Due: March 2

1. Give context-free grammars for the following languages:
   (a) \{a^n b^2 c^m d^3 m | n, m \geq 0\}.
   (b) \{a^n b^m c^n | n, m \geq 0\}.
   (c) \{a^n b^m c^m d^n | n, m \geq 0\}.
   (d) \{x_1 \# x_2 \# x_3 | x_1, x_2, x_3 \in \{a, b\}^* \text{ and either } x_1 = x_2^R \text{ or } x_1 = x_3^R\}.

2. Give right linear grammars for the following languages:
   (a) \{w \in \{0, 1\}^* | \text{the first and last symbols of } w \text{ are different}\}.
   (b) \{0^n 1^m | n, m \text{ are both even}\}.
   (c) \{w \in \{0, 1\}^* | w \text{ contains an odd number of } 0\text{s and exactly one } 1\}.

3. Using the method from class, convert the DFA given in the solutions to Problem 2 on Homework 1 into a right linear grammar.

4. Using the method from class, transform the following right linear grammar into an NFA
   \[
   S \rightarrow 0T | 1V \\
   T \rightarrow 2T | 2 \\
   V \rightarrow 2V | \epsilon
   \]

5. Let \(L\) be the language of all strings in \(\{a, b\}^*\) that have the same number of \(a\)'s as \(b\)'s. In class, we looked at the following grammar that generates the language \(L\).
   \[
   S \rightarrow aB | bA | \epsilon \\
   A \rightarrow aS | bAA \\
   B \rightarrow bS | aBB
   \]
   Prove that this grammar is ambiguous.

6. Give an unambiguous grammar for the language \(L\) of the previous problem.
   (This is a difficult problem, but give it a try. As a hint, you can use two additional variables other than the start symbol. One variable generates all strings in \(L\) that have the additional property that every prefix has at least as many \(a\)'s as \(b\)'s, and the other variable generates all strings in \(L\) that have the additional property that every prefix has at least as many \(b\)'s as \(a\)'s.)