Division

- Used to answer queries such as:
  - Find sailors who have reserved all boats.

Let $A$ have 2 fields, $x$ and $y$; $B$ have only field $y$:

$$A/B = \{x\} \mid \exists (x, y) \in A \land (y) \in B$$

$A/B$ contains all $x$ tuples (sailors) such that for every $y$ tuple (boat) in $B$, there is an $xy$ tuple in $A$.

Or, if the set of $y$ values (boats) associated with an $x$ value (sailor) in $A$ contains all $y$ values in $B$, the $x$ value is in $A/B$.

In general, $x$ and $y$ can be any sets of fields (not singletons).

Examples of Division $A/B$

### Query 1

"Find the names of sailors who've reserved all boats"

$$\rho(\text{Temp} \langle \pi_{\text{sid}, \text{bid}} \text{Reserves} \rangle / (\pi_{\text{bid}} \text{Boats}))$$

$$\pi_{\text{name}}(\text{Temp} \bowtie \text{Sailors})$$

### Query 2

"Find sailors who've reserved all red boats"

$$\rho(\text{Temp} \langle \pi_{\text{sid}, \text{bid}} \text{Reserves} \rangle / (\pi_{\text{bid}} (\sigma_{\text{color}=\text{red}} \text{Boats})))$$

$$\pi_{\text{name}}(\text{Temp} \bowtie \text{Sailors})$$

Expressing $A/B$ Using Basic Operators

- For $A/B$, compute all $x$ values that are not disqualified by some $y$ value in $B$.
- $x$ value is disqualified if by attaching $y$ value from $B$, we obtain an $xy$ tuple that is not in $A$.

Disqualified $x$ values:

$$\pi_{\chi(A\times B)-A}$$

$A/B$:

$$\pi_{\chi(A)} - \text{all disqualified tuples}$$

$$\pi_{\chi(A)} - \pi_{\chi((\pi_{\chi(A\times B)})-A)}$$
Division in SQL

- Not supported as primitive operator
- Need to use nested queries to express division
  - One of the most subtle queries in SQL
  - Need to pay close attention to writing SQL division queries!
- There are two ways of writing division queries
  - Using the set \texttt{EXCEPT} operator (2-level nesting)
  - Without the \texttt{EXCEPT} operator (3-level nesting)

Division: Solution 1

“Find sailors who’ve reserved all boats.”

\texttt{With EXCEPT:}
\begin{verbatim}
SELECT S.sname FROM Sailors S WHERE NOT EXISTS (SELECT * FROM Boats B WHERE NOT EXISTS (SELECT R.bid FROM Reserves R WHERE R.sid=S.sid))
\end{verbatim}

“Find sailors who’ve reserved all \textbf{red} boats.”

\texttt{With EXCEPT:}
\begin{verbatim}
\end{verbatim}

Another Example

```
Movies (movie_id, title, year, studio)
Actors (actor_id, name, nationality)
StarsIn (actor_id, movie_id, character)
```

“Find names of actors who star in ALL movies produced by Universal in year 1990.”
\begin{verbatim}
SELECT A.name FROM Actors A WHERE NOT EXISTS( SELECT M.movie_id FROM Movies M WHERE M.year=1990 AND M.studio='Universal' EXCEPT SELECT S.movie_id FROM StarsIn S WHERE S.actor_id=A.actor_id )
\end{verbatim}