Name: ________________________________

Put all your answers on the test itself. Be sure to put your name above.

1. Let $M_1$ be the Turing machine of Figure 3.10 on page 173 of the text. For each of the following configurations of $M_1$, show the configuration that the given configuration yields, i.e., the next configuration after the given one. (Do not try to trace the entire computation starting from the given configuration. You are only being asked for the next configuration.)

(a) $q_1011\#011$
   Next configuration: $xq_211\#011$

(b) $x01\#q_3111$
   Next configuration: $x01q_6\#x11$

(c) $xx\#xxq_8$
   Next configuration: $xx\#xx \downarrow q_{accept}$

(d) $xx0q_3\#x01$
   Next configuration: $xx0\#q_5x01$

[24 points]
2. Apply the method from class that decides $E_{DFA}$ to the following DFA and answer the questions below.

(a) List the states you mark in the order they get marked.
$p, q, r, t, u$

(b) Does the DFA belong to $E_{DFA}$? No

(c) How does your answer to (b) follow from your answer to (a)?
An accept state (namely $u$) is marked.

[22 points]
3. Is it possible to \( m \)-reduce \( A_{TM} \) to \( \overline{HALT_{TM}} \)?

Yes ______
No ×

Explain your answer below. (You may use results proven in the book, in class, on the homework, and in the homework solutions without reproving them.)

**Solution:** Suppose that \( A_{TM} \leq_m \overline{HALT_{TM}} \). Then, as shown in class, we have \( \overline{A_{TM}} \leq_m HALT_{TM} \). By Corollary 4.23, we know that \( \overline{A_{TM}} \) is not Turing recognizable, so by Corollary 5.29, we obtain that \( HALT_{TM} \) is not Turing recognizable. This contradicts Problem 2 on Homework 10. Thus, the \( m \)-reduction is not possible.

[18 points]
4. Let $B = \{ \langle M \rangle | M \text{ is Turing machine and } L(M) = 0^*1^* \}$. (In other words, \langle M \rangle belongs to B if the language recognized by M is $0^*1^*$. ) Suppose that you want to show that $HALT_{TM} \leq_m B$ using a reduction $f$ that maps $\langle M, w \rangle$ to $\langle M_1 \rangle$.

(a) Fill in the blanks in the following three statements in a way that states what you have to do to make the reduction work. (In all cases you will be writing down something about the behavior of the Turing machine $M_1$.)

- If $M$ accepts $w$, then $L(M_1) = 0^*1^*$.
- If $M$ rejects $w$, then $L(M_1) = 0^*1^*$.
- If $M$ loops on $w$, then $L(M_1) \neq 0^*1^*$.

(b) Give the definition of the desired Turing machine $M_1$, given $M$ and $w$.

$M_1 = \text{“On input } x$

1. If $x \notin 0^*1^*$, reject.
2. If $x \in 0^*1^*$, run $M$ on $w$.
3. If $M$ halts, accept.”

[16 points]
5. Suppose that $A$ and $B$ are sets with $B$ countably infinite and that $f : A \to B$. Prove that the range of $f$ is countable. (The range of $f$ is $\{f(a) | a \in A\}$.)

Solution: Since $B$ is countably infinite, its elements can be listed without repetition as $b_1, b_2, b_3, \ldots$. The range of $f$ is a subset of $B$. If we take $b_1, b_2, \ldots$ and remove from the list those elements of $B$ that are not in the range of $f$, we either get a finite list $c_1, \ldots, c_n$ for some $n$, of the elements of the range of $f$, or else an infinite list $c_1, c_2, \ldots$ of the elements of the range of $f$. In the first case, the range of $f$ is finite and in the second case, the range of $f$ is countably infinite. In either case, the range of $f$ is countable.

[10 points]
6. Let \( AMBIG_{CFG} = \{ \langle G \rangle | G \text{ is a CFG and } G \text{ is ambiguous} \} \). Show that \( AMBIG_{CFG} \) is Turing recognizable.

**Solution:** \( AMBIG_{CFG} \) is recognized by the following Turing machine \( T \).

\( T = \) “On input \( \langle G \rangle \) where \( G \) is a context-free grammar

1. Generate all possible leftmost derivations in \( G \).
2. Each time a leftmost derivation produces a string of terminals \( w \), add \( w \) to a list and check if \( w \) is already on the list. If \( w \) is already on the list, accept, else continue generating leftmost derivations of \( G \).”

[10 points]