1. Give DFAs that recognize the following languages

(a) \( \{ w \in \{0, 1\}^* \mid \text{every 0 in } w \text{ is immediately followed by a 1} \} \).

Solution:

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(b) \( \{ w \in \{0, 1\}^* \mid w \text{ contains at most two 1's} \} \).

Solution:

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(c) \{11, 110\}^*.

Solution:

\[
\begin{array}{c}
\text{0} \\
1 \\
2 \\
3 \\
4 \\
\end{array}
\]

\[
\begin{array}{c}
1 \\
1 \\
0 \\
0, 1 \\
0, 1 \\
\end{array}
\]

2. Using the complementation construction and one of the DFAs from Exercise 1, give a DFA that recognizes the language

\[ \{ w \in \{0, 1\}^* \mid \text{there is a 0 in } w \text{ that is not immediately followed by a 1}\}. \]

Solution:

\[
\begin{array}{c}
\text{1} \\
\text{2} \\
\text{3} \\
\end{array}
\]

\[
\begin{array}{c}
0, 1 \\
0 \\
1 \\
\end{array}
\]

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3. Using the union construction and two of the DFAs from Exercise 1, give a DFA that recognizes the language

\[ \{ w \in \{0,1\}^* | w \text{ contains at most two 1's or every 0 in } w \text{ is immediately followed by a 1} \} \]

Solution:
4. Using the intersection construction and two of the DFAs from Exercise 1, give a DFA that recognizes the language

\[ \{ w \in \{0, 1\}^* | w \text{ contains at most two 1's and every 0 in } w \text{ is immediately followed by a 1} \}. \]

Solution:

\[ (1, 4) \]

\[ (1, 5) \]

\[ (2, 5) \]

\[ (1, 6) \]

\[ (2, 6) \]

\[ (1, 7) \]

\[ (2, 7) \]

\[ (1, 4) \rightarrow (2, 4) \]

\[ (1, 5) \rightarrow (2, 5) \rightarrow (3, 5) \rightarrow (3, 6) \rightarrow (3, 7) \]

\[ (1, 6) \rightarrow (2, 6) \rightarrow (3, 6) \rightarrow (3, 7) \]

\[ (1, 7) \rightarrow (2, 7) \rightarrow (3, 7) \]

\[ (1, 4) \rightarrow (1, 5) \rightarrow 0 \rightarrow (1, 6) \rightarrow 1 \rightarrow (1, 7) \rightarrow 1 \rightarrow (2, 7) \rightarrow 0 \rightarrow (2, 7) \rightarrow 0, 1 \]

\[ (2, 4) \rightarrow 0 \rightarrow (2, 5) \rightarrow 0 \rightarrow (3, 5) \rightarrow 0 \rightarrow (3, 5) \rightarrow 0 \rightarrow (3, 6) \rightarrow 0 \rightarrow (3, 6) \rightarrow 0 \rightarrow (3, 7) \rightarrow 0, 1 \]