Overview

Introduction

Modelling parallel systems

- Transition systems ⟵
- Modeling hard- and software systems
- Parallelism and communication

Linear Time Properties

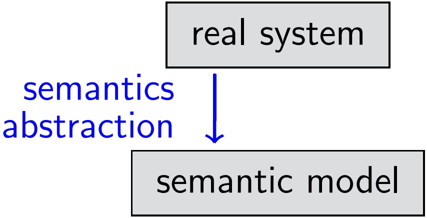
Regular Properties

Linear Temporal Logic

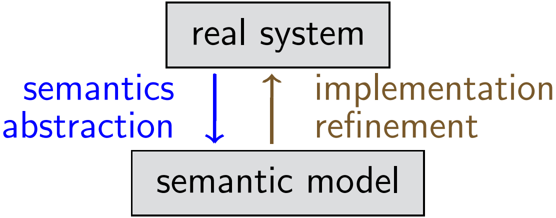
Computation-Tree Logic

Equivalences and Abstraction

Transition systems



Transition systems



Transition systems

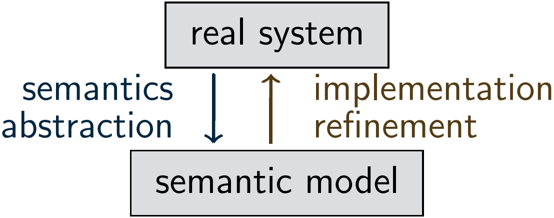
real system

semantics ↓ ↑ implementation
abstraction ↓ ↑ refinement

semantic model

The semantic model yields a formal representation of:

Transition systems



The semantic model yields a formal representation of:

- the states of the system
- the stepwise behaviour
- the initial states

Transition systems

real system

semantics ↓ ↑ implementation

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semantic model

The semantic model yields a formal representation of:

- the states of the system

control component + information on "relevant" data

- the stepwise behaviour
- the initial states

Transition systems $\widehat{=}$ extended digraphs

real system

semantics, abstraction $\downarrow$ $\uparrow$ implementation, refinement

semantic model

The semantic model yields a formal representation of:

- the states of the system $\longleftarrow$ **nodes**

$\boxed{\text{control component} + \text{information on “relevant” data}}$

- the stepwise behaviour $\longleftarrow$ **edges**
- the initial states

Transition systems $\widehat{=}$ extended digraphs

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$\uparrow$

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- the stepwise behaviour $\longleftarrow$ **transitions**
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Transition systems $\widehat{=}$ extended digraphs

real system

semantics / abstraction $\downarrow$ $\uparrow$ implementation / refinement

semantic model

The semantic model yields a formal representation of:

- the states of the system $\longleftarrow$ **nodes**
- the stepwise behaviour $\longleftarrow$ **transitions**
- the initial states
- additional information on
  - communication
  - state properties

Transition systems $\widehat{=}$ extended digraphs

$$\begin{tikzcd} \boxed{\text{real system}} \arrow[d, shift right=2, "\substack{\text{semantics}\\\text{abstraction}}"'] \\ \boxed{\text{semantic model}} \arrow[u, shift right=2, "\substack{\text{implementation}\\\text{refinement}}"'] \end{tikzcd}$$

The semantic model yields a formal representation of:

- the states of the system $\longleftarrow$ **nodes**
- the stepwise behaviour $\longleftarrow$ **transitions**
- the initial states
- additional information on
  communication $\longleftarrow$ **actions**
  state properties $\longleftarrow$ **atomic proposition**

Transition system (TS)

A transition system is a tuple

$$\mathcal{T} = (S, Act, \longrightarrow, S_0, AP, L)$$

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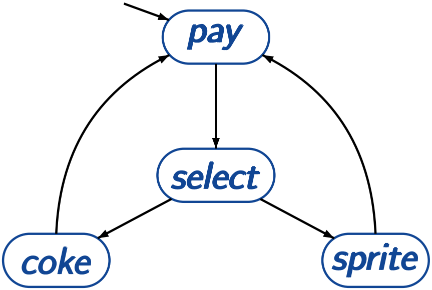
- $S_0 \subseteq S$ the set of initial states,
- AP a set of atomic propositions,
- $L : S \to 2^{AP}$ the labeling function

Transition system for beverage machine



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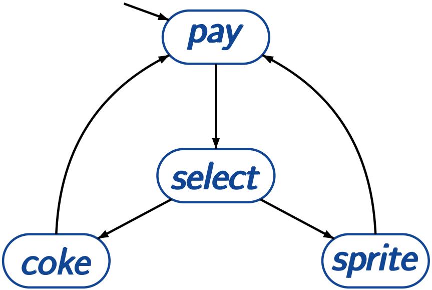
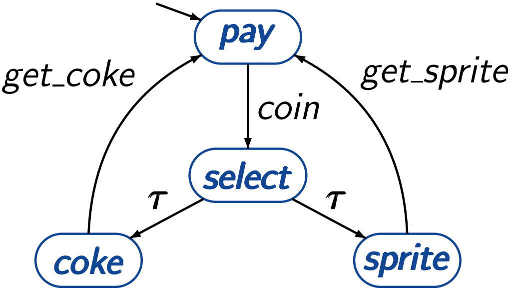


state space $S = \{pay, select, coke, sprite\}$

set of initial states: $S_0 = \{pay\}$

Transition system for beverage machine



actions:
coin
τ
get_sprite
get_coke

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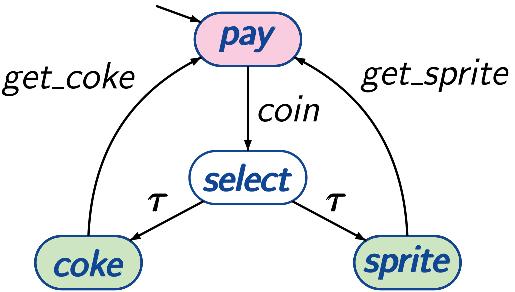
Transition system for beverage machine



actions:
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get_coke

state space $S = \{pay, select, coke, sprite\}$

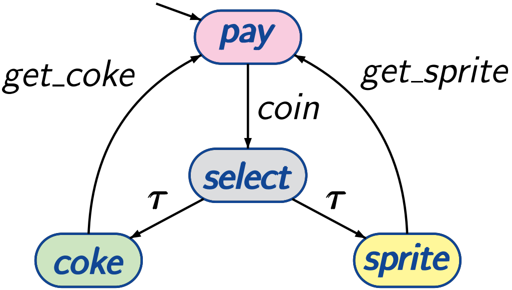
set of initial states: $S_0 = \{pay\}$

set of atomic propositions: $AP = \{pay, drink\}$

labeling function: $L(coke) = L(sprite) = \{drink\}$

$$L(pay) = \{pay\},\ L(select) = \emptyset$$

Transition system for beverage machine



actions:
coin
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get_sprite
get_coke

state space $S = \{pay, select, coke, sprite\}$

set of initial states: $S_0 = \{pay\}$

set of atomic propositions: $AP = S$

labeling function: $L(s) = \{s\}$ for each state s

"Behaviour" of transition systems

possible behaviours of a TS result from:

select nondeterministically an initial state $s \in S_0$
`WHILE` s is non-terminal `DO`
    select nondeterministically a transition $s \xrightarrow{\alpha} s'$
    execute the action α and put $s := s'$
`OD`

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executions: maximal "transition sequences"

$$s_0 \xrightarrow{\alpha_1} s_1 \xrightarrow{\alpha_2} s_2 \xrightarrow{\alpha_3} \ldots \text{ with } s_0 \in S_0$$

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reachable fragment:

$Reach(\mathcal{T})$ = set of all states that are reachable from an initial state through some execution

Possible meanings of nondeterminism in TS

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- (true) concurrency modeled by interleaving
- competition of parallel dependent actions
- implementational freedom, underspecification
- incomplete information on system environment

Transition system for parallel actions

parallel execution of independent actions

parallel execution of dependent actions

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parallel execution of independent actions

e.g. $\underbrace{x := x+1}_{\text{action } \alpha} \;|||\; \underbrace{y := y-3}_{\text{action } \beta}$ α, β independent

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parallel execution of dependent actions

e.g. $\underbrace{x := x+1}_{\text{action } \alpha} \ ||| \ \underbrace{y := 2*x}_{\text{action } \beta}$ $\quad \alpha, \beta$ dependent

Transition system for parallel actions

parallel execution of independent actions $\longleftarrow$ $\boxed{\text{interleaving}}$

e.g. $\underbrace{x := x+1}_{\text{action } \alpha} \; ||| \; \underbrace{y := y-3}_{\text{action } \beta}$ α, β independent

parallel execution of dependent actions $\longleftarrow$ $\boxed{\text{competition}}$

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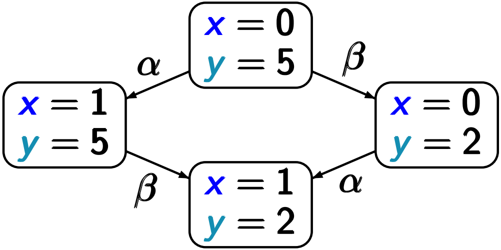

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$$
\begin{tikzcd}
 & \boxed{\begin{array}{l} x = 0 \\ y = 5 \end{array}} \arrow[dl, "\alpha"'] \arrow[dr, "\beta"] & \\
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Possible meanings of nondeterminism in TS

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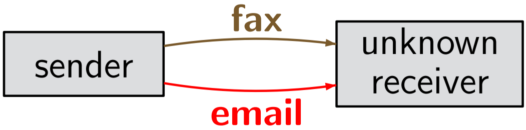
Implementation freedom

… modelled by nondeterminism

Implementation freedom



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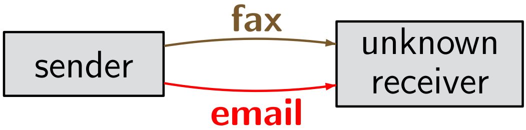
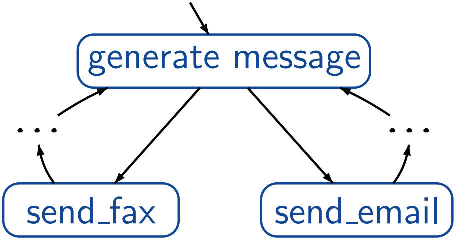


realization by a TS:



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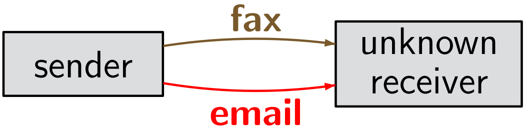

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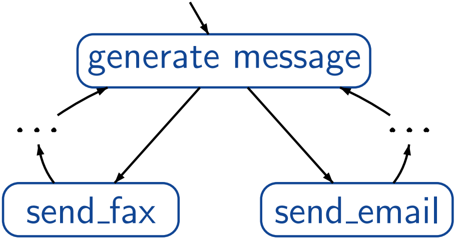
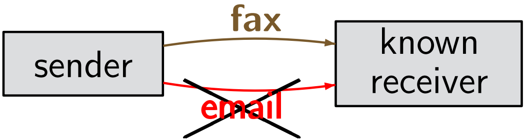


at a future refinement step the nondeterminism is replaced with one of the alternatives

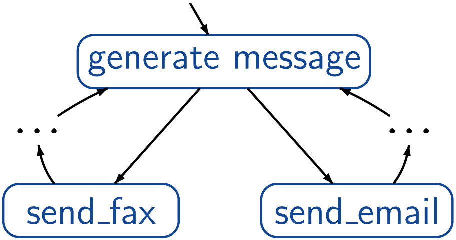
Implementation freedom

TS1.4-5



without
email access

realization by a TS:



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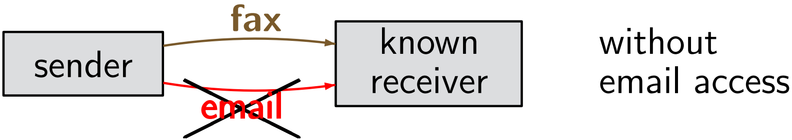
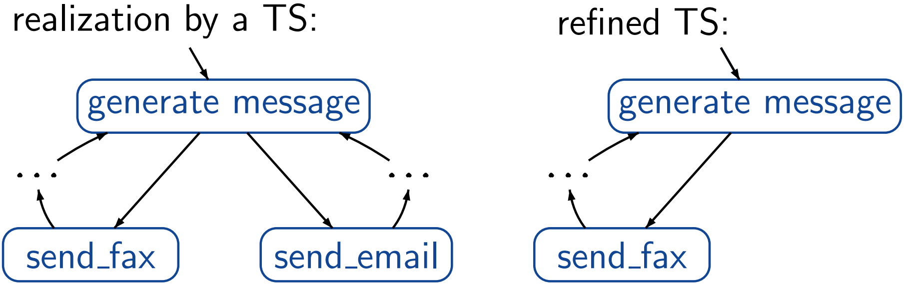
Implementation freedom





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Underspecification

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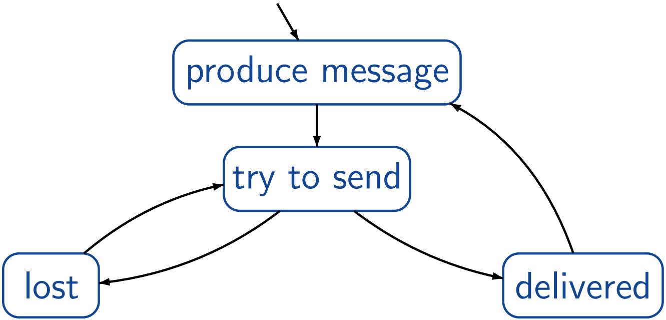
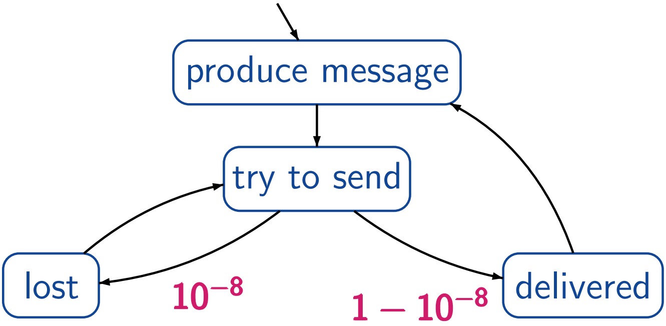


Underspecification



at a future refinement step the nondeterminism is replaced with probabilism

Possible meanings of nondeterminism in TS

- (true) concurrency modeled by interleaving
- competition of parallel dependent actions
- implementational freedom, underspecification
- incomplete information on system environment

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- (true) concurrency modeled by interleaving
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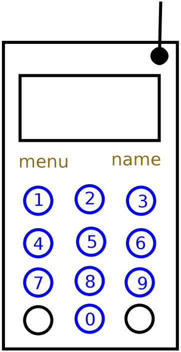
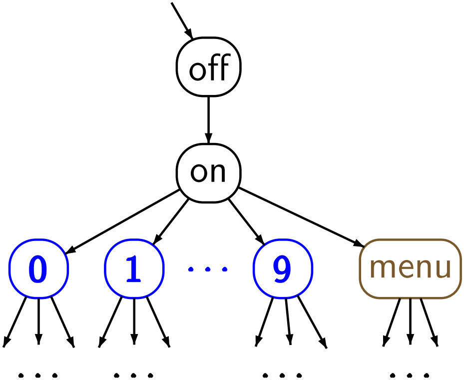
- incomplete information on system environment, e.g., interfaces with other programs, human users, sensors

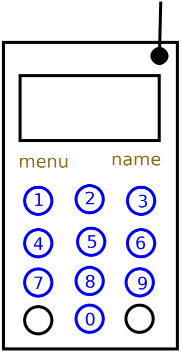
Incomplete information on the environment



mobile phone



Incomplete information on the environment



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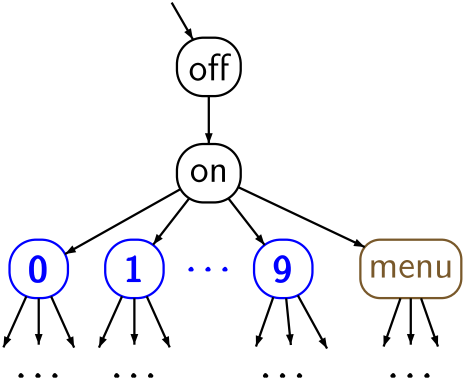
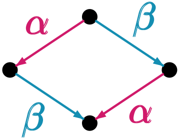


resolution of the nondeterministic choices
by a human user

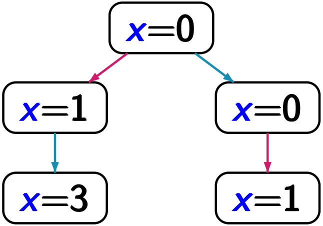
Possible meanings of nondeterminism in TS

concurrency (interleaving)

$\alpha \,|||\, \beta$ is represented by



competitions



to be resolved by a scheduler
e.g. $x{:=}x{+}1 \,\|\, x{:=}3x$

underspecification, implementational freedom

incomplete information on system environment, e.g., interfaces with other programs, human users, sensors

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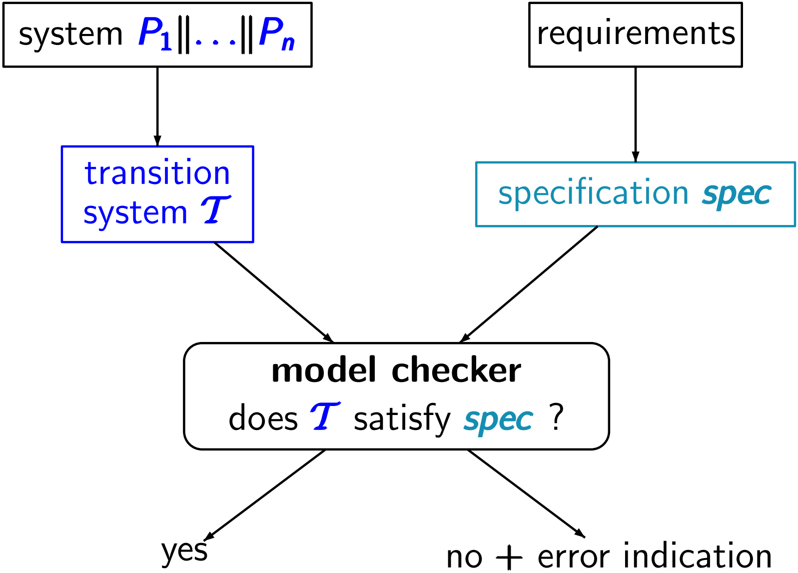
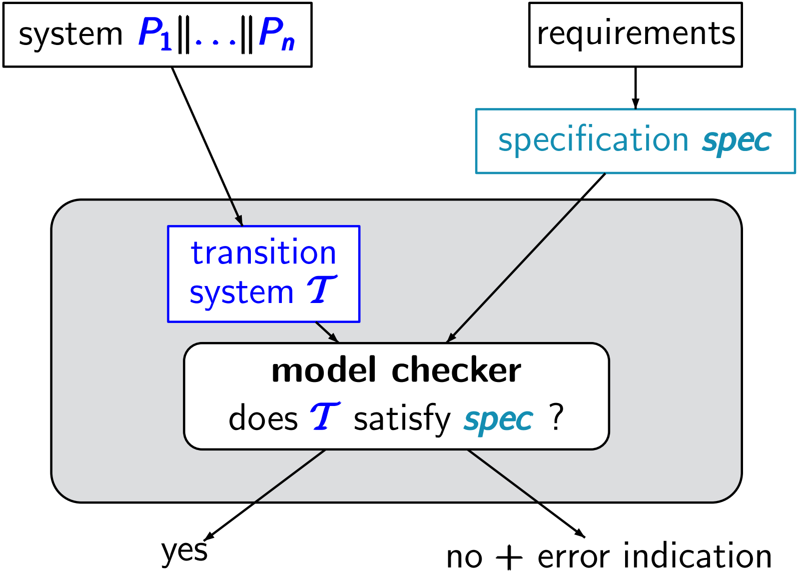
Model checking



Model checking



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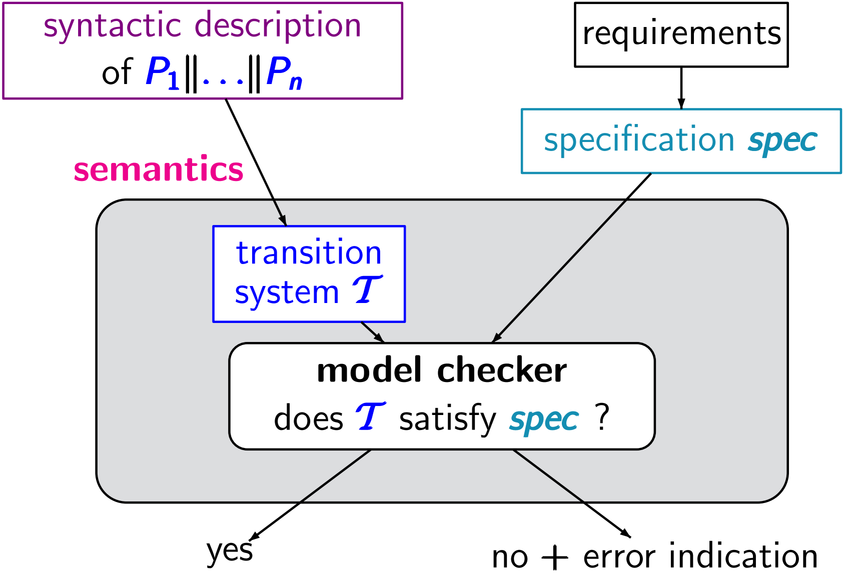
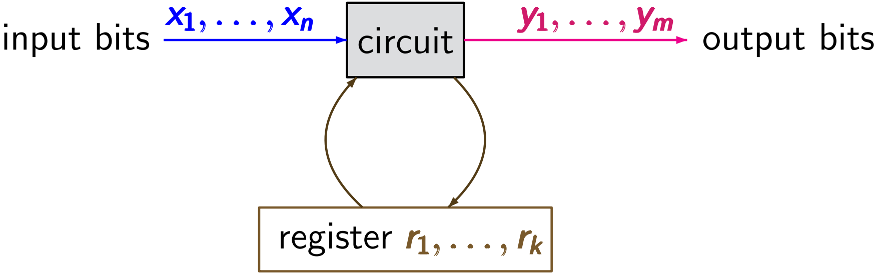


Modelling of sequential circuits by TS



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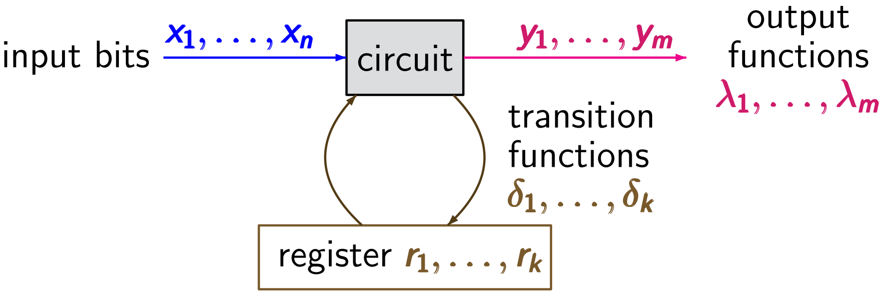

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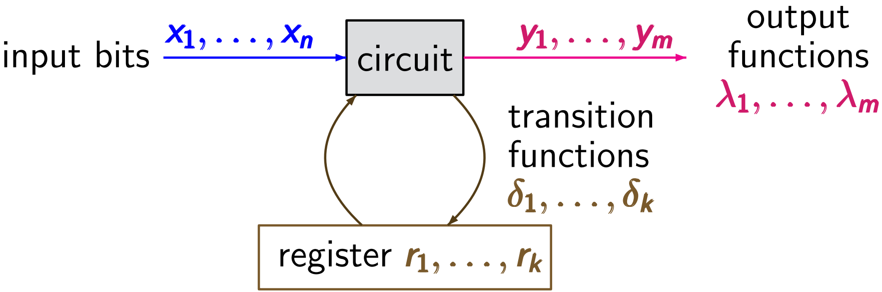
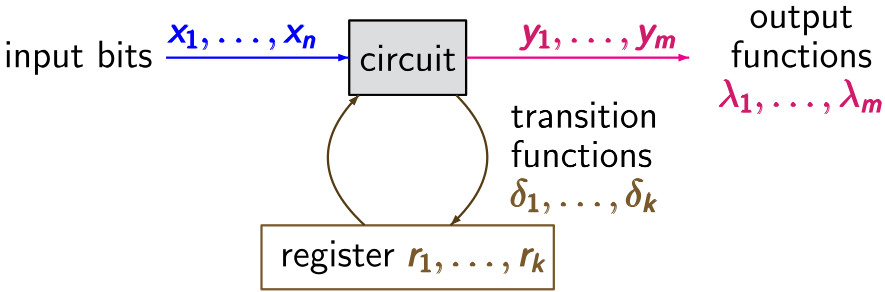


$\delta_j, \lambda_i \ \widehat{=}$ switching functions $\{0,1\}^n \times \{0,1\}^k \longrightarrow \{0,1\}$

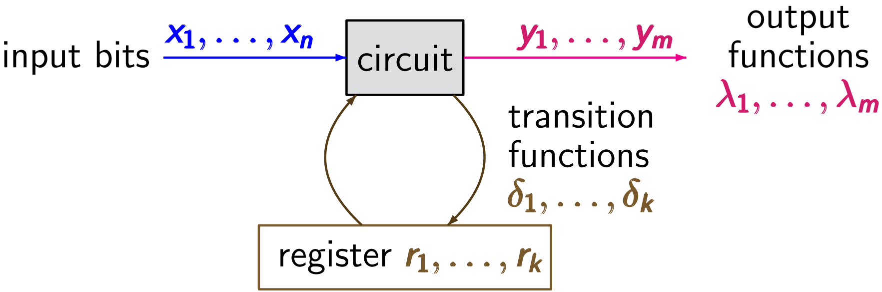
Modelling of sequential circuits by TS



$$\delta_j, \lambda_i \mathrel{\widehat{=}} \text{switching functions } \{0,1\}^n \times \{0,1\}^k \longrightarrow \{0,1\}$$

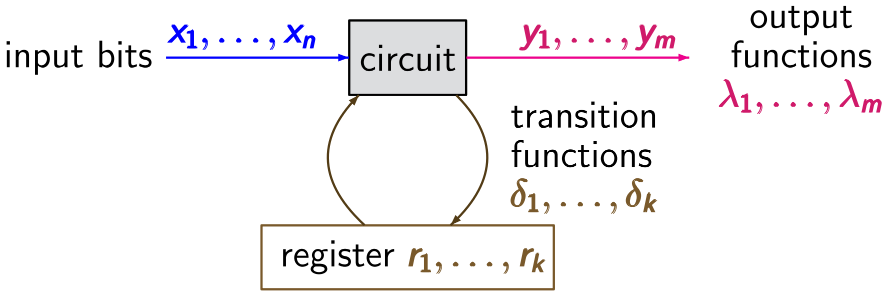
input values $a_1, \ldots, a_n$ for the input variables		output value $\lambda_i(\ldots)$ for output variable y_i
	$\mapsto$	
$+$ current values $c_1, \ldots, c_k$ of the registers		next value $\delta_j(\ldots)$ for register r_j

Modelling of sequential circuits by TS



initial register evaluation $[r_1{=}c_{01}, \ldots, r_k{=}c_{0k}]$

Modelling of sequential circuits by TS



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transition system:

- states: evaluations of $x_1, \ldots, x_n, r_1, \ldots, r_k$

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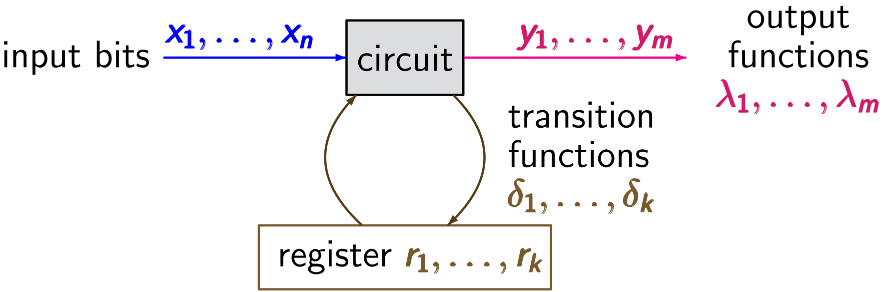

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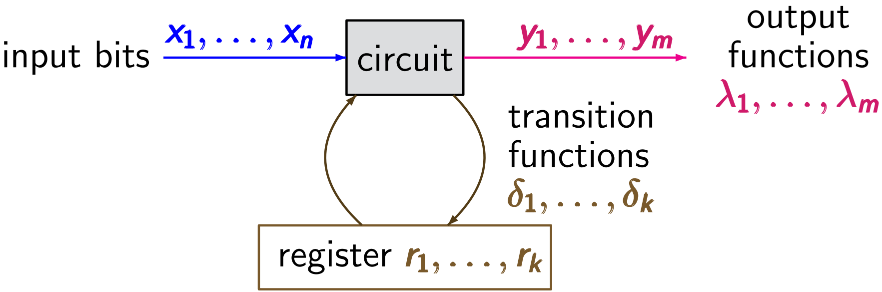

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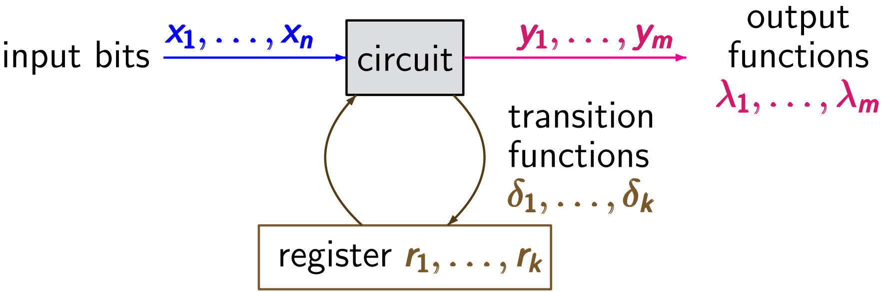
transition system:

- states: evaluations of $x_1, \ldots, x_n, r_1, \ldots, r_k$
- transitions represent the stepwise behavior
- values of input bits change nondeterministically

Modelling of sequential circuits by TS

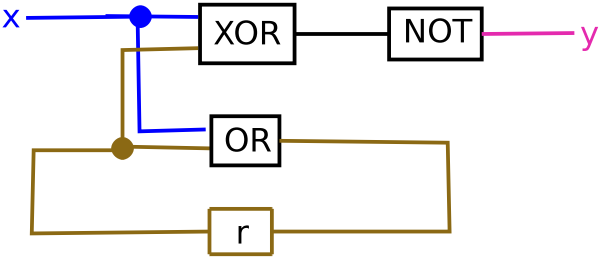


initial register evaluation $[r_1{=}c_{01}, \ldots, r_k{=}c_{0k}]$

transition system:

- states: evaluations of $x_1, \ldots, x_n, r_1, \ldots, r_k$
- transitions represent the stepwise behavior
- values of input bits change nondeterministically
- atomic propositions: $x_1, \ldots, x_n, y_1, \ldots, y_m, r_1, \ldots, r_k$

Example: sequential circuit



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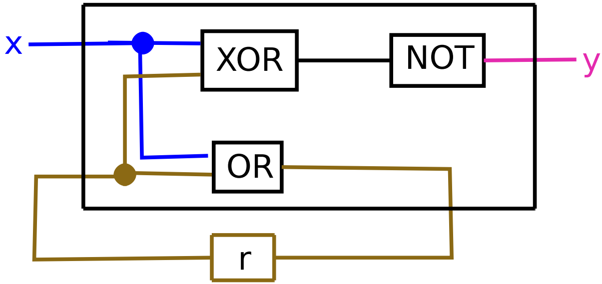
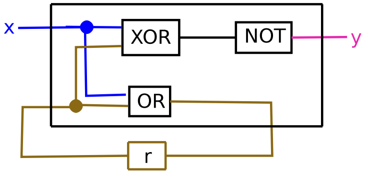


output function: $\lambda_y = \neg(x \oplus r)$

transition function: $\delta_r = x \vee r$

Example: TS for sequential circuit



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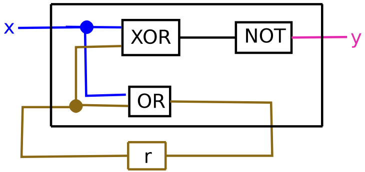

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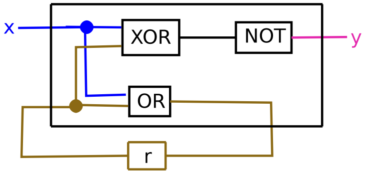

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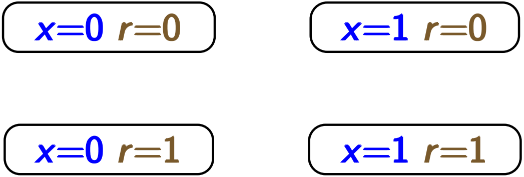

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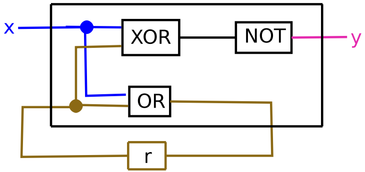

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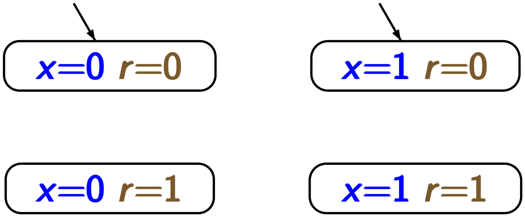

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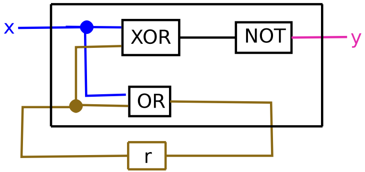

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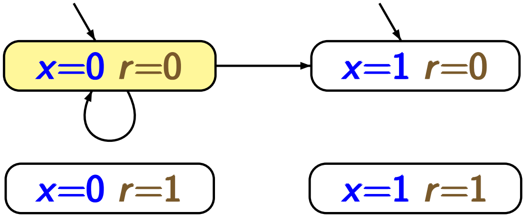

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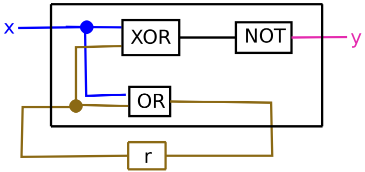

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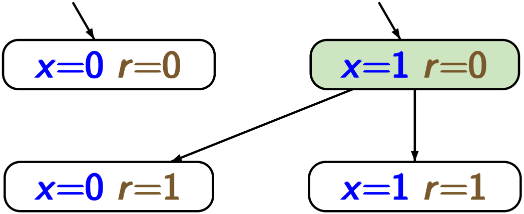

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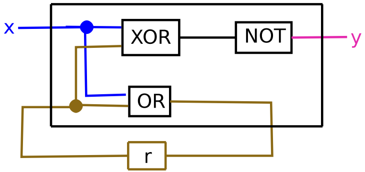

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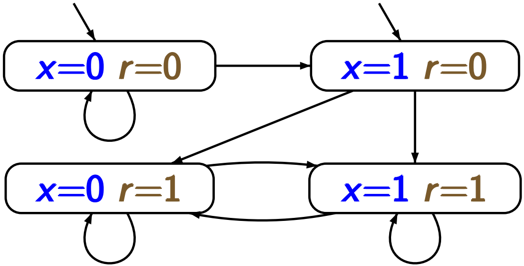

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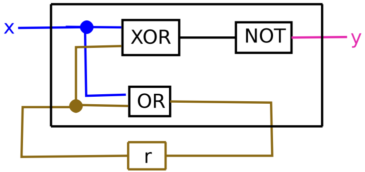

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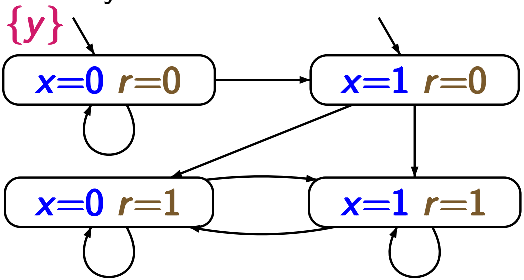

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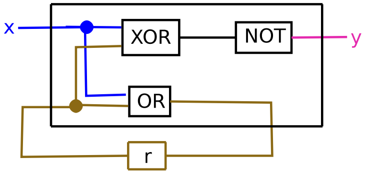

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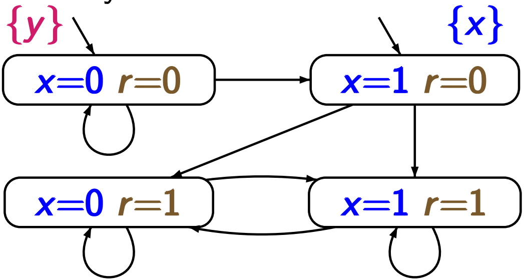

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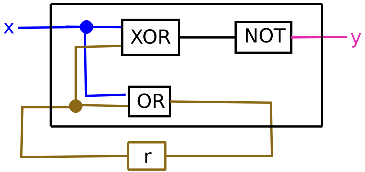

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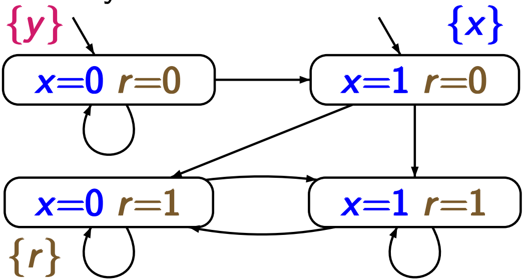

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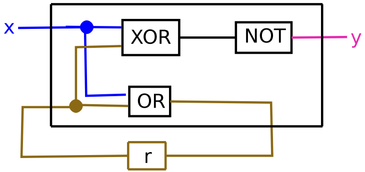

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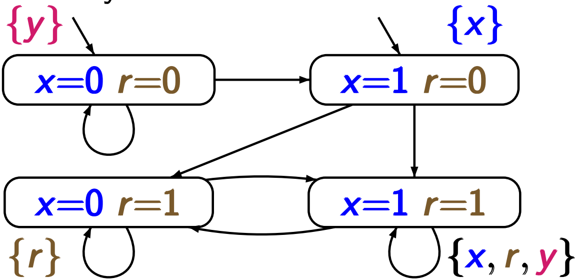

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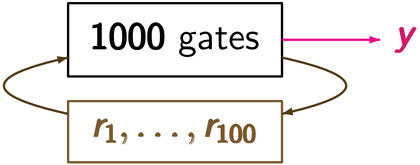

1 output bit
no input
100 registers

How many states . . .

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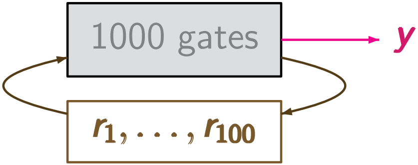

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answer: 2^{100}

How many states ...

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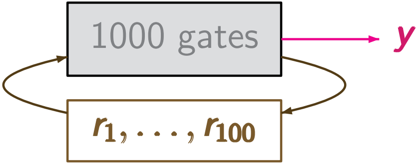

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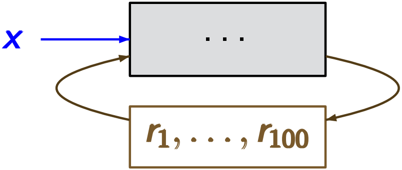

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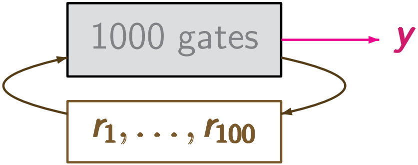

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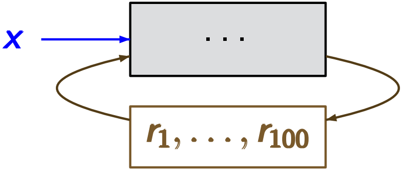

no output
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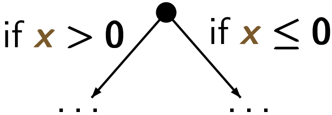
answer: $2^{100} * 2^1 = 2^{101}$

problem: TS-representation of conditional branchings **?**



Data-dependent systems

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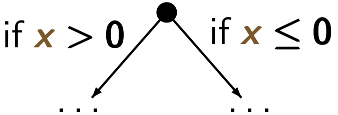

example: sequential program

```
WHILE  x > 0 DO
       x := x−1;
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OD
...
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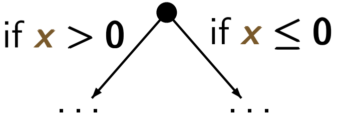

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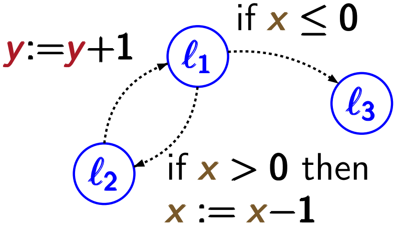

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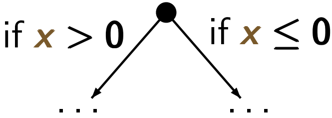

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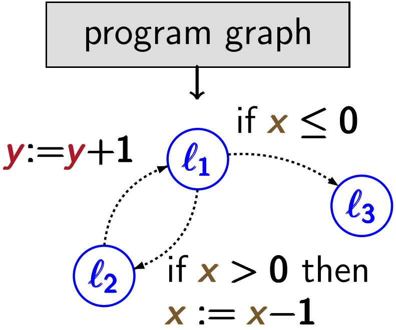

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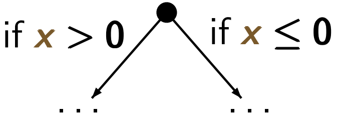

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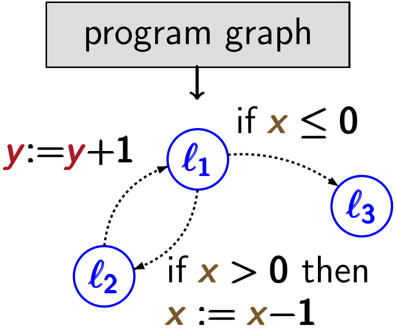

ℓ_1, ℓ_2, ℓ_3 are locations,
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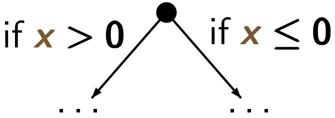

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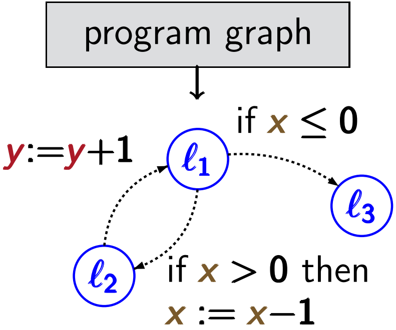

states of the transition system:

locations + relevant data (*here:* values for x and y)

initially: $x = 2$, $y = 0$

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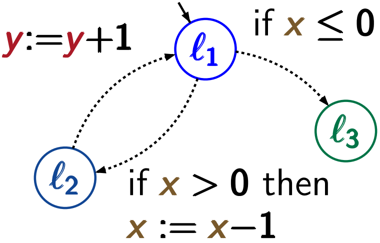
program graph



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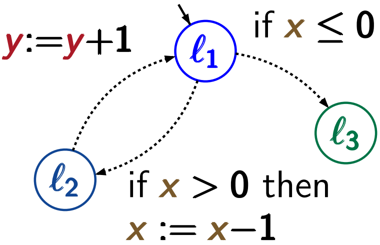

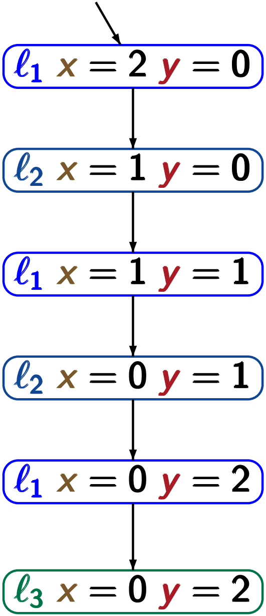

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$x := x - 1 \quad \longleftarrow$ $\boxed{\text{action } \alpha}$

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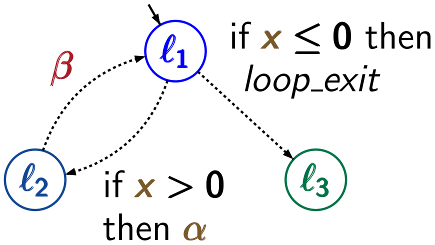

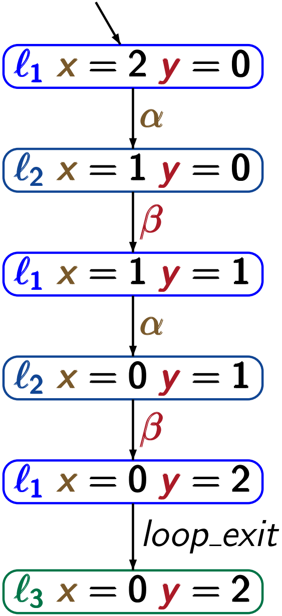

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Notation: $Eval(Var)$ = set of evaluations for Var

Conditions on typed variables

If *Var* is a set of typed variables then

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Example: $(\neg x \wedge y<z+3) \vee w=red$

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satisfaction relation $\models$ for evaluations and conditions

Example:

$$[x{=}0, y{=}3, z{=}6] \models \neg x \wedge y{<}z$$

$$[x{=}0, y{=}3, z{=}6] \not\models x \vee y{=}z$$

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if γ is "$(x, y) := (2x{+}y, 1{-}x)$" then:

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Let Var be a set of typed variables.

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Let ***Var*** be a set of typed variables.

A *program graph* over ***Var*** is a tuple

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function that formalizes the effect of the actions

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$\uparrow$

function that formalizes the effect of the actions

example: if α is the assignment $x{:=}x+y$ then

$$Effect(\alpha, [x{=}1, y{=}7]) = [x{=}8, y{=}7]$$

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ℓ, ℓ' are locations, $g \in \mathit{Cond}(\mathit{Var})$, $\alpha \in \mathit{Act}$

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TS-semantics of a program graph

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program graph $\mathcal{P}$ over Var

$$\Downarrow$$

transition system $\mathcal{T}_{\mathcal{P}}$

TS-semantics of a program graph

$$\boxed{\begin{array}{c}\text{program graph } \mathcal{P} \text{ over } \textit{Var} \\ \Downarrow \\ \text{transition system } \mathcal{T}_{\mathcal{P}}\end{array}}$$

states in $\mathcal{T}_{\mathcal{P}}$ have the form

$$\langle \ell, \eta \rangle$$

location variable evaluation

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Let $\mathcal{P} = (Loc, Act, Effect, \hookrightarrow, Loc_0, g_0)$ be a PG.
The transition system of $\mathcal{P}$ is:

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The transition relation $\longrightarrow$ is given by the following rule:

$$\frac{\ell \overset{g:\alpha}{\hookrightarrow} \ell' \ \wedge \ \eta \models g}{\langle \ell, \eta \rangle \xrightarrow{\alpha} \langle \ell', Effect(\alpha, \eta) \rangle}$$

Structured operational semantics (SOS)

The transition system of a program graph $\mathcal{P}$ is

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is a shortform notation in **SOS**-style.

$$\frac{premise}{conclusion}$$

Structured operational semantics (SOS)

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$\mathcal{T}_{\mathcal{P}} = (S, Act, \longrightarrow, S_0, AP, L)$ where

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is a shortform notation in **SOS**-style.

It means that $\longrightarrow$ is the smallest relation such that:

$$\boxed{\text{if } \ell \overset{g:\alpha}{\hookrightarrow} \ell' \ \wedge \ \eta \models g \text{ then } \langle \ell, \eta \rangle \overset{\alpha}{\longrightarrow} \langle \ell', Effect(\alpha, \eta) \rangle}$$

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Labeling of the states

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$$L(\langle \ell, \eta \rangle) \ = \ \{\ell\} \cup \big\{ g \in Cond(Var) : \eta \models g \big\}$$

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Guarded Command Language (GCL)

by Dijkstra

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- high-level modeling language that contains features of imperative languages and nondeterministic choice

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- high-level modeling language that contains features of imperative languages and nondeterministic choice
- semantics:

GCL-program

↓

program graph

↓

transition system

Guarded Command Language (GCL)

guarded command $g \Rightarrow$ ***stmt***

g	:	guard, i.e., Boolean condition on the program variables
stmt	:	statement

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DO :: $g \Rightarrow \mathit{stmt}$ OD

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$\textit{stmt}$	:	statement

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repetitive command/loop:

`DO` :: $g \Rightarrow \mathit{stmt}$ `OD` ⟵ `WHILE` g `DO` stmt `OD`

conditional command:

`IF` :: $g \Rightarrow \mathit{stmt}_1$

:: $\neg g \Rightarrow \mathit{stmt}_2$

`FI`

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conditional command:

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:: $\neg g \Rightarrow \textit{stmt}_2$
FI

$\longleftarrow$

IF g THEN $\textit{stmt}_1$
ELSE $\textit{stmt}_2$
FI

Guarded Command Language (GCL)

guarded command $g \Rightarrow stmt$ $\longleftarrow$ $\boxed{\text{enabled if } g \text{ is true}}$

repetitive command/loop:

$\mathtt{DO}\ :: g \Rightarrow stmt\ \mathtt{OD}$ $\longleftarrow$ $\boxed{\mathtt{WHILE}\ g\ \mathtt{DO}\ stmt\ \mathtt{OD}}$

conditional command:

$$\begin{array}{lll} \mathtt{IF} & :: & g \Rightarrow stmt_1 \\ & :: & \neg g \Rightarrow stmt_2 \\ \mathtt{FI} & & \end{array} \longleftarrow \boxed{\begin{array}{lll} \mathtt{IF}\ g & \mathtt{THEN} & stmt_1 \\ & \mathtt{ELSE} & stmt_2 \\ \mathtt{FI} & & \end{array}}$$

symbol :: stands for the nondeterministic choice
between enabled guarded commands

Guarded Command Language (GCL)

modeling language with nondeterministic choice

$$
\begin{array}{rcl}
\mathit{stmt} & \stackrel{\text{def}}{=} & x := \mathit{expr} \quad \big| \quad \mathit{stmt}_1 ; \mathit{stmt}_2 \quad \big| \\
& & \texttt{DO} :: g_1 \Rightarrow \mathit{stmt}_1 \quad \ldots \quad :: g_n \Rightarrow \mathit{stmt}_n \ \texttt{OD} \\
& & \texttt{IF} :: g_1 \Rightarrow \mathit{stmt}_1 \quad \ldots \quad :: g_n \Rightarrow \mathit{stmt}_n \ \texttt{FI} \\
& & \vdots
\end{array}
$$

where x is a typed variable and expr an expression of the same type

Guarded Command Language (GCL)

modeling language with nondeterministic choice

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\begin{array}{lll}
\mathit{stmt} & \stackrel{\text{def}}{=} & x := \mathit{expr} \quad \big| \quad \mathit{stmt}_1; \mathit{stmt}_2 \quad \big| \\
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 & & \vdots
\end{array}
$$

where x is a typed variable and *expr* an expression of the same type

semantics of a **GCL**-program: program graph

GCL-program for beverage machine

GCL-program for beverage machine

uses two variables $\#\mathit{sprite}, \#\mathit{coke} \in \{0, 1, \ldots, \mathit{max}\}$ for the number of available drinks (sprite or coke)

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uses the following actions:

	enabled	effect
get_coke	if $\#coke > 0$	$\#coke := \#coke - 1$
get_sprite	if $\#sprite > 0$	$\#sprite := \#sprite - 1$

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refill	any time	$\#sprite := max$ $\#coke := max$

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insert_coin	any time	no effect on variables

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uses the following actions:

	enabled	effect
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get_sprite	if $\#sprite > 0$	$\#sprite := \#sprite - 1$
refill	any time	$\#sprite := max$ $\#coke := max$
insert_coin	any time	no effect on variables
return_coin	if machine is empty and user has entered a coin (no effect on variables)	

```
DO :: true ⇒ insert_coin;
      IF :: #sprite = #coke = 0 ⇒ return_coin

         :: #coke > 0 ⇒ #coke := #coke − 1

         :: #sprite > 0 ⇒ #sprite := #sprite − 1

      FI
   :: true ⇒ #sprite := max; #coke := max
OD
```

GCL-program for beverage machine

DO :: true $\Rightarrow$ insert_coin; (* user inserts a coin *)

IF :: $\#sprite = \#coke = 0 \Rightarrow$ return_coin

:: $\#coke > 0 \Rightarrow \#coke := \#coke - 1$

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FI

:: true $\Rightarrow \#sprite := max; \#coke := max$

OD

GCL-program for beverage machine

$\mathtt{DO} \;::\; \text{true} \Rightarrow \text{insert\_coin}$; (* user inserts a coin *)

$\mathtt{IF} \;::\; \#sprite = \#coke = 0 \Rightarrow \text{return\_coin}$

(* no beverage available *)

$:: \;\#coke > 0 \Rightarrow \#coke := \#coke - 1$

$:: \;\#sprite > 0 \Rightarrow \#sprite := \#sprite - 1$

$\mathtt{FI}$

$:: \;\text{true} \Rightarrow \#sprite := max; \#coke := max$

$\mathtt{OD}$

GCL-program for beverage machine


```
DO :: true ⇒ insert_coin; (* user inserts a coin *)
      IF :: #sprite = #coke = 0 ⇒ return_coin
                                    (* no beverage available *)
         :: #coke > 0 ⇒ #coke := #coke − 1
                                    (* user selects coke *)
         :: #sprite > 0 ⇒ #sprite := #sprite−1
                                    (* user selects sprite *)
      FI
   :: true ⇒ #sprite := max; #coke := max
                                    (* refilling of the machine *)
OD
```

GCL-program for beverage machine


```
DO :: true ⇒ insert_coin; (* user inserts a coin *)
      IF :: #sprite = #coke = 0 ⇒ return_coin
                              (* no beverage available *)
         :: #coke > 0 ⇒ get_coke
                              (* user selects coke *)
         :: #sprite > 0 ⇒ get_sprite
                              (* user selects sprite *)
      FI
   :: true ⇒ refill
                              (* refilling of the machine *)
OD
```

GCL-program for beverage machine

$$
\begin{array}{lll}
\texttt{DO} & :: & \text{true} \Rightarrow \text{insert\_coin}; \\
& & \texttt{IF} \ :: \ \#\textit{sprite} = \#\textit{coke} = 0 \Rightarrow \text{return\_coin} \\
& & \phantom{\texttt{IF}} \ :: \ \#\textit{coke} > 0 \Rightarrow \text{get\_coke} \\
& & \phantom{\texttt{IF}} \ :: \ \#\textit{sprite} > 0 \Rightarrow \text{get\_sprite} \\
& & \texttt{FI} \\
& :: & \text{true} \Rightarrow \text{refill} \\
\texttt{OD} & &
\end{array}
$$

```
DO  ::  true ⇒ insert_coin;
        IF  ::  #sprite = #coke = 0
                              ⇒ return_coin
            ::  #coke   > 0 ⇒ get_coke
            ::  #sprite > 0 ⇒ get_sprite
        FI
    ::  true ⇒ refill
OD
```

... yields a program graph with

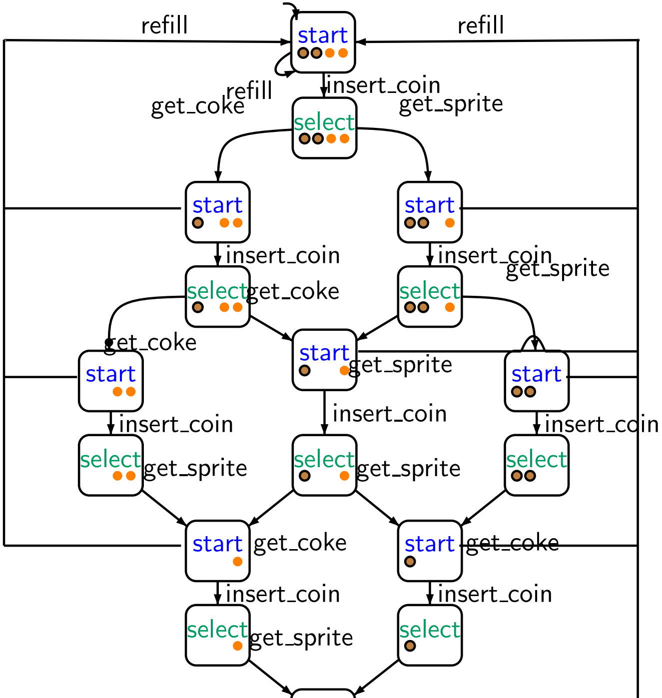
- two variables $\#sprite, \#coke \in \{0, 1, \ldots, max\}$

GCL-program for beverage machine

$$
\begin{array}{lll}
\textit{start} \longrightarrow \texttt{DO} & :: & \text{true} \Rightarrow \text{insert\_coin}; \\
\textit{select} \longrightarrow & & \texttt{IF} \; :: \; \#\textit{sprite} = \#\textit{coke} = 0 \\
& & \qquad\qquad \Rightarrow \text{return\_coin} \\
& & \qquad :: \; \#\textit{coke} \; > 0 \Rightarrow \text{get\_coke} \\
& & \qquad :: \; \#\textit{sprite} > 0 \Rightarrow \text{get\_sprite} \\
& & \texttt{FI} \\
& :: & \text{true} \Rightarrow \text{refill} \\
\quad \texttt{OD} & &
\end{array}
$$

... yields a program graph with

- two variables $\#\textit{sprite}, \#\textit{coke} \in \{0, 1, \ldots, \textit{max}\}$
- two locations ***start*** and ***select***


refill
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get_sprite
select

Summary

- We defined transition system
- We showed how to transform a sequential circuit into a transition system.
- We showed how to transform a program in Dijkstra's guarded command language into a program graph.
- We showed how to convert a program graph into a transition system.