CS 720 Fall 2016
Take-Home Final

Due: December 21, 5 PM
Do not discuss this test with anyone else except me.

Please sign the statement below and include this sheet when you hand in the test.

I have not given or received any help on this test which is not allowed. I have not discussed this test with anyone else.

Signature: ____

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1. Let TS_1 and TS_3 be the transition systems given on page 589 of the text. Explain directly from the definition of bisimulation equivalence why TS_1 and TS_3 are not bisimulation equivalent.
   [Hint: If there is a bisimulation between the two transition systems, then the initial state s_1 of TS_1 has to be related to some initial state of TS_3. Consider each possibility in turn and rule it out based on the definition of bisimulation equivalence.] [20 points]

2. In the LTL model checking algorithm of Theorem 5.37, we assumed that the LTL formula used only the temporal operators $\bigcirc$ and U. Other operators such as $\Diamond$, $\Box$ and W can be treated by replacing them with equivalent formulas that use only $\bigcirc$ and U. If is more efficient however to treat these operators as basic operators and allow $\Diamond\varphi$, $\Box\varphi$ and $\varphi W \psi$ as elements of elementary sets of formulas and redefine the GNBA $\mathcal{G}_\varphi$ to take account of these operators.

   Explain how this is done by answering the following questions.

   (a) How is the definition of elementary set changed, if it all, if $\Diamond$ is allowed in the definition of LTL formula?

   (b) How is the definition of the transition relation of the GNBA $\mathcal{G}_\varphi$ changed if $\Diamond$ is allowed in the definition of LTL formula?

   (c) How is the definition of the acceptance conditions $\mathcal{F}$ of the GNBA $\mathcal{G}_\varphi$ changed, if at all, if $\Diamond$ is allowed in the definition of LTL formula?

   (d) Answer the same three questions for the operator $\Box$.

   (e) Answer the same three questions for the operator W.

   [30 points]

3. Give an algorithm "Computation of $Sat_{wfair}(\exists\Box a)$" meeting this specification:

   Input: A finite transition system TS with no terminal states, $a \in AP$, $fair = \bigwedge_{1 \le i \le k} wfair_i$, with $wfair_i = \Diamond\Box a_i \rightarrow \Box\Diamond b_i$.
   Output: $\{s \in S | s \models_{fair} \exists\Box a\}$
   [Hint: You can take Algorithm 18 as a starting point, but the algorithm you have to produce is much simpler. In particular, the complicated recursive CheckFair can be made non recursive, and the code for CheckFair could even be incorporated into the main algorithm instead of making it a separate procedure.] [10 points]

4. On Homework 7, you considered the temporal operator L for LTL. Similarly, in CTL, for two state formulas Φ and Ψ, we can define the path

formula $\Phi L \Psi$ where for a path $\pi = s_0 s_1 \cdots$, we have $\pi \models \Phi L \Psi$ if and only if for every $i \geq 0$, the following condition holds:

$$\text{If } s_0 \models \Psi, s_1 \models \Psi, \ldots, s_i \models \Psi, \text{ then } s_i \models \Phi.$$

We also define a temporal operator M by $\Phi M \Psi \equiv \Phi L \Psi \wedge \Diamond \neg \Psi$.

Then both $\kappa = \exists(\Phi L \Psi)$ and $\kappa = \exists(\Phi M \Psi)$ satisfy the expansion rule

$$\kappa \equiv \neg \Psi \vee (\Phi \wedge \Psi \wedge \exists \bigcirc \kappa).$$

(a) Let TS be a finite transition system without terminal states. Write down a monotone function $F : P(S) \to P(S)$ such that $Sat(\exists(\Phi L \Psi))$ and $Sat(\exists(\Phi M \Psi))$ are both fixed points of F.
(Assume that you have already calculated $Sat(\Phi)$ and $Sat(\Psi)$ and can use them in the definition of F. Your function F will be a direct translation of the equivalence of formulas given above.)

(b) Which of $Sat(\exists(\Phi L \Psi))$ and $Sat(\exists(\Phi M \Psi))$ is the least fixed point of F and which is the greatest fixed point? Prove your answers.
[You can express L and M in terms of operators we have discussed in class, and this can help you figure out which is the least and which is the greatest fixed point, but your proofs have to be based only on the definitions of L and M and not use any equivalent formulations of these operators.]

(c) As discussed in class, when you have least and greatest fixed points of a monotone function, you automatically get pseudocode to compute the fixed points. Write down the pseudocode you get to compute $Sat(\exists(\Phi L \Psi))$ and $Sat(\exists(\Phi M \Psi))$. Use the actual definition of F in your pseudocode instead of just writing $F(X)$.

(d) Write more explicit pseudocode for computing the sets $Sat(\exists(\Phi L \Psi))$ and $Sat(\exists(\Phi M \Psi))$ in the style of Algorithms 15 and 16. Try to write elegant code and to have a linear running time. (You can get partial credit for a quadratic running time.)

[30 points]

5. Slide 58 of the lecture notes for October 31 states that if E is a regular safety property and $\mathcal{A}$ is a non-blocking NFA with $\mathcal{L}(\mathcal{A})$ equal to the set of bad prefixes of E, then $\mathcal{L}_\omega(\mathcal{A}) = \overline{E} = (2^{AP})^\omega \setminus E$. This claim is true if $\mathcal{A}$ is a DFA or if each accept state of $\mathcal{A}$ has a self-loop labeled *true*, but if neither of these conditions is met, the claim does not always hold. The purpose of this problem is for you to disprove the claim.

   Let $AP = \{A, B\}$, and let $E = \{A^\omega\}$. Then E is a regular safety property. Give a non-blocking NFA $\mathcal{A}$ such that $\mathcal{L}(\mathcal{A})$ is the set of bad prefixes of E, but $\mathcal{L}_\omega(\mathcal{A})$ is $\emptyset$ (so $\mathcal{L}_\omega(\mathcal{A})$ is not $\overline{E}$).
   [Of course, your $\mathcal{A}$ cannot be deterministic, and it cannot have self-loops labeled *true* on each accept state.] [10 points]