVR and one high-definition cable box use an average of 446 kilowatt hours a year, about 10 percent more than a 21-cubic-foot energy-efficient refrigerator, a recent study found.

These set-top boxes are energy hogs mostly because their drives, tuners and other components are generally running full tilt, or nearly so, 24 hours a day, even when not in active use. The recent study, by the Natural Resources Defense Council, concluded that the boxes consumed $3 billion in electricity per year in the United States – and that 66 percent of that power is wasted when no one is watching and shows are not being recorded. That is more power than the state of Maryland uses over 12 months.

The graphic in Figure 7.1 accompanied the article.

Hiawatha Bray’s Stop power leaks; smile at savings from The Boston Globe on January 15, 2009 refers you to http://standby.lbl.gov/standby.html a site maintained by the Lawrence Berkeley National Laboratory, at which you can find out lots of useful stuff about standby power consumption. This reference to the Hiawatha Bray article is the first of several possible exercises on the same theme – if you combine them and assign them and let us know what happened we’ll rewrite them.

2 http://www.boston.com/business/technology/articles/2009/01/15/stop_power_leaks_smile_at_savings/
3 See comment in instructor’s manual.
Many electronic devices use a small amount of electricity even when they are turned off. A TV, for example, is always in standby mode unless it is unplugged, and so is drawing a small amount of electricity. Similarly, computers, modems, cell phones and other devices draw electricity even when turned off. This is known as “leaking electricity”. An article in 1998 estimated that this accounts for about 45 billion kilowatt-hours of electricity consumed in the United States each year, or about 5% of the total electricity use by individuals. (see http://www.aceee.org/pubs/a981.htm)

(a) Choose one electric item in your home and research how much electricity it uses when it is off. You may be able to find this in the documentation for the device, or on the web. Make an estimate if you can’t find the exact specifications.

(b) Suppose you unplugged this device at night, when you are done using it. Estimate how much electricity this will save and how much money you will save in a year.

Exercise 7.8.41. [N] Romney Says His Effective Tax Rate Is 'Probably' 15%

An Associated Press article in the Houston Chronicle on January 18, 2012 about the tax returns of then Presidential candidate Mitt Romney said the following:

Speaking to reporters after a campaign stop in South Carolina, Romney said most of his income comes from investments, not regular wages and salary. The tax rate on investment income is 15 percent, much lower than the 35 percent rate applied to wages for those in the highest tax bracket.

“What’s the effective rate I’ve been paying? It’s probably closer to the 15
percent rate than anything,” Romney said.

(a) The median net income of an American family in 2012 was about $50,000. What is the effective tax rate for someone with that income?

(b) How does Romney’s tax rate of “probably” 15% compare to the “average” American?

Exercise 7.8.42. [U][C][N] Does virtual save energy?

MOST PEOPLE take for granted the Earth-friendly nature of electronic communication. Paperless, ink-free, no shipping supplies, no gas for transportation: the environmental benefits of virtual communication are obvious. But the reality is more complicated, at least according to a growing number of concerned technology experts and scientists. Vast stockpiles of digital data waste energy, too.

Everyday emails aren’t to blame. But large photo and video attachments, cluttered inboxes, and massive email forwards may be. Some analysts estimate that emailing a 4.7-megabyte attachment – the equivalent of four large digital photos – can use as much energy as it takes to boil about 17 kettles of water. The problem is magnified when large emails are forwarded to many people and left in inboxes undeleted. As long as emails remain in your inbox, the data they create is physically stored somewhere.

And that’s where the problems arise: The total amount of digital storage worldwide is approaching 1 zettabyte, or 1 million times the contents of the Earth’s largest library. Currently, that information is archived on equipment with a mass equivalent to 20 percent of Manhattan. Global data storage is expected to reach 35 zettabytes by 2020, which means more equipment, land, and energy. The information industry already accounts for approximately 2 percent of global carbon dioxide emissions. That’s the same amount as the airline industry blasts into the atmosphere. Coupled with the rapid increase in stored data, it’s an unsustainable scenario.

Technology firms must create systems that store data with less energy, and governments should provide incentives for them to do so. Just as important, consumers must demand products that save energy, and use websites like Flickr and MediaFire that allow them to share large files without emailing. Better still, they could consider keeping some of those embarrassing photos and home videos to themselves.

Exercise 7.8.43. [U][N] The Gas Is Greener

http://www.boston.com/bostonglobe/editorial_opinion/editorials/articles/2010/09/07/dont_forward_those_photos/

**Exercise 7.8.44.** [U][N] Every little bit counts.

On March 5, 2012 The Boston Globe reported on the Ocean Renewable Power Company’s plans to install tidal powered generators in Maine:

The first unit capable of powering 20 to 25 homes will be hooked up to the grid this summer, and four more units will be installed next year at a total cost of $21 million . . .

Eventually, Ocean Renewable hopes to install more units to bring its electrical output to 4 megawatts.

**Exercise 7.8.45.** [N] Power, Pollution and the Internet

On September 23, 2012 The New York Times printed an article with that headline. It’s full of numbers that make for interesting explorations. More will be forthcoming:

This is the first article in a series about the physical structures that make up the cloud, and their impact on our environment.

Nationwide, data centers used about 76 billion kilowatt-hours in 2010, or roughly 2 percent of all electricity used in the country that year,

Here’s one of the comments (many are very interesting).

This article is weakened by hysterical reporting that relies on meaningless unscaled scare statistics rather than a balanced presentation of the issues.

Buried deep in the article, one finds a more meaningful statistic: data centers use only 2 percent of the electricity used in the country, hardly a huge figure, given their importance in today’s economy.

You also fail to consider the cost of alternatives. How much does it cost to sort and deliver a first class letter? To print and distribute a paperback book? To answer a phone bank call? To drive to a movie theater? It is likely that the data center is saving energy, not costing it.

No doubt energy efficiency can be improved, and the Times is to be lauded for pointing that out. But no one who has ever worked with this kind of technology

www.bostonglobe.com/business/2012/03/05/maine-company-ready-install-tidal-power-unit/daJ3ivfrUUNh0ejo0t2Iqj/story.html
can suppose that facilities of this kind can be built without backup generators, batteries, and air conditioning: so crucial are these servers to today’s economy that when a major service goes down for an hour, it is front page news. And in the absence of context, the implication that data centers are disproportionately responsible for wasting energy is questionable indeed.

**Exercise 7.8.46.** [N][Section 7.6] [Goal 7.7] Graph gives calculations for force of a bomb

![Graph](image)

**Figure 7.2: Energy and Power from an Atomic Bomb**

This graph is good for discussing the difference between energy and power (the power energy curve is the accumulation of the power curve) but as evidence about the Iranian nuclear program it’s quite suspect – see [http://www.guardian.co.uk/commentisfree/2012/nov/28/ap-iran-nuclear-bomb](http://www.guardian.co.uk/commentisfree/2012/nov/28/ap-iran-nuclear-bomb)
Chapter 8

Climate Change – Linear Models

Section 8.1, page 191

The students may all be interested in this topic, so they will want to think about it. The consensus among climate scientists is that it’s real and anthropogenic, but the real science is complex.

We do teach how to find regression lines (using Excel). But you can’t draw reliable conclusions from simple regressions like the ones here. So treat this material respectfully and cautiously. Our approach stresses skepticism throughout. Rather than teaching regression as a tool they can use, we treat it as a tool often misused.

This part of this chapter, like the start of the last one, is written as an Excel tutorial. If possible, students should follow along, checking the steps using Excel as they read or as you lecture.

Section 8.1, page 191

If you are working on this section in a classroom which allows you to project a spreadsheet onto a screen you can reach you can eyeball the regression line with a yardstick.

Section 8.1, page 193

This error surprised us when it occurred during a class we hadn’t prepared carefully. That
turned out to be useful – the students saw their teacher seeing that a number made no sense, then looking for an explanation.

Section 8.1, page 195

This is an important point – one worth emphasizing whenever it comes up. Encourage your students to use more digits in any intermediate calculation – in particular, when working with the trendline equation. Even better, encourage them to think about the numbers and the graph. That’s the best approach.

Section 8.1, page 195

When we reviewed this chapter, we ended up arguing about the validity of a prediction 10 years into the future. Students who know a bit about extrapolation may raise that issue; others may ask why we bothered to predict to 2010 if we already had the data. Here’s part of our dialogue:

Maura: Philosophically, I’m more comfortable with a prediction into the next year as opposed to a prediction 10 years in the future. That’s the other reason why I’d like to include the data through 2010 in the graph.

Ethan: The temperature data is so erratic that any prediction is likely to be wrong. I picked 10 years because we have actual data that far out so I could use a part of the data as an experiment. One year wouldn’t be good visually.

Section 8.3, page 197

We have deliberately omitted any discussion of the correlation coefficient $R$. We found when we taught that material from an early draft of Common Sense Mathematics we used up a lot of class time on material that did not meet our “what should students remember ten years from now?” criterion for inclusion. We think that thinking qualitatively about $R^2$ is sufficient.

Section 8.4, page 197

Do spend some time teaching the students to scrape data using cut and paste. That and the fact that Excel can read .csv files will save them grief in this course and whenever

Comments on the Exercises

Section 8.5, page 199

1 This is an interesting exercise to work in class.

Extra (ideas for) Exercises

8.5

Exercise 8.5.27. [U][C][Section 8.1][Goal 8.1] Polarization

Note: The data need work before we can ask the students to deal with them.

Figure 8.1 appeared in The Boston Globe on November 6, 2010. We extracted the numerical data from the graph; you can find it at http://www.cs.umb.edu/~eb/qrbook/Instructor/polarization.csv.

![Figure 8.1: Income Disparity and Political Polarization](image)

(a) Find the trendline modeling a linear relationship between the income share of the top 1 percent of the population and the political Polarization index.

(b) Find the trendline modeling a linear relationship between the income share of the top 1 percent of the population in a year and the political Polarization index four years earlier.

Exercise 8.5.28. [U][C] Do the math on overrides

Barry Bluestone and Anna Gartsman wrote an op-ed with that headline in The Boston Globe on June 4, 2010. Implicit in what they propose are several linear dependencies among statistics describing towns in Massachusetts:

We decided to test this theory by simulating the impact on home values of a change in school spending due to a Prop. 2 override, controlling for other factors. We obtained data on housing values in 2005, the change in housing values between 2005 and 2010, and two measures of perceived school quality: school-wide SAT scores and per pupil expenditures. We found complete data for 176 of the 351 cities and towns in the Commonwealth.

According to our analysis, which controlled for initial home value in 2005, a municipality with SAT scores and per pupil spending levels 20 percent higher than average experienced a 24 percent increase in nominal home value between 2005 and 2010. In contrast, a municipality with SAT scores and per pupil spending 20 percent below average experienced a loss in home value of 11 percent.

So, how much difference would the passage of the Hull override have potentially meant for home values in that community? Hull’s 2005 average home value of $366,343 was near the mean for the communities in our study. The average SAT score in the Hull public schools was 961 compared with an average of 1047 for all the study communities. Average per pupil expenditure in Hull was $11,491, some $1,500 higher than average. Based on our home value model, the predicted increase in home values in Hull between 2005 and 2010 was 3.85 percent.

Now what would likely have happened to the average home value in Hull if the recent proposed $1.9 million override had been passed back in 2005? This tax increase would have cost the average homeowner in Hull $506 per year. Over five years, it would have totaled $2,530. However, that tax increase would have resulted in an additional $1,442 spent per pupil. This increase would result in a predicted increase in home value of 6.57 percent rather than the increase of 3.85 percent. The difference between the two predicted values results in an average increase in home value in Hull of $9,970.²

These figures are probably the result of a regression study. Identify the slopes of the regression lines involved, and verify the predictions.

²http://www.boston.com/bostonglobe/editorial_opinion/oped/articles/2010/06/24/do_the_math_on_overrides/
Exercise 8.5.29. [U][C] Say “Cheese”

Figure 8.2 appeared in The New York Times on November 7, 2010.

![Cheese Consumption](image)

The graphs suggest a linear model. Explore.

Exercise 8.5.30. [N] Climate changes

The Economist published Figure 8.3 on May 12, 2010, along with the following paragraph:

How global surface temperature, ocean heat and atmospheric CO2 levels have risen since 1960.

THE record of atmospheric carbon-dioxide levels started by the late Dave Keeling of the Scripps Institute of Oceanography is one of the most crucial of the data sets dealing with global warming. When the measurements started in 1959 the annual average level was 315 parts per million, and it has gone up every year since. To begin with it went up by roughly one part per million per year. Now it is more like two parts per million per year. The figure for 2011 is 391.6. More carbon dioxide in the atmosphere means a stronger greenhouse effect, and various measurements speak to this. Global surface temperature records show a warming over the same period, though because of fluctuations in the climate, air pollution, volcanic eruptions and other confounding factors the rise is nothing like as smooth. A steadier rise can be seen in the heat content of the oceans, measured in terms of the energy stored, rather than the temperature.

Exercise 8.5.31. [N] Start with a graph

This is a placeholder for Exercises as suggested by a Cengage reviewer:
I’d like to see more homework problems here that begin from a graph and trend line, rather than beginning from a data set. Given that it is so easy to mislead with graphs, this would help students to develop those “defensive reading” skills that appear to be one of the goals of this chapter.
Chapter 9

Compound Interest – Exponential Growth

Section 9.4, page 216

Of course you don’t need to rely on experiments to know that the doubling time is independent of the initial value. It’s very easy to prove with a little bit of algebra. But this is a book about quantitative reasoning, not about algebra. For its intended audience the experiments are more convincing than the more formal mathematics many people find mysterious.

Section 9.4, page 218

If we were teaching algebra and not quantitative reasoning we might use a negative exponent to write \((\frac{1}{2})^{10}\) as \(2^{-10}\). But we’re not, so we don’t, so we avoid the time it would take to remind students about working with negative exponents.

Section 9.5, page 218

The material in this section on bacterial growth is at the edge of what we think students in a quantitative reasoning course need. It deals with real data, not the artificial doubling time problems in most books. Carrying through the discussion in sufficient detail to allow them to solve similar problems would take time better spent on other topics. But if there’s time in the syllabus there are ideas here worth exploring. They tie together all the themes of the chapter.
Comments on the Exercises

Section 9.6, page 225

14 We could construct the Fermi problems based on this radioactive waste data ourselves, and ask the students to solve them. But by this time in the course we hope they can start from the numbers and create their own.

Extra (ideas for) Exercises

9.6

Exercise 9.6.39. [S] So many words!


> [F]or the last 300 years, the number of words published annually grew exponentially by about 3 percent per year. From about 20 million words for 1700, the annual word count grew to several trillion for 2000. 

(a) Check the authors’ arithmetic.

(b) If the growth continues at the same rate how many words will be published in the year 3000?

(c) How much confidence do you have in your prediction?

(a) Check the authors’ arithmetic.

The coolest way to do this is with the rule of 70. A 3% annual increase has a doubling time of \(70/3 \approx 25\) years. The question asks about 300 years of growth, which would be 12 doublings.

But to get from 20 million to 20 trillion you must multiply by \(10^6\). Since \(2^{10} \approx 1,000\), that’s 20 doublings. It would take 19 doublings to get to 10 trillion, and 16 to get to 1 trillion. So 12 is not enough.

\[\text{www.nytimes.com/2012/12/02/opinion/sunday/science-and-buzzwords.htm}\]
You can, of course, solve this problem the boring way with some arithmetic:

\[(20 \text{ million}) \times (1.03)^{300} \approx 140,000 \text{ million } = 140 \text{ billion}.\]

That’s at least an order of magnitude short of “several trillion”.

To get to 2 trillion you’d need 19 doublings in 300 years. That’s a doubling time of about 16 years. Then the rule of 70 says you’d have to have had an annual growth rate of about 4.3%.

(b) If the growth continues at the same rate how many words will be published in the year 3000?

Another 100 years at 4.3% would be about 6 more doublings, so each trillion words would grow to \(2^6 = 64\) trillion.

(c) How much confidence do you have in your prediction?

Not much!

**Exercise 9.6.40. [S]** World population

According to a Harvard School of Public Health press release The world’s population has grown slowly for most of human history. It took until 1800 for the population to hit 1 billion. However, in the past half-century, population jumped from 3 to 7 billion. In 2011, approximately 135 million people will be born and 57 million will die, a net increase of 78 million people.

(a) By what percent did world population increase in 2011?

(b) Write the equation for a mathematical model for world population growth for years since 2011 if the annual net increase seen in 2011 remains constant for the remainder of the century. (Use 7 billion as the 2011 population.)

(c) Write the equation for a mathematical model for world population growth for years since 2011 if the annual percentage increase seen in 2011 remains constant for the remainder of the century. (Use 7 billion as the 2011 population.)

(d) Construct an Excel spreadsheet predicting world population for the years through 2100 using both models. Graph both predictions on the same chart.

(e) The table below gives the United Nations’ high estimate for world population growth during the remainder of this century.

<table>
<thead>
<tr>
<th>Year</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>7 billion</td>
</tr>
<tr>
<td>2025</td>
<td>8.5 billion</td>
</tr>
<tr>
<td>2050</td>
<td>10.6 billion</td>
</tr>
<tr>
<td>2100</td>
<td>15.8 billion</td>
</tr>
</tbody>
</table>
Which of your models most closely matches the UN high estimate?

(a) By what percent did world population increase in 2011?
The percentage increase was
\[
\frac{78 \times 10^6}{7 \times 10^9} = 0.01114285714 \approx 1.1\%.
\]

(b) Write the equation for a mathematical model for world population growth for years since 2011 if the annual net increase seen in 2011 remains constant for the remainder of the century. (Use 7 billion as the 2011 population.)
Let \( Y \) be the number of years since 2011 and \( P \) the world population, in billions. Then the equation is
\[
P = 7 + 0.078Y.
\]

(c) Write the equation for a mathematical model for world population growth for years since 2011 if the annual percentage increase seen in 2011 remains constant for the remainder of the century. (Use 7 billion as the 2011 population.)
With the same variables as above, the exponential equation is
\[
P = 7 \times (1.011)^Y.
\]

(d) Construct an Excel spreadsheet predicting world population for the years through 2100 using both models. Graph both predictions on the same chart.

(e) The table below gives the United Nations’ high estimate for world population growth during the remainder of this century.
Which of your models most closely matches the UN high estimate?
I’ve added the predictions to the table. Numbers are in billions. The last two columns show the relative errors in the predictions.

<table>
<thead>
<tr>
<th>Year</th>
<th>UN</th>
<th>linear</th>
<th>exp</th>
<th>lin/UN</th>
<th>exp/UN</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>7.0</td>
<td>7.00</td>
<td>7.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>2025</td>
<td>8.5</td>
<td>8.19</td>
<td>8.17</td>
<td>0.96</td>
<td>0.99</td>
</tr>
<tr>
<td>2050</td>
<td>10.6</td>
<td>10.32</td>
<td>10.78</td>
<td>0.97</td>
<td>1.02</td>
</tr>
<tr>
<td>2100</td>
<td>15.8</td>
<td>14.57</td>
<td>18.77</td>
<td>0.92</td>
<td>1.19</td>
</tr>
</tbody>
</table>

The linear and exponential predictions are both pretty close for the first half of the 21st century. By 2100 the linear prediction is 8% lower than the UN’s, the exponential prediction 19% higher.
Exercise 9.6.41. [N] Making it into the Hall of Fame.

On January 12, 2013, Nate Silver wrote in his blog at *The New York Times* that

> Based on an analysis of Hall of Fame voting between 1967 and 2011, I found that the increase in a player’s vote total is typically proportional to his percentage from the previous year. In his second year on the ballot, for example, the typical player’s vote share increases by a multiple of about 1.1. Thus, a player who received 10 percent of the vote in his first year would be expected to receive about 11 percent on his second try, while a player who got 50 percent of the vote would go up to 55 percent.

Explain why this is exponential growth. Look up the original. Make some projections.

Exercise 9.6.42. [U][Section 9.2] [Goal 9.1] Computing gets cheaper

The web site [http://www.freeby50.com/2009/04/cost-of-computers-over-time.html](http://www.freeby50.com/2009/04/cost-of-computers-over-time.html) offers Figure 9.1 along with the text

> The Bureau of Labor Statistics tracks consumer price data. Computer equipment is one of the price categories that they track. As you can see the price of computers has dropped drastically over the years. From 1999 to 2003 the index dropped over 20% every year. Lately the rate of price drop has slowed but the prices for computers are still dropping. The index has decreased at a rate of 11-12% annually for the last 3 years.

![Computer Price Index](https://example.com/computer-price-index.png)

**Figure 9.1: The Cost of Computers**

(a) Reproduce this chart in Excel.

(b) Why do you think this chart was created using Excel?

(c) Check the writer’s claim about the percentage decreases.

(d) What happens when you adjust the data to take inflation into account?

Exercise 9.6.43. [N] The Richter scale

Exercise 9.6.44. [U] [N] Massive Increase in Ethanol Production

Figure 9.2 graphs the annual U.S. production of ethanol in the years since 1980. It’s from http://www.graphoftheweek.org/2012/02/massive-increase-in-ethanol-production.html where you can read this:

In 2010, the United States produced 13.2 billion gallons of ethanol. That sounds like a lot until you compare it with the amount of imported oil (180.8 billion gallons for the same year). In other words, for each gallon of ethanol produced locally, the U.S. imports nearly 14 gallons of crude oil.

Questions:
1) Will production of ethanol continue to increase exponentially?
2) By how much can we reduce our dependence on foreign oil, in practical terms?
3) How has this affected agriculture in the United States?

![Figure 9.2: Ethanol Use](http://www.graphoftheweek.org/2012/02/massive-increase-in-ethanol-production.html)

Exercise 9.6.45. [U][N] The 1% More Savings Calculator


Exercise 9.6.46. [N] Chicken or beef?

http://www.ers.usda.gov/amberwaves/april06/findings/chicken.htm

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Exercise 9.6.47. [U][N]  London ready to complete Olympic triple play

On July 22, 2012 The Boston Globe wrote about the third Olympic games to be hosted by London:

The Games have grown geometrically during the past 104 years – from 2,023 athletes representing 22 countries competing in 109 events in 24 sports in 1908 to 4,064 athletes, 59 countries, 136 events, and 19 sports in 1948 to 10,500 athletes, 204 countries, 302 events, and 37 sports in 2012. [Note: “geometric” is a synonym for “exponential”.

Possible questions: find exponential regression curves for the numbers of athletes, countries and events.

Exercise 9.6.48. [N]  Monk parakeets

Figure 9.3 shows the bird population as a function of time. (We have a better picture, the raw data, and a reference.)

Figure 9.3: Monk Parakeets
Chapter 10

Borrowing and Saving

Section 10.5, page 241

If you want to take this just a little bit further you can tell the class that the doubling time for continuous compounding is $\frac{\ln 2}{r} \approx \frac{0.6931}{r}$, hence the rule of 70.

Comments on the Exercises

Extra (ideas for) Exercises

10.6

Exercise 10.6.16. [N][U] Debit Card Trap

On August 20, 2009 The New York Times editorialized that

A study by the Center for Responsible Lending, a nonpartisan research and policy group, describes what it calls the “overdraft domino effect.” One college student whose bank records were analyzed by the center made seven small purchases including coffee and school supplies that totaled $16.55 and was hit with overdraft fees that totaled $245.

Some bankers claim the system benefits debit card users, allowing them to keep spending when they are out of money. But interest rate calculations tell a different story. Credit card companies, for example, were rightly criticized when
some drove up interest rates to 30 percent or more. According to a 2008 study by the F.D.I.C., overdraft fees for debit cards can carry an annualized interest rate that exceeds 3,500 percent.  

We haven’t made up any questions yet to go with this interesting quote.

**Exercise 10.6.17.** [S][Section 10.4] [Goal 10.1] [Goal 10.2] [Goal 10.4] Regulating the credit card industry.

_The Boston Globe_ reported on May 13, 2009 that the _Senate might consider cap on card interest rates_. There you can read

> The Senate legislation would require lenders to apply credit card payments to balances with the highest interest rates first.

and

> Senator Bernie Sanders, a Vermont independent, will offer an amendment limiting all lenders to 15 percent interest rates, he said in a written statement. That’s the same limit that applies to cards offered by credit unions, according to the statement.

Suppose a credit card user has a balance of $100 at 24% for purchases and $1000 at 0% for a debt she transferred from another credit card. She makes no new purchases, and pays off her loan at the rate of $100 per month.

(a) When will her loan be paid off and how much interest will she have paid under the 2009 rules?

(b) Under the new rules, which are now law?

(c) If Sanders’ amendment had passed? (It didn’t.)

(a) When will her loan be paid off and how much interest will she have paid under the 2009 rules?

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Under the 2009 rules it will take her 10 months to pay off the $1,000 transfer. During that time her $100 balance will have been accruing interest at the rate of 2% per month. Then she will owe $100(1.02)^{10} = $121.90. (I could have found the same answer with the spreadsheet.) Then she will pay that off in two months, so she’ll have one month’s interest on the unpaid balance of $21.90: another 0.02 \times 21.90 = 0.44. Her total interest payments will be $21.90 + 0.44 = 22.34

(b) Under the new rules she will pay off the high interest part of her bill in the first month, with no interest charge, and then the rest in 10 months, again with no interest charge.

(c) If Sanders’ amendment had passed her interest rate would be capped at 15% annually, so if she pays off the zero interest balance first her unpaid balance will become $100 \times (1 + 0.15/12)^{10} = 113.23$. The last interest charged will be just $(0.15/12) \times 13.23 = 0.17$ for a total interest payment of $113.40.

She’ll pay no interest if she pays off the purchases first.


In The Boston Globe on December 18, 2009, Candice Choi wrote about credit card reward programs:

http://www.boston.com/business/personalfinance/articles/2009/12/18/rewards_cards_may_be_a_bit_less_rewarding_after_you_consider_the_higher_fees/.

Exercise 10.6.19. [U][N] Section 10.4 [Goal 7.6] [Goal 10.4] IRS gives taxpayers more time to file, not to pay

On April 16, 2012 The Boston Globe carried an Associated Press story with that headline and this information:

If you file your tax return on time or get an extension, but fail to pay, the IRS will charge you interest on unpaid taxes. That rate currently works out to about 3.25 percent and is compounded daily.

The IRS also will charge you a late payment penalty of one-half of 1 percent of any tax not paid by April 17. That translates to a $25 penalty if you owe $5,000. It is charged each month or part of a month the tax goes unpaid, up to 25 percent, or $1,250 on that $5,000.

That interest rate can jump to 1 percent, however, if the tax bill hasn’t been paid within 10 days after the IRS issues a notice of intent to levy. But if you work out a payment plan with the IRS, it will reduce the rate to one-quarter of 1 percent.

The last thing you want to do is miss the tax-filing deadline and not ask for an extension. That triggers a penalty of 5 percent of the tax owed for each month the tax return is late, for up to five months. It’s best not to try to calculate what
you might owe in penalties and interest; the IRS will send a bill if you underpay.

Work out the total owed the government on a $5,000 tax bill under various late payment scenarios.

In each case compare the penalty and the interest. Which contributes more?
Chapter 11

Probability – Counting, Betting, Insurance

Section 11.0, page 249

Probability is hard, often counterintuitive. We deal with it in three chapters. This one is about the basic quantitative notion, focusing first on the easy cases coming from games of chance, but not spending significant time on the combinatorics. In the next chapter we take on repeated independent events, the bell curve, and rare events. In the one after that we take on conditional probability, but without formulas. Throughout the discussion we often find that there are ideas about probability that should be thought about but that don’t fit nicely into simple numerical examples, either real or imagined.

Our choice of “invented” instead of “discovered” mathematics in the chapter introduction is deliberate. You might want to discuss that philosophical question in class.

Section 11.2, page 251

Consider not even mentioning the formulas for converting from odds to probabilities and back lest the students latch on to them as more important than they are.

Comments on the Exercises
.3 Ben Bolker suggests analyzing this hiring dilemma using a payoff matrix, with utilities associated with each state (awful, ok, great). Then we could compute an expected value for each action (hire known, hire unknown) in terms of the various probability and payoff assumptions. This would be cool in Excel.

**Extra (ideas for) Exercises**

11.9


From the Atlantic:


See Exercise 5.7.16 (page 123).
Chapter 12

Break the Bank – Independent Events

Section 12.1, page 267

We’ve deliberately avoided the technical term “sample space”. We do present the idea, but don’t think combinatorial computations that require formal reasoning belong in a course at this level.

Section 12.5, page 274

Of course

\[ 0.99^{100} = 0.366032341 \approx \frac{1}{e} = 0.367879441 \]

but you don’t want to go there with this class.

Comments on the Exercises

Extra (ideas for) Exercises

12.7

Exercise 12.7.21. [N] What are the chances of six double-yolkers?

http://www.bbc.co.uk/news/magazine-16118149
Exercise 12.7.22. [N] Gladwell and success


Here’s another example. A few years ago, I criticized the following passage from Gladwell:

It’s one thing to argue that being an outsider can be strategically useful. But Andrew Carnegie went farther. He believed that poverty provided a better preparation for success than wealth did; that, at root, compensating for disadvantage was more useful, developmentally, than capitalizing on advantage.

I argued that Gladwell was making a statistical fallacy:

At some level, there’s got to be some truth to this: you learn things from the school of hard knocks that you’ll never learn in the Ivy League, and so forth. But . . . there are so many more poor people than rich people out there. Isn’t this just a story about a denominator? Here’s my hypothesis:

Pr (success — privileged background) \( \leq \) Pr (success — humble background)

\( \frac{\# \text{ people with privileged background}}{\# \text{ people with humble background}} \)

Multiply these together, and you might find that many extremely successful people have humble backgrounds, but it does not mean that being an outsider is actually an advantage.

The comments on that post are worth reading!

Exercise 12.7.23. [U][N][Section 12.1] [Goal 12.1] What’s the problem with stories and the story with probability?

This material appears in both http://www.irishtimes.com/newspaper/finance/2012/0808/1224321714580.html and http://www.ft.com/cms/s/0/bc86b5dc-dfcb-11e1-9bb7-00144feab49b.html#axzz239alNGMW

The mark of a first-rate intelligence is the ability to hold conflicting ideas in the mind at the same time and still function.

LINDA IS single, outspoken and deeply engaged with social issues. Which of the following is more likely? That Linda is a bank manager or that Linda is a bank manager who is an active feminist?

This is one of the best-known problems in behavioural economics. Many people say the second option is more likely. Yet, the standard response goes, this cannot be. The rules of probability tell us the probability that both A and B are true cannot exceed the probability that either A or B is true. It is less likely that someone is a female Jamaican Olympic gold medallist than that a person is female, or that a person is Jamaican, or that a person is a gold medallist. Yet even people trained in probability make a mistake with the Linda problem.

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Or is it a mistake? Little introspection is required to understand what is going on. Respondents do not interpret the question as one about probability. They think it is a question about believability. The description of Linda that ends with the statement Linda is a bank manager is designed to be incongruous. The addition of who is an active feminist begins to restore coherence. The story becomes more believable, even if less probable. No one gets the Jamaican problem wrong because every part of the story makes sense.

We do not often, or easily, think in terms of probabilities, because there are not many situations in which this style of thinking is useful. Probability theory is a marvellous tool for games of chance, such as spinning a roulette wheel. The structure of the problem is comprehensively defined by the rules of the game. The set of outcomes is well defined and bounded, and we will soon know which outcome has occurred.

An answer/comment at http://www.ft.com/intl/cms/s/0/7b3c497e-e21a-11e1-b3ff-00144feab49a.html#axzz239alNGMW

(“When storytelling leads to an unhappy ending”, August 8). It is that, regardless of the question actually asked, we think that the question concerns conditional probabilities.

Which is the more likely: that Linda is single and outspoken given that Linda is a bank manager; or that Linda is single and outspoken given that she is a bank manager who is an active feminist?

Here our intuition is correct: the latter probability is higher.

Robert Simons, Eudoxus Systems, Leighton Buzzard, Beds, UK

Exercise 12.7.24. [N] Mathematician plots pregnancy probability

Figure [12.1] shows ...

Exercise 12.7.25. [U][N] Is Failure to Predict a Crime?

This oped from The New York Times has an interesting discussion of the difficulty working with small probabilities for rare events.


Exercise 12.7.26. [U] Russian roulette

(a) What is Russian roulette?

(b) What is the probability of surviving 1, 2, 3, 4, 5 or 6 rounds?
Figure 12.1: Odds of conceiving . . .
Chapter 13

How Good Is That Test?

Section 13.0, page 281

This chapter focuses on two way contingency tables in order to discuss several important common logical pitfalls dealing with everyday probabilities. We think that approach makes more sense, and is easier to remember and apply, than an explicit treatment of dependent events and Bayes’ theorem. That’s too technical for our goals in this quantitative reasoning text, and so better left for a full course in probability and statistics. In fact, many of the examples in this chapter employ qualitative rather than quantitative reasoning.

You can even skip the first two sections and the vocabulary of dependent events and start with the section on screening for rare diseases.

If you want to go further into the analysis of dependence (perhaps leading to Bayes’ theorem) consider two way tables as the entry point. Independence corresponds to tables whose rows (and hence columns) are proportional. Those are the only ones that can be modeled using areas of parts of a square, as in the last chapter.

Causation corresponds to tables with a 0 in one quadrant.

Comments on the Exercises

Extra (ideas for) Exercises

13.7


Build the two way contingency tables based on the data there, and discuss the consequences of the data.

Exercise 13.7.17. [U][C][N] Missile defense.

Theodore A. Postel wrote in The Boston Globe on April 15, 2008 that

THE HOUSE Subcommittee on National Security and Foreign Affairs will hold a long-overdue oversight hearing tomorrow on the prospects for national missile defense. The most basic question that needs to be addressed is the inability of the national missile defense to tell the difference between simple warheads and decoys.

... The issue of the effectiveness of decoys against the missile defense is easy to understand. The national missile defense is designed to destroy warheads by hitting them with infrared homing Kill Vehicles while the warheads are in the near vacuum of space. Since there is no air-drag in space, a warhead weighing thousands of pounds and a balloon weighing almost nothing will travel together. Warheads could be placed inside balloons, and many balloons could be deployed along with the warheads. ... Since there would be no way for the Kill Vehicle to know which balloons contain warheads, the chances of actually hitting a warhead would be minuscule.  

Exercise 13.7.18. [N] Asian carp eDNA testing can lead to false positives, study finds

The technical team developed a strategy to test for false positives. Water samples were collected in April from a metro lake that served as a negative control (very little chance Asian carp could be present). Twenty samples were sent to the Corps of Engineers laboratory in Vicksburg and 20 were sent to the private contractor that did the 2011 analysis. All of the samples from the Corps of Engineers lab tested negative, while one sample from the private contractor tested positive for silver carp. This sample was tested again and the positive was verified.

http://www.boston.com/bostonglobe/editorial_opinion/oped/articles/2008/04/15/troubling_questions_about_missile_defense/
There is a high likelihood this is a false positive which creates uncertainty about previous results. The percentage of positives in the 2012 samples was much lower than previous samples suggesting there may have been a mix of real and false positive samples in 2011. This does not minimize eDNA testing as an important tool for detecting Asian carp, but it does emphasize the need to determine the source of false positives and to review and modify sampling and analytical procedures. In addition, we have collected live Asian carp from the St. Croix and Mississippi Rivers which are definitive evidence these fish are present and pose a threat to Minnesota.

*The posting has no numbers, so it’s hard to write questions.*

**Exercise 13.7.19.** [N] *Shaky Foundations for the New Mammogram Economy*

From Bloomberg news, on August 1, 2012:

Part of the problem is that, on a mammogram, a noncancerous abnormality can look very much like cancer. . . . This causes 10 percent to 15 percent of screened women in the U.S. to be recalled for more evaluation. Most (95 percent) screening-detected abnormalities are ultimately found to be noncancerous. An American woman who is regularly screened during her 40s has a 61 percent chance of getting a false positive result.

. . .

Now, for every $100 spent on screening, an additional $30 to $33 is spent to evaluate false positive findings. In the Medicare population, the workup of false positive mammogram results is estimated to total $250 million a year.

Also see the report *High Rate of False-Positives with Annual Mammogram* from UCSF:

For the false-positive study, the researchers found that after a decade of annual screening, a majority of women will receive at least one false-positive result, and 7 to 9 percent will receive a false-positive biopsy recommendation.

**Exercise 13.7.20.** [U] *Testing for prostate cancer*

Figure [13.1] from the Department of Family Medicine at Virginia Commonwealth University illustrates the possible outcomes of a PSA screening test for prostate cancer.

This quotation spells out some of the arguments for and against the test:

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There are possible advantages to having a PSA test.

1. A normal PSA test may reassure you.
2. A PSA test may find prostate cancer early before it has spread.
3. Treatment of prostate cancer in early stages may help some men to avoid problems from cancer.
4. Treatment of prostate cancer in early stages may help some men live longer.

There are possible disadvantages to having a PSA test.

1. A normal PSA test may miss some prostate cancers.
2. A false positive PSA test may cause unnecessary anxiety.
3. A false positive PSA test may cause an unneeded prostate biopsy.
4. You may find out that you have prostate cancer, but it may be a cancer that would never cause you any problems.
5. Treatment of prostate cancer may cause you harm. Difficulties with getting erections or problems with controlling your bladder or bowels are some potential harms.

(a) What does “PSA” stand for?
(b) Construct the two way contingency table based on this data.
(c) If your screening test is positive, what is the probability that you have prostate cancer?
(d) If you have prostate cancer, what is the probability that this screening test will detect it?
The figure shows that the overall incidence of prostate cancer is 10%. That is probably an invented statistic, to make the arguments easier to understand. The reality is complex. Here’s a start on it, from the American Cancer Society:

What are the key statistics about prostate cancer?

Other than skin cancer, prostate cancer is the most common cancer in American men. The latest American Cancer Society estimates for prostate cancer in the United States are for 2012:

- About 241,740 new cases of prostate cancer will be diagnosed
- About 28,170 men will die of prostate cancer
- About 1 man in 6 will be diagnosed with prostate cancer during his lifetime.
- Prostate cancer occurs mainly in older men. Nearly two thirds are diagnosed in men aged 65 or older, and it is rare before age 40. The average age at the time of diagnosis is about 67.
- Prostate cancer is the second leading cause of cancer death in American men, behind only lung cancer. About 1 man in 36 will die of prostate cancer.
- Prostate cancer can be a serious disease, but most men diagnosed with prostate cancer do not die from it. In fact, more than 2.5 million men in the United States who have been diagnosed with prostate cancer at some point are still alive today.

http://www.cancer.org/Cancer/ProstateCancer/DetailedGuide/prostate-cancer-key-statistics
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