Q1.

a.

\[ \pi_{\text{grade}}(\text{Grades} \bowtie (\sigma_{\text{age}=20}\text{Students}) \bowtie (\sigma_{\text{credits}=4}\text{Courses})) \]

b.

\[ \pi_{\text{name}}((\pi_{\text{sid}}(\text{Grades} \bowtie (\sigma_{\text{cname}=\text{calculus}}\text{Courses})) - \pi_{\text{sid}}(\sigma_{\text{grade}=\text{C}}\text{Grades})) \bowtie \text{Students}) \]

c.

\[ \pi_{\text{name}}\left((\pi_{\text{cid}}(\sigma_{\text{credits}=3}\text{Courses})) \bowtie (\sigma_{\text{grade}=\text{A}}\text{Grades}) \bowtie \text{Students}\right) \]

\[ \cup \pi_{\text{name}}\left((\pi_{\text{sid}}(\sigma_{\text{grade}=\text{B}}\text{Grades})) \bowtie \text{Students}\right) \]

d.

\[ \rho\left(\text{TMP1},((\pi_{\text{sid}}((\sigma_{\text{cname}=\text{calculus}}\text{Courses}) \bowtie \text{Grades})) \bowtie \text{Students})\right) \]

\[ \rho(\text{TMP2},\text{TMP1}) \]

\[ \rho(\text{TMP3},\pi_{\text{sid}}\text{TMP1} - \pi_{\text{TMP1,sid}}(\text{TMP1} \bowtie \text{TMP1,age< TMP2,age} \text{TMP2})) \]

\[ \pi_{\text{age}}(\text{TMP3} \bowtie \text{Students}) \]
Q2.

a.

```sql
SELECT DISTINCT S.age
FROM Students S, Courses C, Grades G
WHERE S.sid = G.sid AND G.cid = C.cid AND C.cname = 'CS310'
```

b.

```sql
SELECT S.sname
FROM Students S, Grades G
WHERE S.sid = G.sid AND S.sid NOT IN
  (SELECT G1.sid
   FROM Grades G1, Courses C
   WHERE G1.cid = C.cid AND C.credits <> 4)
```

c.

```sql
SELECT G.cid, AVG(G.grade)
FROM Grades G
GROUP BY G.cid
HAVING 10 <= (SELECT COUNT(*)
               FROM Students S, Grades G1
               WHERE S.sid = G1.sid AND S.age >= 25 AND G1.cid = G.cid)
```

d.

```sql
SELECT S.sname
FROM Students S
WHERE NOT EXISTS (SELECT C.cid
                  FROM Courses C
                  WHERE C.credits = 4
                  MINUS
                  SELECT G.cid
                  FROM Grades G
                  WHERE G.sid = S.sid)
```

e.

```sql
SELECT G.sid, G.cid
FROM Grades G
WHERE G.grade =
  (SELECT MAX(G1.grade) FROM Grades G1 where G1.cid = G.cid);
```
Q3.

```sql
CREATE VIEW TopStudents (ID, Name, GPA) AS
    SELECT S.sid, S.sname, AVG(G.grade)
    FROM Student S, Grades G
    WHERE S.sid = G.sid
    GROUP BY S.sid, S.sname
    HAVING AVG(G.grade) > 3.0;
```

Q4.

a)

b)

c)
Q5.

After building the table with attribute closure for each attribute combination, we get as key $AB$.

$F^+ = \{AB \rightarrow C, AB \rightarrow D, BC \rightarrow D\}$

First two FDs are not violations, because they contain $AB$ on the LHS. Last one is a violation of BCNF as $BC$ does not include a key, and also a violation of 3NF as $D$ is not included in any key (only key is $AB$). So the relation is neither BCNF, nor 3NF.

**Decomposition in BCNF**

We decompose according to violating FD $BC \rightarrow D$. 
BCNF decomposition is $ABC, BCD$. It can be shown immediately that it is also dependency-preserving through transitivity.
After Step 4:

After Step 5:

C can still exercise P, E not.